

Phonon Hall Viscosity and the Intrinsic Thermal Hall Effect of α -RuCl₃

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Abstract

The thermal Hall effect has been observed in a wide variety of magnetic insulators, yet its origin remains controversial. While some studies attribute it to intrinsic origins—such as heat carriers with Berry curvature—others propose extrinsic origins—such as heat carriers scattering off crystal defects. Even the nature of the heat carriers is unknown: magnons, phonons, and fractionalized spin excitations have all been proposed. These questions are significant for the study of quantum spin liquids and are particularly relevant for α -RuCl₃, where a quantized thermal Hall effect has been attributed to Majorana edge modes. Here, we use ultrasonic measurements of the acoustic Faraday effect to demonstrate that the phonons in α -RuCl₃ have Hall viscosity—a non-dissipative viscosity that rotates phonon polarizations and deflects phonon heat currents. We show that phonon Hall viscosity produces an intrinsic thermal Hall effect that quantitatively accounts for a significant fraction of the measured thermal Hall effect in α -RuCl₃. More broadly,

we demonstrate that the acoustic Faraday effect is a powerful tool for detecting phonon Hall viscosity and the associated phonon Berry curvature, offering a new way to uncover and study exotic states of matter that elude conventional experiments.

Introduction

Thermal transport has emerged as the dominant technique in the search for exotic, charge-neutral quasiparticles^{1,2}. The observation of a thermal Hall effect—where a temperature gradient builds up perpendicular to both a magnetic field and a heat current—has been used as evidence for fractionalized spin excitations in quantum magnets such as $\text{Tb}_2\text{Ti}_2\text{O}_7$ ³, $\alpha\text{-RuCl}_3$ ^{4,5}, and several Kagome antiferromagnets^{6–8}. Central to these claims is the assumption that phonons do not themselves generate a thermal Hall effect. This is a reasonable assumption, as phonons carry neither electric charge nor spin and thus should not interact directly with magnetic fields.

This assumption has recently been called into question due to the discovery of relatively large thermal Hall effects in conventional antiferromagnets that are unlikely to host spinons^{9–11}, as well as in non-magnetic insulators where phonons are the only heat carrier^{12–14}. This suggests that phonons may be responsible for the thermal Hall effect in at least a subset of these materials.

How can phonons generate a thermal Hall effect? Intrinsic mechanisms invoke phonon Berry curvature^{15–18}, whereas extrinsic mechanisms invoke phonon skew scattering^{19–21}. While a large number of theoretical proposals have been put forward^{15–24}, the only experimental probe thus far has been thermal transport—applying heat currents and measuring the resultant temperature gradients. A key limitation of thermal transport is its inability to distinguish between intrinsic and extrinsic mechanisms, and this has made it difficult to distinguish between proposed origins at even a coarse level.

Here, we take advantage of the fact that phonon Berry curvature not only causes an intrinsic thermal Hall effect, but also generates a dissipationless viscous force known as

phonon Hall viscosity^{25,26}. Phonon Hall viscosity rotates the polarization of transverse-polarized phonons as they propagate (see Fig. 1). This is called the acoustic Faraday effect and is analogous to how the optical Faraday effect rotates the polarization of light. Where thermal transport measurements use phase-incoherent heat currents, acoustic Faraday measurements use phase-coherent phonons that are only sensitive to intrinsic effects and not to phase-incoherent scattering.

We measure the acoustic Faraday effect in α -RuCl₃ and extract the phonon Hall viscosity. We find that the Hall viscosity is peaked near the magnetic phase boundary where the thermal Hall effect is maximized. The existence of phonon Hall viscosity in α -RuCl₃ suggests a non-zero phonon contribution to the thermal Hall effect, challenging interpretations of the thermal Hall effect solely in terms of Majorana fermions^{4,27,28} or other fractionalized spin excitations⁵. We suggest that phonons acquire Hall viscosity in α -RuCl₃ by coupling to spin excitations. This general mechanism is likely responsible for the unusual thermal Hall effects reported in many other magnetic insulators.

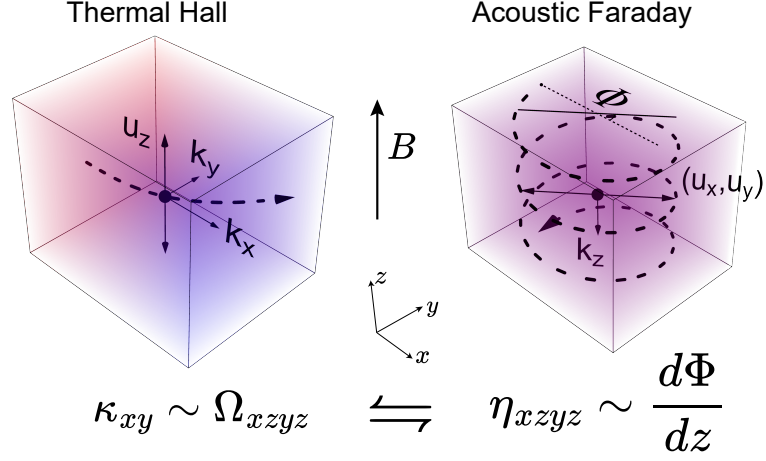


Fig. 1 | The thermal Hall and acoustic Faraday effects, and how they are related. Applying a magnetic field B parallel to z breaks time reversal symmetry and allows for both phonon Berry curvature, Ω_{xzyz} , and phonon Hall viscosity, η_{xzyz} . Phonon Hall viscosity can be thought of as a long-wavelength manifestation of the microscopic Berry curvature—the double-headed arrow indicates that these two quantities are equivalent^{25,26}. On application of a temperature gradient along x (indicated by the red-blue color gradient), phonon Berry curvature deflects phonons carrying heat along x and generates a heat current along y (the dashed line indicates the trajectory of a phonon in the presence of Berry curvature). Similarly, the polarization of acoustic phonons traveling along z rotates at a rate that is proportional to the Hall viscosity (Φ indicates the angle of the polarization away from the x axis, and the dashed line indicates the path of the polarization vector as the phonon propagates along z).

Results

Measuring the acoustic Faraday effect

We first describe how phonon Hall viscosity alters sound propagation in solids, giving rise to the acoustic Faraday effect. In general, three dimensional solids have three distinct speeds of sound for waves propagating along any given direction. In crystals with high enough symmetry, and for high-symmetry propagation directions, the two transverse modes—modes with their polarization vector of atomic displacement perpendicular to their propagation vector—become degenerate, with the same speed of sound. This is the case for sound propagating

along the c axis of α -RuCl₃. The degeneracy between the two modes is lifted by Hall viscosity, which can arise, for example, through spin-lattice coupling. As a result, the left- and right-circularly polarized combinations of the transverse modes then propagate at different speeds, giving rise to an acoustic Faraday effect (see Fig. 1). This phenomenon is well documented in materials like yttrium iron garnet (YIG) and Cr₂O₃, where it arises from hybridization between acoustic phonons and magnons^{29,31,34}; in Tb₃Ga₅O₁₂, where it arises from coupling between acoustic and optical phonons³² and which also has a phonon thermal Hall effect³³; in CeAl₂ and Ni-doped MgO, where it arises from crystal field splitting^{30,35}; and in superfluid ³He-B, where it emerges due to the shear stiffness of the Fermi liquid³⁶.

The acoustic Faraday effect is measured by generating linearly polarized, transverse sound waves using shear-polarized piezoelectric transducers attached to a sample (see Fig. 2a). A radio-frequency voltage pulse actuates the piezoelectric transducer, generating a sound wave that travels through the sample, and is detected by a second shear-polarized piezoelectric transducer. The amplitude of sound detected by the receive transducer depends on the strength of Faraday rotation, which rotates the polarization of the incoming wave: the amplitude is maximum when the sound wave's polarization aligns with the traducer's polarization, and minimum when they are perpendicular. In general, varying the external magnetic field changes the Hall viscosity, which modifies the amount of Faraday rotation and, consequently, the amplitude of sound detected by the receive transducer.

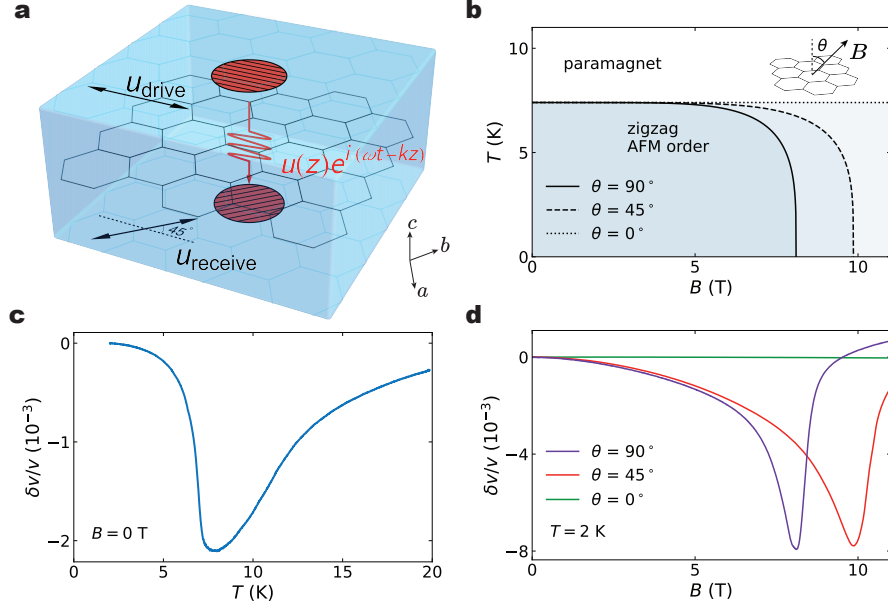


Fig. 2 | Experimental geometry and sample characterization. **a**, A piezoelectric drive transducer (upper red circle) sends a pulse of traverse sound (red line) along the c axis of a single-crystal α - RuCl_3 sample (indicated schematically by the honeycomb). The pulse is detected by a second, receive transducer (lower red circle) polarized 45° relative to the drive transducer. **b**, Schematic temperature-field phase diagram of α - RuCl_3 for different field orientations. The inset shows that θ is defined as the angle between the magnetic field B and the c axis (i.e. rotation is in the bc plane). We show the phase boundary for field along the b axis ($\theta = 90^\circ$, sold line), field along the c axis ($\theta = 0^\circ$, short dashed line), and field 45° between the b and c axes (long dashed line). **c**, The relative change in sound velocity as a function of temperature across the ordering temperature of $T_N = 7.5$ K. The sample exhibits a single magnetic phase transition, with no features near 14 K that would indicate a secondary structural and magnetic phase^{37,38}. **d**, The relative change in sound velocity as a function of magnetic field across the critical field of $B_c = 8$ T for $B \parallel b$, $B_c = 10$ T for $\theta = 45^\circ$, and for $B \parallel c$ the sample remains in the ordered phase up to 12 tesla. The single phase transition indicates that the field is well-aligned with the b axis³⁹.

One complication is that the amplitude of sound detected by the receive transducer depends not only on the amount of Faraday rotation, but also on sound attenuation. The attenuation can vary significantly with magnetic field, especially near a magnetic phase transition. This is particularly relevant for α - RuCl_3 , where a spin-liquid state has been proposed near the critical field where long-range magnetic order is suppressed (≈ 8 tesla for in-plane magnetic

fields, see Fig. 2b). Importantly for our experiment, sound attenuation is an *even* function of magnetic field—it depends on only the magnitude of the magnetic field, and not on the sign. In contrast, the acoustic Faraday effect is an *odd* function of magnetic field, changing sign with field reversal. To isolate the Faraday contribution, we measure the received sound amplitude for both positive and negative magnetic fields and then antisymmetrize the data. To address the limitation that a traditional pulse-echo reflection experiment cannot distinguish right-handed from left-handed rotations, we rotated the polarization of the receive transducer 45° with respect to that of the drive transducer (see Fig. 2a). This configuration also enables an important experimental check: the field-antisymmetrized signal should switch sign when the sound propagation direction is reversed by swapping the drive and receive transducers.

Single-crystal samples of α -RuCl₃ were grown using the methods described in Kim *et al.*⁴⁰. We select large single crystals with roughly parallel faces perpendicular to the c axis. We remove material by cleaving the sample until we produce two pristine, parallel faces, with overlapping area and no visible stacking faults. On two opposite c -axis faces, we sputter platinum bottom electrodes with titanium adhesion layers, followed by 1 μm thick, ZnO, mixed-mode (shear and longitudinal) piezoelectric transducers, followed by platinum top electrodes on titanium adhesion layers. We excite the drive transducer using 50 ns bursts of ≈ 1.5 GHz radiofrequency voltage, amplify the signal detected at the receive transducer, and digitize it on an oscilloscope. We extract the amplitude and phase of the received signal using digital lockin. Further experimental details can be found in the Methods. The high quality of our samples is demonstrated by their single phase transition above 7 kelvin in zero magnetic field (Fig. 2c), with no additional phase transitions at higher temperatures.

The acoustic Faraday effect of α -RuCl₃ for $B \parallel c$

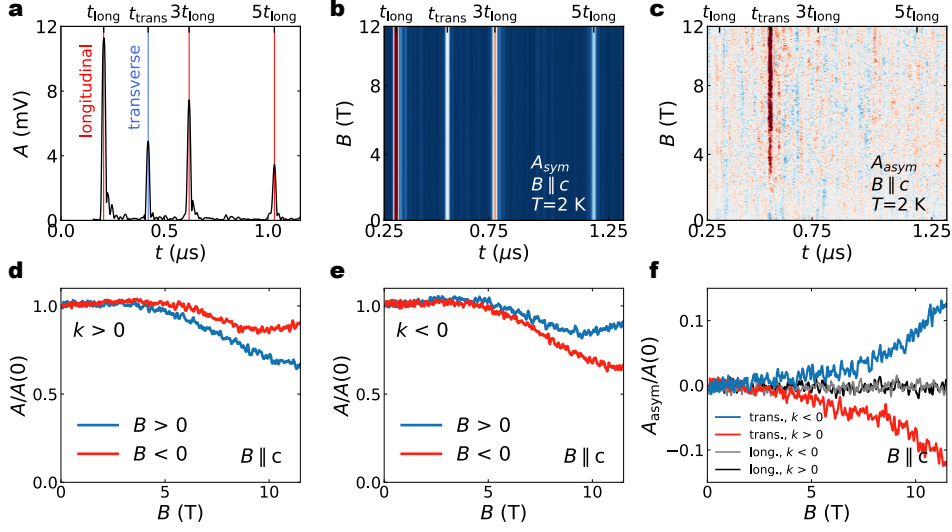


Fig. 3 | Isolating the acoustic Faraday effect. **a**, The raw signal detected using the setup shown in Fig. 2a, for $B \parallel c = 10$ T and $T = 2$ K. Longitudinal and transverse pulses are identified based on their known speeds of sound^{41,46}. The first longitudinal pulse arrives at t_{long} , with echoes of this signal arriving at $3t_{\text{long}}$ and $5t_{\text{long}}$. The transverse pulse arrives at t_{trans} and is clearly separated in the time domain from the longitudinal signal. **b**, The field-symmetrized data show all longitudinal and transverse signals from 0 to 12 tesla. **c**, The field-antisymmetrized data contains only transverse signal because the Faraday effect rotates the transverse sound polarization in opposite directions for $\pm B$, whereas it cannot rotate longitudinal polarization. **d**, The amplitude of transverse sound, normalized to its zero-field value, as a function of magnetic field for $B \parallel c$ at $T = 2$ K. The magnetic field suppresses the amplitude more for positive field than for negative field. **e**, This behaviour switches when the propagation direction is switched, as expected for a Faraday effect. **f**, The field-antisymmetrized amplitude for both propagation directions. The transverse signal shows the characteristics of a Faraday effect, going to zero at $B = 0$, and switching sign when $k \rightarrow -k$. The longitudinal signal, in contrast, is zero at all fields for both propagation directions.

We first show data taken at 2 kelvin as a function of magnetic field from zero to 12 tesla applied along the c axis. In this configuration, the critical field to suppress magnetic order is over 30 tesla^{42,43}, and the entire measurement is deep within the ordered antiferromagnetic state. This allows us to first demonstrate the acoustic Faraday effect in α -RuCl₃ without the

additional complication of the magnetic phase transition (this configuration also has a thermal Hall effect, see Le Francois *et al.*⁴⁴). Fig. 3a shows the amplitude of transmitted sound as a function of time at a fixed field of $B = 10$ T along the c -axis and in the magnetically ordered state at $T = 2$ K. The mixed-mode transducers generate and detect both longitudinal and transverse sound waves, which travel at different speeds. The longitudinal and transverse modes are visible as distinct pulses in the time-series data, and we use both to validate our experimental method. Fig. 3b (c) shows data that have been symmetrized (antisymmetrized) in magnetic field. Antisymmetrization completely removes the longitudinal signal, whereas transverse signal remains. This is consistent with the transverse signal arising due to the acoustic Faraday effect.

Fig. 3d shows the amplitude of the transverse signal (at $t = t_{\text{trans.}}$) as a function of magnetic field for both positive and negative field directions ($\pm B$), and panel e shows the same but for the opposite sound propagation direction. The asymmetry between $+B$ and $-B$ switches sign when the sound propagation direction k is switched to $-k$ (by swapping the drive and receive cables at the top of the cryostat). The change in sign of the antisymmetric signal when changing k to $-k$, the fact that the antisymmetric signal is non-zero only for transverse sound, and the vanishing of the antisymmetric signal at zero magnetic field, all confirm that the antisymmetric signal is caused by the acoustic Faraday effect.

Having demonstrated that our procedure isolates the acoustic Faraday effect, we perform similar measurements in other experimental configurations: 1) we measure at temperatures up to 40 kelvin to test whether the Faraday effect is confined to the ordered state below 7 kelvin; 2) we measure with the magnetic field rotated 55° away from the c axis towards the b axis to determine how crossing the magnetic phase boundary affects the Faraday rotation; 3) we measure with magnetic field purely in the honeycomb plane to ensure that the Faraday signal disappears in this configuration; 4) we measure two additional samples of different thicknesses to ensure reproducibility; 5) we measure at additional ultrasonic frequencies. Data

sets from (1) and (2) are analyzed below to extract the Hall viscosity of α -RuCl₃. Data sets from (3), (4), and (5) are shown in the Extended Data.

Extracting the Hall viscosity of α -RuCl₃

We now extract the phonon Hall viscosity by modeling our acoustic Faraday data using the elastic wave equation with viscous contributions included. The relationship between stress, σ , and strain, ε , in a solid—Hooke’s law—is modified at finite frequency by introducing the viscosity tensor $\hat{\eta}$:

$$\sigma_{ij} = c_{ijkl}\varepsilon_{kl} + \eta_{ijkl}\dot{\varepsilon}_{kl}, \quad (1)$$

where $\hat{c}$ is the elastic tensor and $\dot{\varepsilon}$ is the time derivative of strain. While the most familiar viscosity components are dissipative, time-reversal symmetry breaking—either intrinsic or from an applied magnetic field—allows for additional, *non*-dissipative “Hall” viscosities. A magnetic field component along the c axis of α -RuCl₃ activates the viscosity component η_{xzyz} .

When incorporated into the elastic wave equation, this viscosity couples transverse waves polarized along y to transverse waves polarized along x for waves propagating along the z direction. This coupling rotates linearly polarized transverse waves and produces an acoustic Faraday effect. Note that η_{xzyz} is allowed as long as there is *any* component of field along c . Other Hall viscosities are allowed when other field components are present—see Methods for a full symmetry analysis of the allowed Hall viscosities. The wave equation with Hall viscosity included is

$$\rho\omega^2\vec{u} = k^2 \begin{bmatrix} c_{xxzz} & i\omega\eta_{xzyz} \\ -i\omega\eta_{xzyz} & c_{yyzz} \end{bmatrix} \vec{u}, \quad (2)$$

where ρ is the material density, ω is the sound frequency, $\vec{u}$ is the sound polarization vector, k is the sound wavenumber, and $c_{xxzz} \equiv c_{55}$ and $c_{yyzz} \equiv c_{44}$ are elastic constants.

Fig. 4a and c show the antisymmetrized Faraday rotation data taken with $B||c$ and B rotated 55° toward b , respectively. Both data sets are taken at 2 kelvin: the $B||c$ data is entirely in the magnetically ordered state, whereas the $\theta = 55^\circ$ data crosses the antiferromagnetic-to-paramagnetic transition at $B_c = 8.5$ T.

For $B||c$, $c_{44} = c_{55}$ and we measure this elastic constant independently using the time-of-flight between echoes (see Fig. 3a). The Hall viscosity η_{xzyz} is the only unknown in Equation 2 and is therefore directly determined by the antisymmetric amplitude (recall that field-symmetric changes in amplitude are removed through the antisymmetrization procedure).

When the magnetic field is rotated away from the c axis, the symmetry of the lattice is broken and c_{44} is no longer equal to c_{55} —an effect known as acoustic *birefringence*. Even though acoustic birefringence is even in magnetic field, it cannot be removed entirely through antisymmetrization in the presence of Hall viscosity. This likely produces the sharp spike in the antisymmetric signal just above B_c in Fig. 4c, where the acoustic birefringence is largest. Because the effects of birefringence are difficult to disentangle from the viscous contribution near B_c , we exclude this narrow field region from the analysis. Note that birefringence alone cannot produce an antisymmetric signal; field-antisymmetric signals require Hall viscosity.

Fig. 4b and d show the phonon Hall viscosity extracted from the antisymmetric data using Equation 2. For $B||c$, the Hall viscosity increases continuously as a function of magnetic field up to $\eta_{xzyz} = 0.03$ mPa·s at 12 T. For $\theta = 55^\circ$, the Hall viscosity peaks just above the critical field of 8.5 T, reaching a value of approximately 0.013 mPa·s.

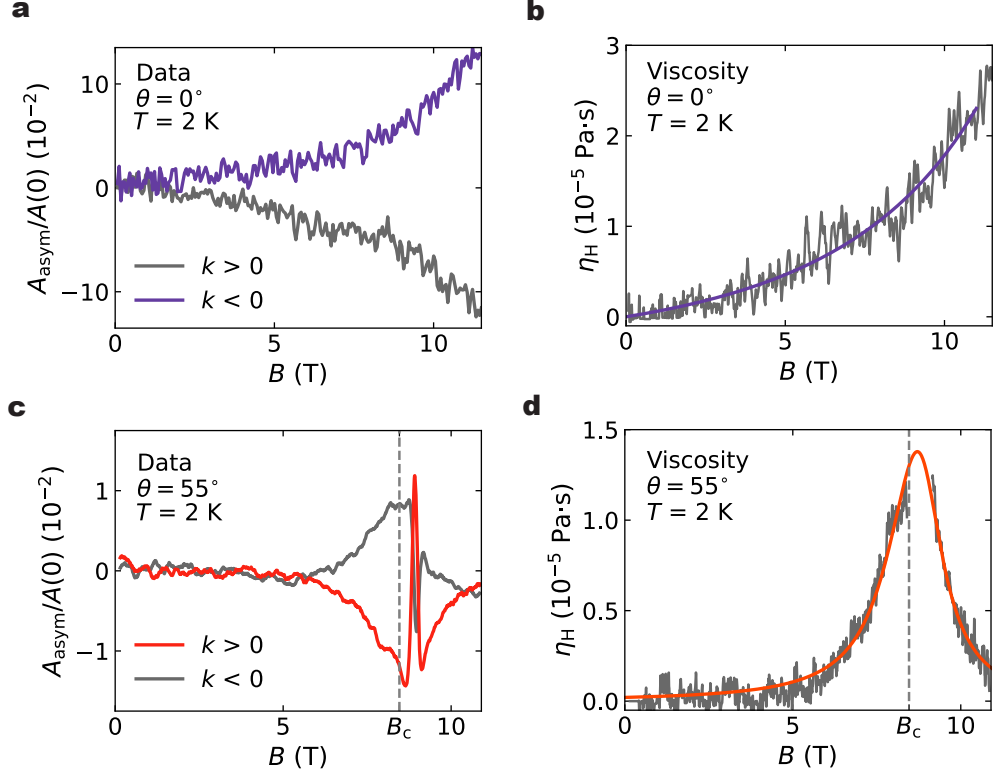


Fig. 4 | The phonon Hall viscosity of α -RuCl₃. **a**, The field-antisymmetrized Faraday signal normalized to the total zero-field signal, as a function of magnetic field for $B \parallel c$, at $T = 2$ K, for $\pm k$. **b**, The Hall viscosity extracted from the antisymmetric signal using Equation 2. The solid line is a guide to the eye. Panels **c** and **d** show the same data and analysis, but for magnetic field rotated 55° toward the b axis. The sharp feature slightly above B_c in panel **c** is likely a result of the small absolute signal size and the rapid change in speed of sound near this field. We have truncated this feature from the viscosity in panel **d**.

Comparison to the thermal Hall effect

How do our measurements of phonon Hall viscosity relate to the thermal Hall effect in α -RuCl₃? The Hall viscosity can be interpreted in two ways. In an acoustic Faraday experiment with magnetic field along z , η_{zxzy} mixes a sound wave propagating along z and polarized along x with a sound wave propagating along z and polarized along y . In a thermal transport experiment with magnetic field along z , η_{xzyz} takes phonon heat flow along x and deflects it

to heat flow along y . By symmetry, $\eta_{zxzy} = \eta_{xzyz}$, and thus our observation of phonon Hall viscosity suggests that phonons contribute to the thermal Hall effect in α -RuCl₃.

We can estimate the phonon contribution to the thermal Hall effect using our measurement of η_{xzyz} and the equations for acoustic energy transport in the presence of Hall viscosity—a detailed derivation is given in the Methods. The result is that Hall viscosity produces a thermal Hall conductivity: $\kappa_{xy} = \frac{\eta_{xzyz}}{\rho}C$, where ρ and C are α -RuCl₃'s density and specific heat, respectively. Note that the phonon Hall conductivity is independent of the phonon mean free path—it is intrinsic. The Hall conductivity can be compared with the longitudinal thermal conductivity: $\kappa_{xx} = \bar{v}_s l C$, where $\bar{v}_s$ is the average speed of sound and l is the phonon mean free path. The ratio of phonon Hall to phonon longitudinal thermal conductivities is then

$$\frac{\kappa_{xy}}{\kappa_{xx}} = \frac{\eta_{xzyz}}{\rho v_s l} = \frac{C}{\kappa_{xx}} \frac{\eta_{xzyz}}{\rho}. \quad (3)$$

Using our measured value of $\eta_{xzyz} = 1.3 \times 10^{-5}$ Pa·s, and the measured values of κ_{xx} and the specific heat at 10 K with $\mathbf{B}||c$ ^{44,45}, we estimate $\kappa_{xy}/\kappa_{xx} = 10^{-4}$ —only a factor of 4 smaller than what is observed in thermal transport experiments under the same conditions by LeFrancois *et al.*⁴⁴. Note that this estimate is a lower bound on the total phonon contribution to κ_{xy} : the full κ_{xy} will receive contributions from all three acoustic branches (here we have only accounted for one), and from all 4 viscosity tensor elements that are allowed for this magnetic field orientation (e.g. η_{xyxz} , etc.) There is no reason to expect that the different viscosities are substantially different, and first-principles calculations by Dhakal *et al.* indeed suggest they are all similar in magnitude⁴⁷. In addition, finite Hall viscosity enables phonon side-jump scattering that will also contribute to κ_{xy} . Thus the total κ_{xy} contribution from phonon Hall viscosity could easily be a factor of 10 larger than what we estimate here.

Next, we analyze the temperature dependence of the Hall viscosity for $B||c$. Fig. 5a shows that, aside from a sharp drop near the magnetic phase transition at 7.5 K, the Hall viscosity is only weakly temperature dependent up to 40 kelvin. This clearly demonstrates that

phonon Hall viscosity in α -RuCl₃ is not due to the coherent hybridization of acoustic phonons and magnons into chiral “magnetopolarons”²⁹—the conventional mechanism of the acoustic Faraday effect. We can compare the temperature dependence of the Hall viscosity to the thermal Hall effect by plotting η_{xzyz} alongside $\rho\kappa_{xy}/C$, which has units of viscosity. These quantities are the same order of magnitude and have qualitatively similar temperature dependencies: both showing minima near T_N and extending to temperatures much greater than T_N . The drop in η_{xzyz} near T_N may be intrinsic, or it may be due to strong phonon scattering due to thermal fluctuations of the order parameter at the antiferromagnetic phase transition. Although η_{xzyz} itself does not depend on the phonon mean free path, our ability to measure it does when the mean free path of our ultrasonic phonons becomes much shorter than the sample size, which happens near T_N .

Finally, in Fig. 5b we compare the magnetic field dependence of the Hall viscosity at $\theta = 55^\circ$ to the field dependence of the thermal Hall effect measured with magnetic field purely along the a axis. Both quantities begin to rise above 5 tesla and are peaked *above* the critical field where long-range order is suppressed. As both the static spin susceptibility and inelastic neutron scattering show that the magnetic state becomes soft at B_c ^{48,49}, this suggests that the phonon Hall viscosity originates from coupling between phonons and the spin degrees of freedom. Note that the thermal Hall effect in this field configuration—with $B||a$ —is sensitive to many more viscosity tensor elements than the Faraday effect is: our measurement always probes η_{xzyz} , whereas κ_{xy} for $B||a$ is sensitive to η_{zxxy} , η_{xyxx} , η_{yzxx} , η_{yyyz} and η_{yyxz} . Thus, a direct quantitative comparison between these two quantities is not valid for this field orientation; the purpose of Fig. 5b is to show that both quantities are sensitive to the proximity of the critical field where order is suppressed. Also note that, while crystal symmetry constrains κ_{xy} to be zero when the magnetic field purely along b , the same symmetry argument does not require all viscosity components to vanish for this field orientation (this is discussed further in Dhakal *et al.*⁴⁷).

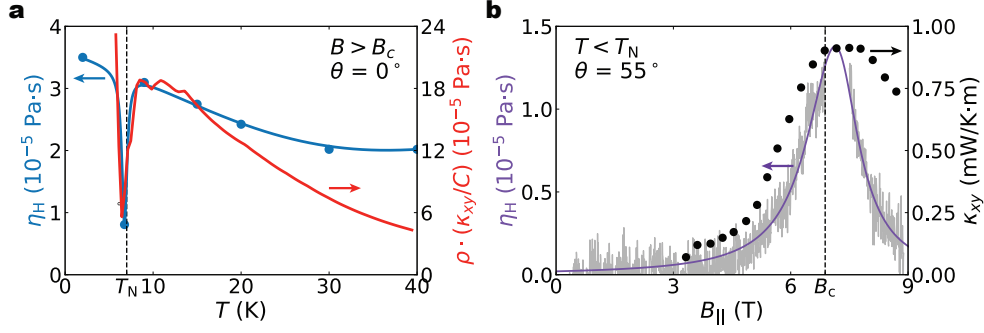


Fig. 5 | Temperature and field dependence of the phonon Hall viscosity. **a**, The phonon Hall viscosity as a function of temperature for $\mathbf{B}||c = 12$ T. Blue circles are measured data points, and the blue line is a guide to the eye. The quantity $\rho\kappa_{xy}/C$, calculated using κ_{xy} from LeFrancois *et al.*⁴⁴ and C from Widman *et al.*⁵⁰, is plotted in red for comparison. **b**, The phonon Hall viscosity measured with $\mathbf{B}$ rotated 55° from c toward b , plotted as a function of in-plane magnetic field at $T = 2$ K. The κ_{xy} data are measured with $\mathbf{B}||a$ and are taken from Czajka *et al.*⁵¹.

Discussion

Having shown that phonons can account for a significant fraction—if not all—of the observed thermal Hall effect in α -RuCl₃, we turn to the question of *why* the phonons in α -RuCl₃ have Hall viscosity. As noted above, the observation of Hall viscosity at temperatures well above the antiferromagnetic phase transition rules out the conventional mechanism of magnon-phonon hybridization²⁹. Indeed, neutron scattering finds a broad continuum of excitations at low energy and momentum rather than distinct, sharp magnon bands^{52,53}.

Can incoherent magnetic excitations produce phonon Hall viscosity? Strong spin-lattice coupling is clearly present in α -RuCl₃, as evidenced by the large changes in sound velocity at T_N and B_c (Fig. 2 and Hauspurg *et al.*⁴¹), which are two orders of magnitude larger than those typically seen at superconducting phase transitions^{54,55}. Dhakal *et al.*⁴⁷ used a first-principles based approach to calculate the microscopic spin-lattice couplings in α -RuCl₃, and they demonstrated that the resultant phonon Hall viscosities can account for the full magnitude of the measured thermal Hall effect. Although they use a two-dimensional model

that does not access the same viscosity component that we do in our experiment (η_{xzyz}), they find that all five in-plane viscosity components are the same order of magnitude—and, strikingly, the same order of magnitude as the value we measure for η_{xzyz} . Our measurements therefore provide experimental evidence that the phonon Hall viscosity in α -RuCl₃ is indeed large enough to account for the thermal Hall effect, calling into question the existence of a quantized contribution from Majorana edge modes. Phonon Hall viscosity also likely explains the unusual thermal Hall effects found in many other magnetic insulators. However, non-magnetic materials, such as SrTiO₃, likely require a different mechanism.

Beyond clarifying the origin of the thermal Hall effect in α -RuCl₃, our measurements provide a direct experimental link between phonon Hall viscosity and phonon Berry curvature—connecting two concepts that have, until now, remained largely theoretical^{16–18,25,26}. Conventionally, viscosity is thought of as a long-wavelength property of sound, whereas Berry curvature is associated with the microscopic wave functions of phonons. The connection between phonon Hall viscosity and Berry curvature can be demonstrated by examining how the Hamiltonian of the system, $\mathcal{H}$, changes under an applied strain: $\frac{\partial \mathcal{H}}{\partial \varepsilon_{ij}}$. The stress of a state $|\Psi\rangle$ is the expectation value of this derivative: $\sigma_{ij} = \langle \Psi | \frac{\partial \mathcal{H}}{\partial \varepsilon_{ij}} | \Psi \rangle$. If the state of the system varies trivially with strain, then this reduces to the classical definition of stress: $\sigma_{ij} = \partial E / \partial \varepsilon_{ij} \equiv c_{ijkl} \varepsilon_{kl}$, where E is the state's energy. If, however, the system gains a geometric phase when traversing a path in strain space, then the expectation value acquires Berry curvature,

$$\sigma_{ij} = c_{ijkl} \varepsilon_{kl} + \Omega_{ijkl} \dot{\varepsilon}_{kl}, \quad (4)$$

where the full expression for Ω_{ijkl} in terms of strain is given by Avron *et al.*²⁵. This expression can be compared to Equation 1,

$$\sigma_{ij} = c_{ijkl} \varepsilon_{kl} + \eta_{ijkl} \dot{\varepsilon}_{kl},$$

making the correspondence between Ω_{ijkl} and η_{ijkl} clear. It can be helpful to interpret Ω_{ijkl} as a “real-space” Berry curvature—it gives rise to a geometric phase when the crystal lattice is moved around a closed loop by an oscillating strain, such as the one created by a sound wave. This is analogous to the “momentum-space” Berry curvature of Bloch electrons, which gain geometric phases when traversing closed paths in momentum space. Where Bloch electrons experience a non-dissipative force from Berry curvature that leads to the anomalous Hall effect, the lattice (i.e. phonons and sound waves) experiences a non-dissipative Hall viscosity that leads to the thermal Hall and acoustic Faraday effects.

In general, a variety of mechanisms can generate phonon Berry curvature and Hall viscosity. For example, electron-phonon coupling can generate phonon Berry curvature and Hall viscosity in quantum Hall systems, topological insulators, and topological superconductors²⁶. Similarly for spin-lattice coupling in chiral spin liquids¹⁶. Thus, our measurements do not rule out the possibility of spinon excitations in α -RuCl₃. They do, however, require that phonons account for at least a significant fraction of the thermal Hall effect. More generally, we have demonstrated that the acoustic Faraday effect is a direct probe of phonon Hall viscosity or, equivalently, phonon Berry curvature. Because phonon Hall viscosity is a property of many exotic phases, including topological superconductors and 3D quantum Hall states, acoustic Faraday rotation measurements will be a valuable tool for discovering and characterizing these states of matter.

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Methods

Crystal growth and selection

Single crystals of $\alpha\text{-RuCl}_3$ were grown using the method described in Kim *et al.*⁴⁰.

We first sort crystals by eye for untwinned specimens. Once identified, we further select for crystals without visible stacking faults by examining the crystal morphology. Sections that appear to have faults are removed by cleaving the sample. It is also necessary to ensure that there is enough overlapping area between the top and bottom surfaces so that there is a direct path along the c axis for sound to travel between two transducers. Finally, samples are always handled using brushes—never tweezers—to ensure minimal structural degradation.

To confirm our results we measure three samples of varying thickness, all grown by the same method. A representative piece of $\alpha\text{-RuCl}_3$ used for the ultrasound experiment is shown in Extended Data Fig. 1a. The naming convention is summarized in Extended Data Table 1.

Transducer fabrication

We perform all depositions in a 4-gun Angstrom Engineering NexDep magnetron sputtering system, with the sample mounted above the sputtering targets.

We sputter thin-film ZnO piezoelectric transducers on cleaved (001) planes of single-domain $\alpha\text{-RuCl}_3$ crystals. The transducer consist of three layers: a bottom Ti/Pt layer (8nm/100nm), an $\sim 1\ \mu\text{m}$ layer of piezoelectric ZnO, and a top layer of Ti/Pt (8nm/100nm). The bottom metallic layer ensures adhesion of the ZnO to the sample surface and functions

as a ground electrode for the transducer. The top metallic stack serves as an electrode for excitation and read out. Both metal layers are DC sputtered in a pure argon environment.

We deposit the piezoelectric ZnO layer by RF sputtering with a 2 inch diameter, 99.99% purity ZnO target. We use a 1:3 ratio of oxygen to argon flow rates at a total pressure of 3 mTorr. The deposition rate is approximately 0.2 Å/s using a power density of 21 W/in².

The polarization of the transducer is determined by the *c* axis orientation of the ZnO layer. For shear-polarized transducers, we use “glancing angle deposition”⁵⁶ to orient the ZnO *c* axis away from sample surface normal. To achieve this, we position the sample as far from the sputtering target as our chamber allows, and keep the sample stage fixed (i.e. no rotation) during deposition. The angle of incidence of ZnO ions on the sample surface in our sputtering chamber is approximately 70° away from the surface normal. The resulting ZnO films have their *c* axis tilted away from the sample surface normal along the straight line connecting the sample and target. This process results in a transducer that generates both compressional and shear stress in the sample. The polarization of the shear stresses is in the direction of the ZnO *c* axis tilt.

To facilitate antisymmetrization of the data as a function of magnetic field, the polarization of one transducer is rotated 45° with respect to the polarization of the other transducer: this allows differentiation between “rotation left” vs “rotation right”. A precise 45° misalignment is not necessary to perform the antisymmetrization, but maximizes the size of antisymmetric signal in the presence of viscosity.

Sample mounting

We mount the α -RuCl₃ samples on custom-designed, two-port PCBs using GE varnish. The attachment point is far from the ultrasonic transducer to minimize unwanted strain effects in the measurement. To minimize possible motion of the sample in an applied magnetic field, we further support the sample with pieces of microscope slide. We make electrical contact to the transducer bottom electrode with silver paint, connecting the electrode directly to the

PCB ground plane. We wire the top electrode of the transducer to a coplanar waveguide using 25 μm diameter 99% purity silver wire. Waveguides to both transducers terminate in MMCX connectors. Extended Data Fig. 2b and c show samples S1 and S2 after transducer deposition and mounting on the PCB for measurement.

Ultrasound Measurements

Fridge/Magnet

Measurements are performed in an Oxford Instruments Teslatron system, with a 12 T superconducting magnet and variable temperature insert. The sample is loaded onto an Oxford Heliox He³ probe equipped with custom low-loss cupronickle coaxial cables.

Measurement Electronics

Extended Data Fig. 2 shows the measurement circuit for the ultrasound experiment. We use a Tektronix TSG 4106A signal generator to generate RF pulses. We set the pulse position, width, and repetition frequency by external pulse modulation of the RF source supplied by a Tektronix AFG 3100 arbitrary waveform generator. Before arriving at the drive transducer the RF pulse is amplified by a Mini Circuits ZHL-42W+ power amplifier. The transmitted ultrasound excites the receive transducer, which is connected to a low noise Mini Circuits ZX60-83LN-S+ amplifier in series with a ZHL-42W+ power amplifier. We record the amplified signal from the receive transducer on a Tektronix MSO 6 Series oscilloscope. We use Mini Circuits ZFWA2-63DR+ switches to isolate the RF source and pulse amplifier from the oscilloscope. The switch logic is controlled by the AFG 3100 waveform generator.

Extracting amplitude and speed of sound

Our raw data at each value of temperature and applied magnetic field is the voltage across the receive transducer as a function of time. Each voltage trace consists of a series of RF pulses—echoes—separated by the transit time of a strain pulse through the sample, typically ~ 100 ns.

We perform digital lock-in at the drive frequency to extract the relative change in the speed of sound and transmitted ultrasound amplitude.

We measure the change in phase difference between two echoes as a function of temperature or applied magnetic field to track the change in speed of sound. At drive frequency f , the total phase accumulated as the sound wave travels for a time t is $\phi = 2\pi ft$. The fractional change in the phase, $\delta\phi/\phi$ is then related to the fractional change in the phase velocity, $\delta v/v$, of the ultrasound by

$$\frac{\delta\phi}{\phi} = \frac{\delta t}{t} = -\frac{\delta v}{v}. \quad (5)$$

The Faraday rotation is related to the amplitude of the received sound. Because each transducer is a polarization-specific detector, a change in the polarization direction of incoming transverse sound results in a change in amplitude of the voltage at the receive transducer. The amplitude of the received signal goes as $|\cos(\theta)|$, where θ is the angle between the transducer polarization and the incoming sound wave polarization.

Changes in signal amplitude as a function of temperature and magnetic field can also result from effects unrelated to polarization rotation. Energy is lost due to ultrasound attenuation, which can change dramatically e.g. across a magnetic phase transition. Additionally, experimental artifacts can cause changes in the received amplitude: loss of collimation of the strain pulse during propagation results in interference which changes with the wavelength of the ultrasound. These interference effects can also be pronounced when there are large changes in the speed of sound, for example across a phase transition. Crucially for our experiment, these effects are all time-reversal-*even*—they remain the same whether the magnetic field is up or down.

To detect the presence of time-reversal-*odd* polarization rotation (i.e. Faraday rotation), we antisymmetrize the received amplitude with respect to the applied magnetic field. In addition to isolating the time-odd contribution to the signal, this process gives us several checks to confirm the consistency of the experimental protocol, described below.

Data analysis—isolating the Faraday rotation

Antisymmetrization: transmission/reflection and compression/shear

Our experimental protocol—45° misaligned transducer polarizations, antisymmetrizing with respect to magnetic field direction, and swapping ultrasound propagation direction—aims to isolate the antisymmetric viscous contribution to the signal. The experimental protocol also has two built-in checks to verify that the antisymmetric signal is not artificial. First, transmitted *longitudinal* sound should have no antisymmetric component. Second, the antisymmetric component of the signal should disappear for both longitudinal and transverse ultrasound if the experiment is run in reflection mode where a single transducer is used for both exciting and detecting sound waves. This occurs because, even though the Faraday rotation angle accumulates over multiple reflections (and does not “unwind” when the propagation direction is reversed), a single transducer used to excite and detect ultrasound cannot distinguish between clockwise and counterclockwise polarization rotations.

Extended Data Fig. 3 shows the amplitude of transmitted longitudinal sound and reflected transverse sound, antisymmetrized in magnetic field for two magnetic field orientations. In both cases, within our resolution, only transmitted, transverse sound contains a field-antisymmetric component. This result is consistent with Faraday rotation and validates our method for isolating the antisymmetric signal due to Hall viscosity.

Angle dependence

As explained below, we expect the Hall viscosity component we are sensitive to— η_{xzy} —to vanish with the magnetic field applied entirely in the honeycomb plane. Extended Data Fig. 4 shows a comparison of the antisymmetric-in-field signal for magnetic field in the honeycomb plane compared with magnetic field with a component along the c axis. Within our experimental precision, we find no antisymmetric signal due to viscosity with magnetic field entirely in the honeycomb plane.

Our claim that the Hall viscosity of α -RuCl₃ is related to the coupling of phonons to spins is largely based on the observation that the Hall viscosity in our samples peaks near the critical magnetic field B_c . Extended Data Fig. 5 shows that the peak in the Hall viscosity follows the critical magnetic field at two different tilt angles.

Frequency dependence

Extended Data Fig. 6 shows the antisymmetric amplitude in sample 1 (210 μ m thickness) measured at several frequencies. We find no appreciable frequency dependence over our limited available bandwidth.

Antisymmetric Signal at Elevated Temperatures

We find that the Hall viscosity persists to temperatures above the Néel temperature of $T_N \approx 7.5$ K. Extended Data Fig. 7 shows the antisymmetric-in-field data used to estimate the Hall viscosity at temperatures well inside the antiferromagnetic phase, just below the Néel transition, and well above the Néel transition in the paramagnetic phase.

Data analysis – estimating the viscosity

Wave equation with viscosity and birefringence

To estimate the magnitude of the Hall viscosity from our pulse echo data, we use the wave equation for transverse sound waves propagating along the c axis of α -RuCl₃ in the presence of Hall viscosity. The Lagrangian for small displacements $\vec{u}$ is

$$\mathcal{L} = \frac{\rho}{2} \dot{u}^2 - \frac{c_{ijkl}}{2} \varepsilon_{ij} \varepsilon_{kl} - \frac{\eta_{ijkl}}{2} \varepsilon_{ij} \dot{\varepsilon}_{kl}, \quad (6)$$

where ρ is the material density, $\varepsilon_{ij} = (\partial_i u_j + \partial_j u_i)/2$ are the lattice strains, c_{ijkl} are the elastic moduli, and η_{ijkl} are the antisymmetric Hall viscosities. For harmonic displacements

with wave vector k along the c axis, and with Hall viscosity $\eta \equiv \eta_{xzyz}$ to which we are sensitive in our experiment, Equation 6 becomes

$$\mathcal{L} = \frac{\rho}{2}\dot{u}^2 - \frac{c_{44}}{2}k^2u_y^2 - \frac{c_{55}}{2}k^2u_x^2 - \frac{\eta}{2}k^2(u_x\dot{u}_y - \dot{u}_xu_y), \quad (7)$$

from which we derive the equation of motion presented in the main text:

$$\rho\omega^2\vec{u} = k^2 \begin{bmatrix} c_{55} & i\eta\omega \\ -i\omega\eta & c_{44} \end{bmatrix} \vec{u}. \quad (8)$$

Note that, while $c_{44} = c_{55}$ in zero magnetic field for α -RuCl₃, this will no longer be the case with a magnetic field component in the honeycomb plane. This in-plane magnetic field dependence between c_{44} and c_{55} is known as acoustic birefringence and, crucially, is even in magnetic field (time reversal even).

We can gain some intuition for the equation of motion by rewriting the matrix on the right-hand side in terms of the average elastic modulus $c = (c_{44} + c_{55})/2$ and the difference $\delta = (c_{44} - c_{55})/2$. Then the equation of motion becomes a sum of Pauli matrices,

$$\begin{bmatrix} c_{55} & i\eta\omega \\ -i\omega\eta & c_{44} \end{bmatrix} = c1 + \delta\sigma_z + \omega\eta\sigma_y \quad (9)$$

The square of the phase velocities, $v_{\pm}^2 = \omega^2/k^2$, are then

$$\rho v_{\pm}^2 = c \pm \sqrt{\delta^2 + \omega^2\eta^2}, \quad (10)$$

so that the two transverse wave speeds are split both by the difference in elastic moduli and by the viscosity. The polarizations are, up to a normalization constant,

$$\mathbf{u}_+ = \frac{1}{\sqrt{\delta^2 + \omega^2 \eta^2}} \begin{bmatrix} \delta + \sqrt{\delta^2 + \omega^2 \eta^2} \\ i\omega\eta \end{bmatrix} \quad \mathbf{u}_- = \frac{1}{\sqrt{\delta^2 + \omega^2 \eta^2}} \begin{bmatrix} i\omega\eta \\ \delta + \sqrt{\delta^2 + \omega^2 \eta^2} \end{bmatrix}. \quad (11)$$

From Equation 11 we can see that when $\eta = 0$, changing δ does not change the polarizations though it does split the degenerate speeds of sound. When $\delta = 0$ adding non-zero viscosity results in circular rather than linear polarizations. Finally, when both $\eta, \delta \neq 0$, we get elliptical polarizations.

Wave equation fits to the data

We use the equations of motion—Equation 8—to model the ultrasound experiment. Our experiment consists of launching a strain wave along the c axis with frequency ω and polarization at angle ϕ_i in the x - y plane: $\mathbf{u}_i = (\cos(\phi_i), \sin(\phi_i))$. To obtain the amplitude that we measure at the receive transducer for a given value of the viscosity, we proceed in three steps.

First, we compute the normal mode wave vectors, $k_{\pm}$, and polarizations $\mathbf{u}_{\pm}$, using Equation 8. The initial polarization is then decomposed into the normal modes:

$$\mathbf{u}_i = \alpha \mathbf{u}_+ + \beta \mathbf{u}_-, \quad (12)$$

from which we can find the coefficients $\alpha = \mathbf{u}_+^\dagger \cdot \mathbf{u}_i$ and $\beta = \mathbf{u}_-^\dagger \cdot \mathbf{u}_i$. The phase of each mode after traveling the length of the sample, ℓ , is $e^{ik_{\pm}\ell}$. We then project each mode on to the polarization of the receive transducer $\mathbf{u}_f = (\cos(\phi_f), \sin(\phi_f))$, giving two contributions to the final (complex) amplitude, $\tilde{A}_{\pm}$ from each mode polarization $\mathbf{u}_{\pm}$

$$\tilde{A}_{\pm} = (\mathbf{u}_{\pm}^\dagger \cdot \mathbf{u}_i) (\mathbf{u}_f^\dagger \cdot \mathbf{u}_{\pm}) e^{ik_{\pm}\ell} \quad (13)$$

The receive transducer sees the sum of the two modes, and the amplitude we measure, A_m is the amplitude of the sum

$$A_m = \left| \tilde{A}_+ + \tilde{A}_- \right| \quad (14)$$

By flipping the sign of η in the model we form symmetrized and antisymmetrized amplitudes as a function of applied magnetic field. Because the antisymmetric amplitude alone is not a meaningful quantity in this context (it depends on the overall amplitude of the initial excitation, for example) we normalize the antisymmetric amplitude in the model and the data by the symmetric amplitude at zero applied magnetic field. To extract the viscosity as a function of applied magnetic field, we perform a one parameter fit to the ratio of the antisymmetric and zero field symmetric amplitudes using the model presented above. The value of viscosity as a function of magnetic field presented in the main text minimizes the squared difference between the model output and the data.

Hall viscosity tensor in α -RuCl₃

The viscosity tensor, η relates stress to the time rate of strain $\dot{\epsilon}$ and gives a contribution to the elastic energy density

$$U_{\text{el}} = \frac{c_{ijkl}}{2} \epsilon_{ij} \epsilon_{kl} + \frac{\eta_{ijkl}}{2} \epsilon_{ij} \dot{\epsilon}_{kl}, \quad (15)$$

where c is the elastic tensor.

Because strain is a symmetric tensor, the viscosity is also symmetric under interchange within the first and second pairs of indices: $i \longleftrightarrow j$ and $k \longleftrightarrow l$. It is also clear from [Equation 15](#) that the viscosity is odd under time reversal.

The viscous contribution to the energy comes from the product of two strains. The viscosity tensor, then, can be divided into symmetric and antisymmetric parts under exchange of those strains – the first and second pair of indices $ij \longleftrightarrow kl$. The symmetric viscosity, $\eta^S = (\eta_{ijkl} + \eta_{klij})/2$, results in energy loss and is even under reversing the sign of an

applied magnetic field. In the context of an ultrasound experiment η^S is proportional to the ultrasonic attenuation. The antisymmetric, Hall viscosity, $\eta^H = (\eta_{ijkl} - \eta_{klij})/2$, generates acoustic Faraday rotation and an intrinsic thermal Hall effect.

When setting out to measure the Hall viscosity, it is important to know when it is required to vanish by symmetry. Counting the symmetry-allowed components of η^H is simplified by working with the irreducible representations of strain. The elastic energy due to the Hall viscosity coupling two irreducible strains ε_Γ and $\varepsilon_{\Gamma'}$ is

$$U_{\text{visc}} = \frac{\eta_{\Gamma\Gamma'}}{2} (\dot{\varepsilon}_\Gamma \varepsilon_{\Gamma'} - \varepsilon_\Gamma \dot{\varepsilon}_{\Gamma'}). \quad (16)$$

The system, assumed to break time reversal symmetry (whether intrinsically or through an applied magnetic field), allows $\eta_{\Gamma\Gamma'}$ to be time odd. Both the energy and tensor element $\eta_{\Gamma\Gamma'}$ are scalars (A_g objects in the S_6 point group), which requires the term in parentheses in [Equation 16](#) to also be a scalar. Because $\eta_{\Gamma\Gamma'}$ is antisymmetric under exchange $\Gamma \longleftrightarrow \Gamma'$, this forces either: 1) that $\varepsilon_\Gamma = \varepsilon_{\Gamma'}$ and that the representation Γ squared contains an antisymmetric scalar (and therefore must two or three dimensional); or 2) that ε_Γ and $\varepsilon_{\Gamma'}$ are distinct strains that belong to the same representation.

When time reversal symmetry is broken by an applied magnetic field, it is more transparent to “factor out” the magnetic field from the Hall viscosity. Then, to identify which magnetic field components generate which Hall viscosity coefficients, we expand the viscosity to linear order in applied magnetic field. The magnetic field splits into representations, B_Σ , and we form the antisymmetric products which each contribute to the elastic energy,

$$U_{\text{visc}}(B_\Sigma) = B_\Sigma \tilde{\eta}_{\Gamma\Gamma'}^\Sigma (\dot{\varepsilon}_\Gamma \varepsilon_{\Gamma'} - \varepsilon_\Gamma \dot{\varepsilon}_{\Gamma'}), \quad (17)$$

where now the coefficient $\tilde{\eta}_{\Gamma\Gamma'}^\Sigma$ is a scalar and has unit of $\text{Pa}\cdot\text{s}\cdot\text{T}^{-1}$, and the antisymmetric term in parentheses transforms as Σ so that the whole expression is a scalar. Note that the

product $\varepsilon_{\Gamma}\varepsilon_{\Gamma'}$ generically decomposes into the sum of multiple representations, not all of which transform like Σ . To avoid cluttering the notation further, we specify which portion of the product is involved in a particular term with the superscript on the coefficient $\tilde{\eta}_{\Gamma\Gamma'}^{\Sigma}$ (but again note that $\tilde{\eta}_{\Gamma\Gamma'}^{\Sigma}$ itself is always a scalar).

Now we take the specific example of α -RuCl₃. Below its structural transition at 150 K, α -RuCl₃ transitions to rhombohedral symmetry with point group S₆. There are four irreducible strains in S₆: two A_g strains and two E_g strains:

$$\varepsilon_{A_g^{(1)}} = \varepsilon_{x^2+y^2}, \quad \varepsilon_{A_g^{(2)}} = \varepsilon_{z^2}, \quad \varepsilon_{E_g^{(1)}} = \{\varepsilon_{x^2-y^2}, \varepsilon_{xy}\}, \quad \varepsilon_{E_g^{(2)}} = \{\varepsilon_{xz}, \varepsilon_{yz}\}.$$

The magnetic field splits into two representations: the out-of-plane component transforms as A_g , and the in-plane component transforms as E_g :

$$B_{A_g} = B_z, \quad B_{E_g} = \{B_x, B_y\}.$$

To simplify the notation in what follows, we drop the g in the representation labels since we only need to consider representations that are even under inversion. As mentioned earlier, because the product of two strains can decompose into the sum of several other strains, we use the notation from [Equation 17](#) to specify which viscosity coefficient goes with which strain in the sum. For example, one such term will be

$$U_{\text{visc}}(B_{A_{1g}}) \sim B_{A_g} \tilde{\eta}_{E_g^{(1)}E_g^{(2)}}^{A_g} (\varepsilon_{E_g^{(1)}} \dot{\varepsilon}_{E_g^{(2)}} - \dot{\varepsilon}_{E_g^{(1)}} \varepsilon_{E_g^{(2)}}) \equiv B_A \tilde{\eta}_{E_1E_2}^A (\varepsilon_{E_1} \dot{\varepsilon}_{E_2} - \dot{\varepsilon}_{E_1} \varepsilon_{E_2}), \quad (18)$$

where $\tilde{\eta}_{E_1E_2}^A$ denotes a viscosity term that couples the A_g portion of the product of $E_g^{(1)}$ and $E_g^{(2)}$ strains to the B_{A_g} magnetic field component.

With an out-of-plane magnetic field B_z , we can form the following four Hall viscosity contributions to the elastic energy:

$$U_{\text{visc}}(B_z) = B_A \left[\tilde{\eta}_{A_1 A_2}^A (\varepsilon_{A_1} \dot{\varepsilon}_{A_2} - \dot{\varepsilon}_{A_1} \varepsilon_{A_2}) + \tilde{\eta}_{E_1 E_2}^A (\varepsilon_{E_1} \dot{\varepsilon}_{E_2} - \dot{\varepsilon}_{E_1} \varepsilon_{E_2}) \right. \\ \left. + \tilde{\eta}_{E_1 E_1}^A (\varepsilon_{E_1} \dot{\varepsilon}_{E_1} - \dot{\varepsilon}_{E_1} \varepsilon_{E_1}) + \tilde{\eta}_{E_2 E_2}^A (\varepsilon_{E_2} \dot{\varepsilon}_{E_2} - \dot{\varepsilon}_{E_2} \varepsilon_{E_2}) \right], \quad (19)$$

where the last two terms, which involve the square of a representation, arise because the product of E_g with itself contains an antisymmetric object which transforms as A_g .

An in-plane magnetic field generates five Hall viscosity terms:

$$U_{\text{visc}}(B_x, B_y) = B_E \left[\tilde{\eta}_{A_1 E_1}^E (\varepsilon_{A_1} \dot{\varepsilon}_{E_1} - \dot{\varepsilon}_{A_1} \varepsilon_{E_1}) + \tilde{\eta}_{A_1 E_2}^E (\varepsilon_{A_1} \dot{\varepsilon}_{E_2} - \dot{\varepsilon}_{A_1} \varepsilon_{E_2}) \right. \\ \tilde{\eta}_{A_2 E_1}^E (\varepsilon_{A_2} \dot{\varepsilon}_{E_1} - \dot{\varepsilon}_{A_2} \varepsilon_{E_1}) + \tilde{\eta}_{A_2 E_2}^E (\varepsilon_{A_2} \dot{\varepsilon}_{E_2} - \dot{\varepsilon}_{A_2} \varepsilon_{E_2}) \\ \left. + \tilde{\eta}_{E_1 E_2}^E (\varepsilon_{E_1} \dot{\varepsilon}_{E_2} - \dot{\varepsilon}_{E_1} \varepsilon_{E_2}) \right]. \quad (20)$$

The Hall viscosity we measure in our experiment shows up here as $B_A \tilde{\eta}_{E_2 E_2}^A = \eta_{xzyz}$ and is only generated by the out-of-plane magnetic field, B_z . When the magnetic field is tilted away from the c axis (but with some component still along z), all nine Hall viscosity coefficients from both [Equation 19](#) and [Equation 20](#) can be generated, including η_{xzyz} .

Heat flux in the presence of Hall viscosity

To estimate the intrinsic contribution to the thermal Hall effect due to Hall viscosity, we consider the transverse energy flux generated by thermal phonons. Thermally generated stress at frequency ω , $\sigma(\omega)$, carries an acoustic energy current

$$j_i = \text{Re}[\sigma_{ij}(\omega) \dot{u}_j] = \text{Re}[(c_{ijkl} \varepsilon_{ij} + \eta_{ijkl} \dot{\varepsilon}_{kl}) \dot{u}_j], \quad (21)$$

where $\dot{u}$ is the time derivative of displacements due to the stress⁵⁷. Longitudinal thermal transport occurs because temperature gradients produce gradients in the amplitude (energy occupations) of thermal phonons. For example, if we consider z polarized thermal strains propagating in the x -direction with wave vector q_x , so that $u = u_z(x)e^{i(\omega t - q_x x)}$, there is a contribution to the energy flux

$$j_x = \text{Re}[\sigma_{xz}\dot{u}_z] = \text{Re}[i\omega c_{55}\varepsilon_{xz}u_z] = c_{55}\omega q_x u_z^2, \quad (22)$$

where we use Hooke's law $\sigma_{xz} = iq_x c_{55}u_z$ for this particular component. Strains involving displacements along x and y also contribute to j_x , bringing in terms proportional to c_{11} and c_{66} respectively. Taken together we can combine these contributions, using an average elastic modulus $\bar{c}$, to obtain

$$j_x = \bar{c} q_x \omega u^2 = \frac{\bar{c}}{\rho v_s} (\rho \omega^2 u^2) = v_s (\rho \omega^2 u^2), \quad (23)$$

where v_s is an average speed of sound and ρ is the density. Because these are thermally generated, random strains, there is no preferred direction for q_x and, in the absence of a thermal gradient, the net energy flux is zero.

However, in the presence of a thermal gradient along x , there is heat transport. Conceptually, the thermal phonons travel a mean free distance ℓ before they scatter and equilibrate with the local lattice temperature. The net acoustic energy flux is then

$$j_{x,\text{net}} = j_x(x + \ell) - j_x(x - \ell) = v_s (\rho \omega^2 u(x + \ell)^2 - \rho \omega^2 u(x - \ell)^2). \quad (24)$$

Recognizing that $\rho\omega^2 u^2$ is the phonon energy density, U , we write

$$j_{x,\text{net}} \approx v_s \ell \frac{\partial U}{\partial x} = v_s \ell C \frac{\partial T}{\partial x} = \kappa_{xx} \frac{\partial T}{\partial x} \quad (25)$$

where C is the heat capacity, and we identify the longitudinal thermal conductivity $\kappa_{xx} = v_s \ell C$. This thermal conductivity is extrinsic in the sense that it is proportional to the phonon mean free path.

Now consider the heat flux when there is a non-zero Hall viscosity. In particular, we consider an applied magnetic field along the c axis of α -RuCl₃. Returning to [Equation 21](#), we find that the Hall viscosity produces a transverse energy flux where, for example, the viscosity η_{xzyz} produces

$$j_y = \text{Re}[2\eta_{xzyz}\dot{\epsilon}_{xz}\dot{u}_z] = \eta_{xzyz}\omega^2 \frac{\partial u_z}{\partial x} u_z = \frac{\eta_{xzyz}}{2\rho} \frac{\partial}{\partial x} (\rho\omega^2 u_z^2), \quad (26)$$

and the other two viscosity components η_{yyxy} and η_{yxxx} contribute similar terms. In the absence of a temperature gradient, there will be no net transverse heat flow, but if we enforce an amplitude gradient by applying a temperature gradient along the x -direction, then we transfer the spatial derivative to the applied temperature gradient and write

$$j_y = \frac{\bar{\eta}}{\rho} C \frac{\partial T}{\partial x}, \quad (27)$$

where $\bar{\eta}$ is an average Hall viscosity. Note that, while the transverse energy flux requires a longitudinal temperature gradient, its magnitude does not depend on the phonon mean free path—this effect is intrinsic.

Finally, the thermal Hall angle—the ratio of the thermal Hall conductivity and longitudinal thermal conductivity—is equal to the ratio of the transverse and longitudinal heat currents.

Using Equation 25 and Equation 27 we recover the estimate

$$\frac{\kappa_{xy}}{\kappa_{xx}} = \frac{\eta}{\rho} \frac{C}{\kappa_{xx}}. \quad (28)$$

Note that using this equation to compare our measurement of η_{xyz} to the thermal Hall effect depends on the Hall viscosity being only weakly dependent on frequency (since thermal measurements access much higher phonon frequencies than do our ultrasound measurements). This assumption is justified by the calculations of Dhakal *et al.*⁴⁷, who find that the Hall viscosity is independent of frequency for small wavevector.

References and Notes

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57. Love, A. E. H. *A Treatise on the Mathematical Theory of Elasticity*. 4th edn, (Dover, Garden City, NY, 1944).

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Author Contributions

B. J. R. and A.S. conceived the experiment. E.H., S.K, and Y.J.K. grew and characterized the samples of α -RuCl₃. A.S. prepared the samples for ultrasound experiments, collected and analyzed the data, and produced the figures. B.J.R. and A.S. wrote the manuscript, with input from all coauthors. B.J.R. supervised the project.

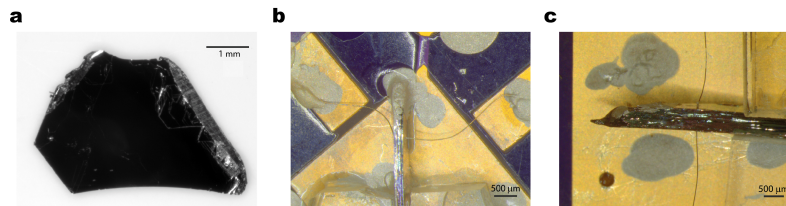
Competing interests:

The authors declare no competing interests.

Data and Code availability:

All data are available from the authors upon request.

Extended Data Figures and Tables

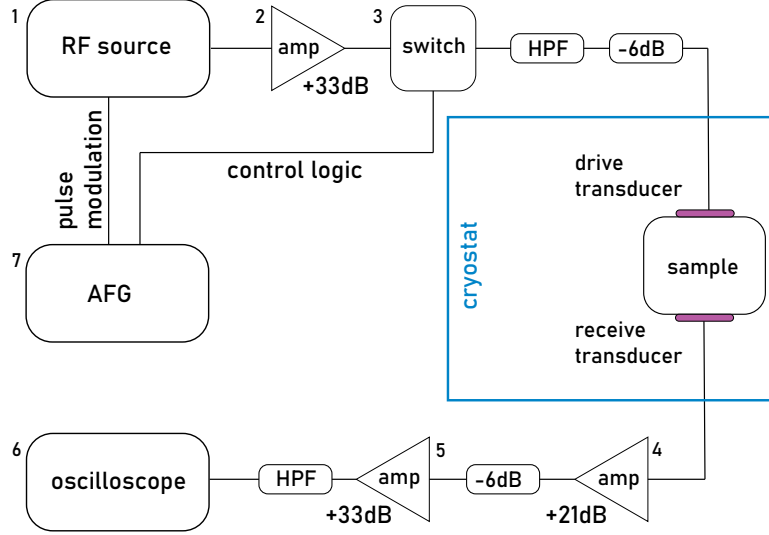


Extended Data Fig. 1 | α -RuCl₃ **Samples:** **a**, shows a representative sample of α -RuCl₃ used for the ultrasound experiment before the piezoelectric ZnO is grown on the sample surface. A pristine (001) face is exposed. The lateral dimensions are several millimeters and the thickness is several hundred microns. **b–c**, show samples 1 and 2 (see Extended Data Table 1) after ultrasonic transducers have been deposited on both exposed (001) surfaces of each sample, and the samples have been mounted on PCBs for the ultrasound transmission measurement. The two wires running to the sample surface are used to excite (read out) the drive (receive) transducer.

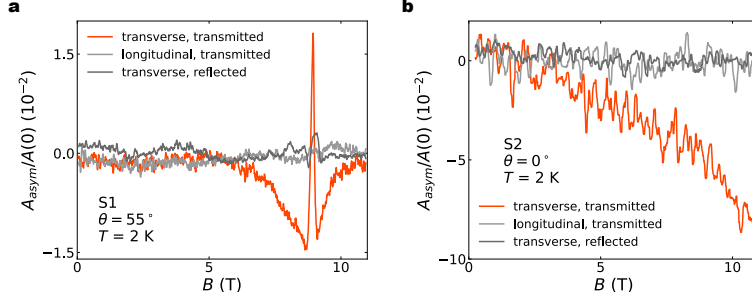
Extended Data Tab. 1 | **Table of samples**

Sample Name	Thickness (μm)	T_N (K)
S1	210	7.6
S2	550	7.7
S3	490	7.7

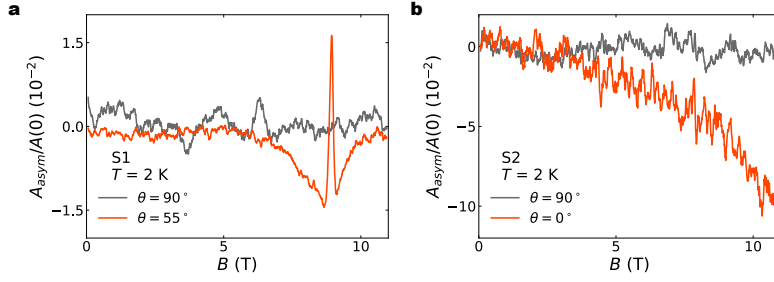
Table 1 | Table of samples



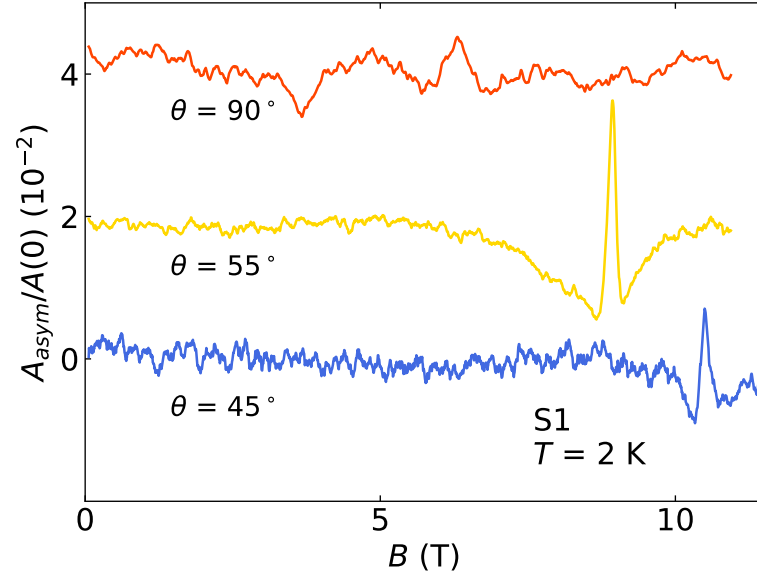
Extended Data Fig. 2 | **Measurement circuit:** Schematic of the measurement circuit, starting from the top left and moving clockwise. We generate RF pulses at the transducer drive frequency (~ 1 GHz) using a Tektronix TSG 4106A signal generator (1). The source is pulse modulated using a Tektronix AFG 3100 waveform generator (7) to output ~ 30 ns square pulses at a repetition frequency of ~ 100 kHz. The RF pulse is further amplified by a Mini Circuits ZHL-42W+ power amplifier with 33 dB gain (2). The pulse is then fed through a Mini Circuits ZFWA2-63DR+ switch (3) to isolate the sample and downstream circuit from the power amplifier after the initial drive pulse is sent to the transducer. The switch logic is controlled by a second channel on the AFG 3100 generator (7). Before reaching the sample, the pulse is high-pass-filtered (HPFed) to remove switching noise and attenuated to minimize unwanted reflections from the transducer. The signal at the receive transducer – the transmitted ultrasound – is amplified twice, first by a low noise Mini Circuits ZX60-83LN-S+ (4) then by a second ZHL-42W+ power amplifier (5) before being filtered and recorded on a Tektronix MSO 6 Series oscilloscope (6).



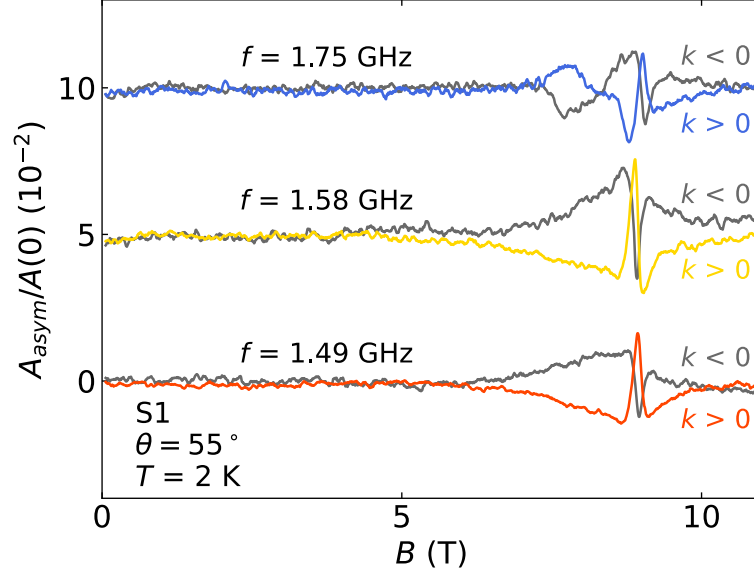
Extended Data Fig. 3 | **Null signal of longitudinal and reflected transverse sound:** To confirm that the antisymmetric signal we measure is not an artifact of the experimental technique, we show the antisymmetric in field amplitude of transmitted longitudinal sound and reflected transverse sound. Compressional sound does not exhibit a Faraday rotation and therefore should have no antisymmetric in field component. Reflected transverse sound can exhibit a Faraday rotation, but the effect disappears on antisymmetrization because when the same transducer is used to excite and detect the sound wave the transducer cannot distinguish between clockwise and counterclockwise rotations. **a**, shows the null signal for sample 1 in a tilted magnetic field and **b**, shows the null signal for sample 2 with the magnetic field along the crystal c axis. In both cases, the antisymmetric signal for transmitted transverse sound exceeds the null signal by an order of magnitude.



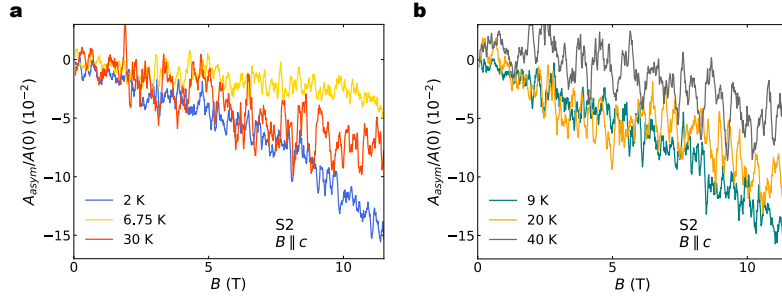
Extended Data Fig. 4 | **Null signal with in-plane magnetic field:** Symmetry constrains the Faraday rotation to occur only when the applied magnetic field has a component along the crystal c axis. Here we show the antisymmetric signal for magnetic field applied entirely in the plane ($\theta = 90^\circ$, where we expect no Faraday rotation) versus with an out-of-plane component of the magnetic field. **a**, shows the result for sample 1 along with the 55° data presented in the main text. **b**, shows the same for sample 2 with the 0° data from the main text. In both cases, we find no evidence of Faraday rotation with the magnetic field applied entirely in the honeycomb plane. Note that this does not imply that there is no thermal Hall effect for this magnetic field orientation; only that the Hall viscosity component we measure, η_{xzyz} , is zero.



Extended Data Fig. 5 | **Magnetic field angle dependence:** To investigate the origin of the Hall viscosity of α -RuCl₃ we measure the Faraday rotation as a function of the out-of-plane tilt angle of the magnetic field. Here we show the antisymmetric in field amplitude for three orientations of the applied magnetic field. In both tilted field orientations the onset and peak of the antisymmetric signal – and therefore the Hall viscosity – tracks the critical magnetic field where zigzag AFM order is destroyed, moving to higher magnetic field as the field is tilted towards the c axis. This suggests that the Hall viscosity of α -RuCl₃ is connected to the spin degrees of freedom in the material.



Extended Data Fig. 6 | **Frequency dependence:** Here we show the antisymmetric in field amplitude over a range of frequencies for both propagation directions.



Extended Data Fig. 7 | **Temperature dependence:** To connect the Hall viscosity to the thermal Hall conductivity of α -RuCl₃ we measure the Faraday rotation with the magnetic field along the crystal c axis as a function of temperature. Here we show the raw data used to extract the viscosity at 12 T and compare with thermal transport measurements in the main text. **a**, shows that the large antisymmetric signal measured at 2 K is reduced just below T_N . **b**, shows that the antisymmetric signal is restored above T_N and decays slowly as the temperature is increased. The data are separated into two panels for visual clarity.