

Impact of projection-induced optical selection bias on the weak lensing mass calibration of galaxy clusters

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Weak gravitational lensing signals of optically identified clusters are impacted by a selection bias—halo triaxiality and large-scale structure along the line of sight simultaneously boost the lensing signal and richness (the inferred number of galaxies associated with a cluster). As a result, a cluster sample selected by richness has a mean lensing signal higher than expected from its mean mass, and the inferred mass will be biased high. This selection bias is currently limiting the accuracy of cosmological parameters derived from optical clusters. In this paper, we quantify the bias in mass calibration due to this selection bias. Using two simulations, MiniUchuu and Cardinal, with different galaxy models and cluster finders, we find that the selection bias leads to an overestimation of lensing mass at a 20 – 50% level, with a larger bias (20 – 80%) for large-scale lensing (> 3 Mpc). Even with a conservative projection model, the impact of selection bias significantly outweighs the impact of other currently known cluster lensing systematics. We urge the cluster community to account for this bias in all future optical cluster cosmology analyses, and we discuss strategies for mitigating this bias.

I. INTRODUCTION

The number counts of galaxy clusters as a function of mass across cosmic time are sensitive to the mean matter density Ω_m , the density fluctuation parameter σ_8 , and dark energy [1–6]. Wide-field optical imaging surveys [7–11] simultaneously identify clusters by their galaxy content and provide weak gravitational lensing signals for mass calibration, or, equivalently, the calibration of the observable–mass relation [12–17]. These surveys can identify clusters down to low masses, providing statistical power comparable to that of cosmic shear [6, 18–20]. In cluster cosmology analyses, the accuracy of cosmological constraints depends on the accuracy of the mass calibration [21–24].

Currently, the accuracy of mass calibration for optically selected clusters is limited by a selection bias caused by line-of-sight projection effects [25–28], described below. In optical surveys, we assign each cluster a *richness* value λ , defined as the membership-weighted number of galaxies associated with a cluster. In the presence of distance uncertainties, galaxies physically unassociated with the cluster but along the line of sight can be mistaken as cluster members and boost cluster richness [29–33]. At the same time, the lensing signal of clusters is determined by the projected matter density [34–36]. As a result, if a cluster happens to have an excess of galaxies and matter along the line of sight (due to halo triaxiality or filaments), its richness and lensing signals will be coherently boosted. If we select clusters above a richness threshold, we preferentially select clusters with boosted lensing. In addition,

since low-mass halos are much more abundant than high-mass halos, clusters in a richness bin tend to be dominated by low-mass halos with up-scattered richness and lensing. As a result, the lensing signal will be biased high compared to a mass-selected sample [27, 28, 37].

In this paper, we use simulated cluster catalogs to quantify the impact of this projection-induced selection bias on weak lensing mass calibration. We use two mock catalogs with different galaxy–halo connection models and cluster finders. The MiniUchuu-based mock uses a halo occupation distribution (HOD) model [38–41] and a counts-in-cylinder cluster finding algorithm [27, 31, 42, 43], while the Cardinal mock [44] is based on the Addgals (Adding Density Dependent GALaxies to Lightcone Simulations) galaxy model [45] and the redMaPPer (red-sequence Matched-filter Probabilistic Percolation) cluster finder [46, 47]. We simulate the stacked lensing signal and perform a mock mass calibration. Our results indicate that the selection bias leads to 20 – 50% bias in mass calibration, making it the dominant systematic uncertainty in optical cluster cosmology.

This paper is organized as follows. We describe the basic formalism of cluster lensing in Sec. II and present the two cluster simulations in Sec. III. Section IV presents our results of weak lensing mass calibration. We summarize and discuss our results in Sec. V. Throughout this paper, Mpc refers to physical Mpc (used in lensing measurements and fitting), while cMpc refers to comoving Mpc (used in N -body simulations).

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II. BASIC FORMALISM FOR CLUSTER LENSING

In this section, we describe the basic formalism for cluster lensing, following [48] and the `cluster_toolkit` software¹. We use the halo–matter correlation function ξ_{hm} to model the mass distribution around clusters. The 1-halo term is described by the Navarro–Frenk–White [49] profile

$$\xi_{1\text{h}}(r, M) = \frac{\rho_{\text{NFW}}(r, M)}{\rho_{\text{m}}} - 1 = \frac{\delta_{\text{m}}}{(r/r_s)(1 + r/r_s)^2} - 1, \quad (1)$$

where r is the 3D distance to the halo center, and ρ_{m} is the mean matter density of the Universe. Here r_s is the scale radius, which is related to halo concentration via $c_{\Delta} = R_{\Delta}/r_s$ for a given spherical overdensity mass definition ($\Delta = 200\text{m}$ or vir); δ_{m} is the characteristic overdensity set by the concentration and the total mass of the halo [50].

The 2-halo term is modeled by the matter correlation function and a linear halo bias

$$\xi_{2\text{h}}(r, M) = b(M)\xi_{\text{mm}}(r), \quad (2)$$

where $b(M)$ is the halo bias calculated with the fitting formula in [51]. The matter correlation function ξ_{mm} is calculated with the `halofit` [52] non-linear matter power spectrum from `class` [53]. For the 1-halo and 2-halo transition, we follow [54],

$$\xi_{\text{hm}} = \max(\xi_{1\text{h}}, \xi_{2\text{h}}). \quad (3)$$

For a given halo–matter correlation function, we compute the surface density profile

$$\Sigma(r_p) = \rho_{\text{m}} \int_{-\infty}^{+\infty} \xi_{\text{hm}}\left(r = \sqrt{r_p^2 + \pi^2}\right) d\pi, \quad (4)$$

where r_p is the 2D projected distance to the halo center, and π is the distance along the line of sight.

The weak lensing signal is given by the excess surface mass density profile

$$\Delta\Sigma(r_p) = \bar{\Sigma}(< r_p) - \Sigma(r_p), \quad (5)$$

where the first term on the right is the mean surface density within r_p ,

$$\bar{\Sigma}(< r_p) = \frac{2}{r_p^2} \int_0^{r_p} r'_p \Sigma(r'_p) dr'_p. \quad (6)$$

To simulate the lensing signal of source galaxies in radial bins, we use the radially averaged profiles

$$\overline{\Delta\Sigma} = \frac{2}{r_{p,1}^2 - r_{p,2}^2} \int_{r_{p,1}}^{r_{p,2}} r'_p \Delta\Sigma(r'_p) dr'_p. \quad (7)$$

III. SIMULATIONS FOR OPTICAL CLUSTER LENSING

We describe the two complementary cluster simulations used in this work: MiniUchuu-based (III A) and Cardinal mocks (III B). The former allows us to test a specific projection model, while the latter lets us study realistic red-sequence cluster finding.

Both simulations are built on N -body simulations. The former simulates galaxies using an HOD model, while the latter simulates galaxies using the Addgals algorithm [45]. The former simulates cluster finding using a conservative distance uncertainty of $30 h^{-1}\text{cMpc}$, while the latter runs the full redMaPPer cluster finder [46, 47], which uses matched filters on galaxy colors, luminosities, and positions to find clusters.

A. MiniUchuu-based mock catalog

MiniUchuu is an N -body simulation from the publicly available Uchuu suite [55]², generated with the 2LPTIC initial condition code [56] and the GREEM N -body code [57]. It has 2560³ particles within a box size of $400 h^{-1}\text{cMpc}$, a dark matter mass resolution of $3.27 \times 10^8 h^{-1}M_{\odot}$, and a force resolution of $4.27 h^{-1}\text{kpc}$. The dark matter halos are identified using the Rockstar halo finder [58]. We focus on the $z = 0.3$ snapshot and use the halo mass definition $M_{200\text{m}}$, a spherical overdensity 200 times the mean matter density of the Universe. We only use isolated halos and exclude subhalos. MiniUchuu is based on a flat ΛCDM cosmology from *Planck* [59]: $\Omega_{\text{m}} = 0.3089$, $\Omega_{\text{b}} = 0.0486$, $\sigma_8 = 0.8159$, $n_s = 0.9667$, and $h = 0.6774$.

To build the galaxy catalog, we use a simple HOD model [40] to assign galaxies to halos in MiniUchuu. For halos with mass above $10^{12} h^{-1}M_{\odot}$, we assign a central galaxy to the halo center. For the number of satellite galaxies, we draw an N_{sat} value based on a Poisson distribution with mean

$$\langle N_{\text{sat}} | M \rangle = \left(\frac{M - M_0}{M_1} \right)^{\alpha}. \quad (8)$$

We use the parameter values from [27], which are designed to reproduce the DES cluster abundance: $M_0 = 10^{11.7} h^{-1}M_{\odot}$, $M_1 = 10^{12.9} h^{-1}M_{\odot}$, and $\alpha = 1$. The satellite positions are drawn based on an NFW profile [49], with the concentration–mass relation from [60].

The next step is to build a mock cluster catalog from these galaxies. We assume clusters are centered on dark matter halos with mass above $10^{12.5} h^{-1}M_{\odot}$. To simulate the observed cluster richness, we use counts-in-cylinders to model the distance uncertainties [25–27]; that is, λ is the total number of galaxies within a cylinder along the line of sight. We use a cylinder length of $\pm 30 h^{-1}\text{cMpc}$ based on the findings in [28]. This length is a conservative choice, and the actual projection effects may be stronger [30, 31, 61]. To take into account that richer clusters tend to have larger radii, we determine cluster

¹ <https://cluster-toolkit.readthedocs.io/>

² <https://www.skiesanduniverses.org/Simulations/Uchuu/>

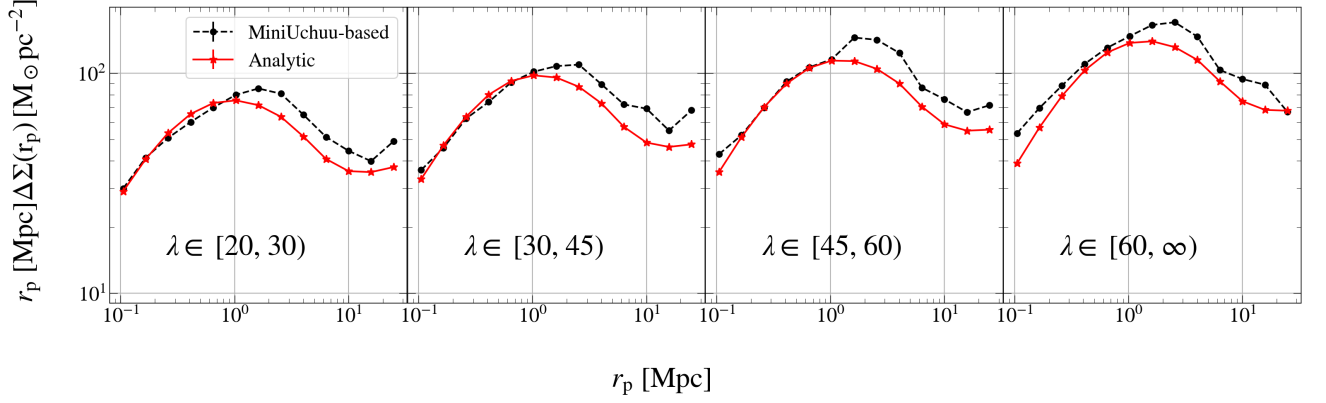


FIG. 1. Stacked lensing signal derived from the MiniUchuu-based mock (black) in four richness bins at $z = 0.3$, with richness modeled by counts-in-cylinders ($\pm 30 h^{-1} \text{cMpc}$ distance uncertainties). The red curves show the analytic predictions based on the mean mass and concentration of halos in each bin. The mean lensing signals of richness-selected clusters are biased high, and the bias is higher on large scales than on small scales.

richness and radius by iteratively solving

$$R_\lambda = \left(\frac{\lambda}{100} \right)^{0.2} h^{-1} \text{Mpc}, \quad (9)$$

an optimal relation found in [62]. If a galaxy is simultaneously contained in the cylinders of multiple halos, we only assign it to the most massive one to avoid double-counting galaxies.

We put our clusters in four richness bins: $[20, 30)$, $[30, 45)$, $[45, 60)$, and $[60, \infty)$. For each richness bin, we calculate the cross-correlation between clusters and dark matter particles to derive the lensing signal $\Delta\Sigma$, using the `corrfunc` software [63] with a projection depth $\pm 100 h^{-1} \text{cMpc}$. This depth is larger than the $\pm 30 h^{-1} \text{cMpc}$ used for cluster finding. Increasing this depth further has a negligible effect on the lensing signal. Figure 1 shows the lensing signal derived from MiniUchuu. In each panel, the black curve shows the MiniUchuu lensing signal in a given richness bin, and the red curve shows the analytical prediction based on the mean mass and concentration of the halos in that bin. The MiniUchuu results are consistent with the analytic expectation below $\lesssim 1 \text{ Mpc}$ but are higher at larger scales. In Sec. IV, we will show how this biased lensing signal leads to a biased mass calibration.

B. Cardinal

Cardinal [44] is a suite of lightcone mock catalogs designed to reproduce observed cluster richness and clustering and is thus uniquely suitable for our study. We use the mock designed for the Dark Energy Survey (DES) with an area of $\approx 5000 \text{ deg}^2$. We use the redshift range $0.2 < z < 0.65$, which is constructed from two N -body simulations: The $z < 0.315$ lightcone has a mass resolution $3.3 \times 10^{10} h^{-1} M_\odot$ and a force resolution $20 h^{-1} \text{kpc}$; the $z > 0.315$ lightcone has a mass resolution $1.6 \times 10^{11} h^{-1} M_\odot$ and a force resolution $35 h^{-1} \text{kpc}$. We use the virial mass definition based on the $\Delta_{\text{vir}}(z)$ from [64] and focus on halos with mass above $10^{13} h^{-1} M_\odot$. Cardinal is

based on a flat ΛCDM cosmology: $\Omega_m = 0.286$, $\Omega_b = 0.047$, $\sigma_8 = 0.82$, $n_s = 0.96$, and $h = 0.7$.

The galaxy positions, magnitudes, and colors are modeled with the Addgals algorithm [45]. The Addgals algorithm calibrates the luminosity–density relation and luminosity–halo mass relation using a high-resolution mock catalog generated with subhalo abundance matching. These relations are then used to populate galaxies in large-volume low-resolution simulations. Each galaxy is then assigned colors based on its distance to massive halos. Cardinal [44] additionally accounts for tidal stripping in the subhalo abundance matching process, which improves the modeling of galaxies in clusters; as a result, Cardinal can reproduce the observed richness and cluster–galaxy cross-correlation functions.

Because Cardinal has the realistic galaxy colors, the redMaPPer cluster finder can be directly applied to it. The redMaPPer algorithm uses the red sequence (the tight color–magnitude relation for galaxies in clusters) to identify clusters and determine their photometric redshifts. It iteratively calibrates the red-sequence model (the color–magnitude relation as a function of redshift) and identifies clusters [46, 47]. We use the redMaPPer catalog with clusters centered on dark matter halos to avoid misidentified cluster centers.

We put our clusters in 3 redshift bins: $[0.2, 0.35)$, $[0.35, 0.5)$, and $[0.5, 0.65)$, and 4 richness bins: $[20, 30)$, $[30, 45)$, $[45, 60)$, and $[60, \infty)$. For clusters in each bin, we calculate the average of the $\Delta\Sigma$ profiles to simulate the stacked lensing signal. Similar to MiniUchuu, we use the dark matter particles from Cardinal’s N -body simulations, again using a projection depth of $\pm 100 h^{-1} \text{cMpc}$.

Figure 2 shows the stacked lensing profiles derived from the Cardinal redMaPPer catalog. Similar to the MiniUchuu calculation, we add an analytical model prediction based on the mean mass and concentration in each bin. Compared with MiniUchuu, the Cardinal cluster lensing signal shows a stronger bias on most scales.

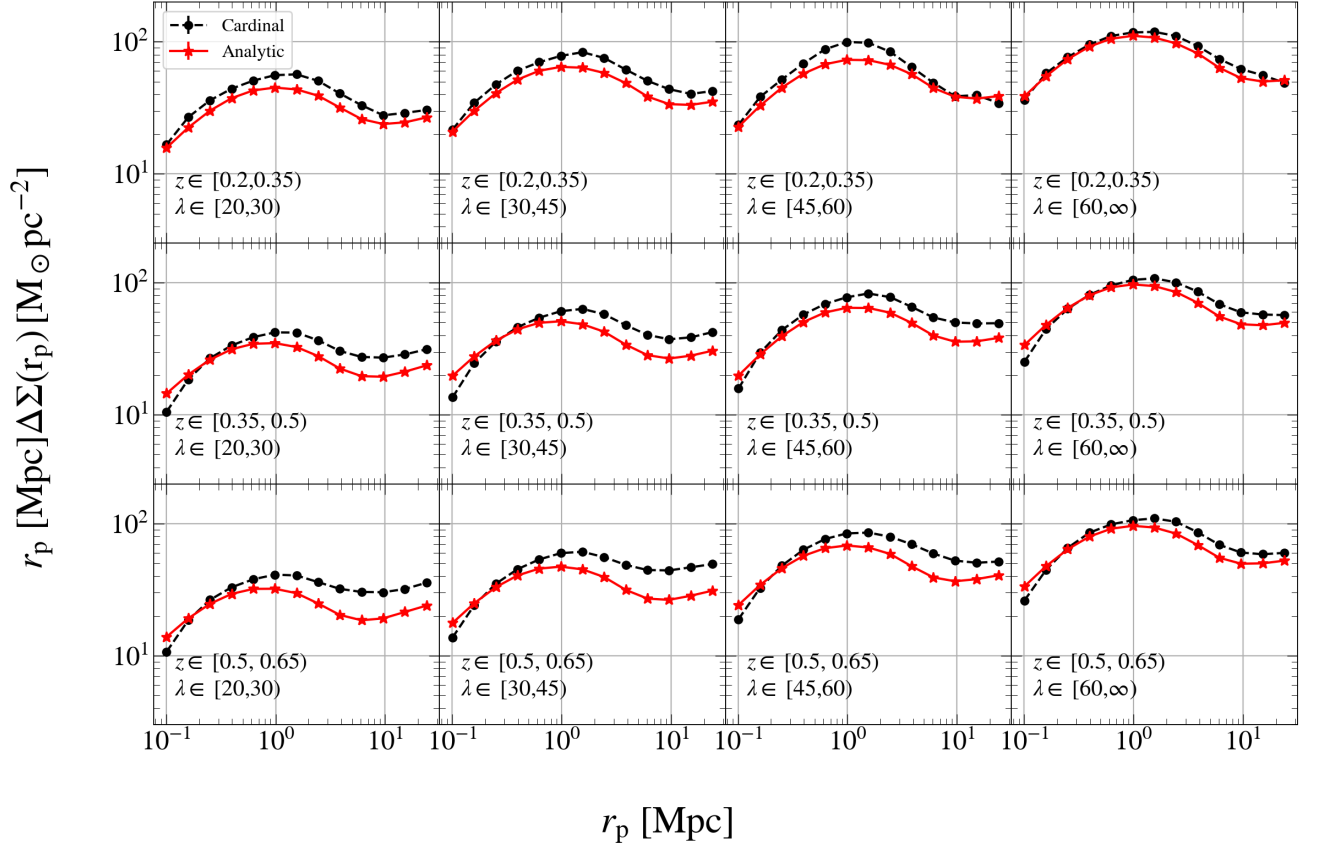


FIG. 2. Analogous to Fig. 1 but for Cardinal, based on the Addgals galaxy model and the redMaPPer cluster finder. The lensing bias persists across all scales and is stronger than that shown in Fig. 1.

IV. MASS CALIBRATION IN THE PRESENCE OF PROJECTION EFFECTS

In this section, we present our mock mass calibration analyses. For a redshift and richness bin k , we use the following likelihood function to perform the fitting:

$$\ln \mathcal{L}(\Delta \Sigma_k | M, c) \propto -\frac{1}{2} \mathbf{d}_k^T \mathbf{C}^{-1} \mathbf{d}_k, \quad (10)$$

where $\mathbf{d}_k = (\Delta \Sigma_{\text{data}} - \Delta \Sigma_{\text{model}})_k$. We use Eq. (7) for $\Delta \Sigma_{\text{model}}$, and the simulated lensing for $\Delta \Sigma_{\text{data}}$. For the covariance matrix $\mathbf{C}$, we use the semi-analytic covariance matrix of DES Year 1 cluster analyses presented in [22], which corresponds to a survey area of $1,437 \text{ deg}^2$ and a source density of 6.28 galaxies per arcmin^2 . Using this covariance matrix allows us to assess the mass bias present in the DES cluster mass calibration [22]. This covariance matrix also provides a conservative estimate for statistical uncertainties, which will be reduced in future analyses.

Following the DES analyses [22, 65], for both MiniUchuu and Cardinal, we measure cluster lensing using 15 logarithmically spaced bins between 0.03 and 30 Mpc. For mass fitting, we use the 11 bins between 0.2 and 30 Mpc. To investigate the impact of projection effects on large-scale and small-scale analyses, we split our lensing data vector into small scales (0.2

to 3 Mpc) and large scales (3 Mpc to 30 Mpc).

We use two free parameters, $\log_{10} M$ and c , in our fitting, and we assume flat priors of [12, 16] and [2, 10], respectively. We use the *emcee* software for Markov chain Monte Carlo [66], with 32 walkers for 10,000 iterations and discarding the first 1,000 iterations as burn-in. Figure 3 shows the difference in the posterior mean mass M_{obs} and the true mean mass M_{true} . The error bars show the 68% credible interval. We show the difference in natural log, which can be interpreted as the fractional difference.

The left-hand panel of Fig. 3 shows the MiniUchuu results. For small scales, [0.2, 3] Mpc, the mass is biased high by $\approx 25\%$. For large scales, [3, 30] Mpc, the bias is $\approx 50\%$. Since the small-scale lensing dominates the statistical power, combining all scales gives a bias of $\approx 25\%$, similar to small scales. For MiniUchuu, the mass bias is nearly independent of richness.

The right-hand panel of Fig. 3 shows the Cardinal results, which have a larger bias than MiniUchuu and show a richness trend. For small scales, the bias is 20 – 40%; for large scales, the bias is 20 – 80%. Combining all scales gives a bias of 20 – 50%, similar to the small-scale analysis. The richness and redshift trend is the strongest for the large-scale analysis, with an 80% bias at $z \geq 0.5$ and $\lambda \approx 20$. This redshift and richness trend is consistent with the findings in [32, 33].

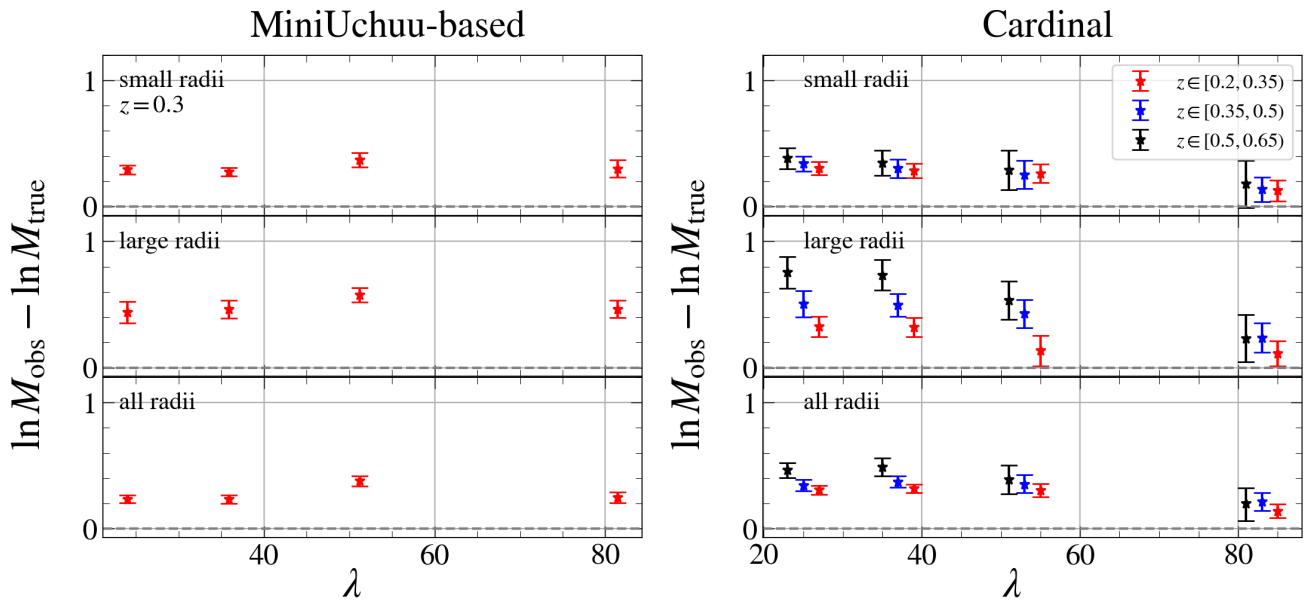


FIG. 3. Fractional mass bias resulted from projection-induced selection bias, for the MiniUchuu mock (left) and Cardinal (right). We fit the analytic model to the mock lensing signals. The points and error bars correspond to the posterior means and 68% credible intervals. The top panels use radial range [0.2 Mpc, 3 Mpc], the middle panels use [3 Mpc, 30 Mpc], while the bottom panels combine both radial ranges. Cardinal tends to have larger mass biases than MiniUchuu. For Cardinal, the bias is higher for high-redshift and low-richness clusters.

MiniUchuu and Cardinal are based on different cosmological parameters, but they have similar σ_8 (0.8159 vs. 0.82). Therefore, the difference in mass bias is not due to different matter clustering but rather different galaxy–halo connection and cluster finding.

V. SUMMARY AND DISCUSSION

A cluster sample selected by optical richness tends to have a mean lensing signal higher than expected from its mean mass, and this bias is caused by the coherent boost of richness and lensing due to projection effects. To assess the impact of this selection bias on cluster mass calibration, we have performed mock mass calibrations using two simulations with different cluster finders: The MiniUchuu-based mock simulates a $30 h^{-1} \text{cMpc}$ distance uncertainty using counts-in-cylinders, and the Cardinal mock uses the redMaPPer cluster finder. We have found a 25% bias in the former and 20 – 50% in the latter, and that large-scale-focused mass calibration ($r_p > 3 \text{ Mpc}$) has a larger bias compared with the small-scale-focused mass calibration. The Cardinal results show that the bias is larger for high-redshift and low-richness clusters.

Our findings indicate that the projection-induced selection bias is the most detrimental among the known systematic biases impacting cluster mass calibration. Other known systematics include the impact of baryons on the mass distribution [67–75], the offset between the inferred cluster centers and the true halo centers (miscentering) [76–79], and the diluted lensing signal due to cluster members (the so-called boost factor) [65, 77]. Ref. [80] has assessed the impact of these systematics

on the weak lensing mass bias for ~ 1000 clusters selected in mm-wave by the South Pole Telescope, which is unaffected by the optical selection bias discussed in this paper. Their weak lensing mass bias is dominated by baryonic effects (for $z \lesssim 0.4$) and the photometric redshift calibration of source galaxies (for $z \gtrsim 0.4$); see their Table IV and Fig. 10. These systematics lead to a mass bias around 5 – 10%; their highest bias is 16%, which occurs at $z \approx 1$. These systematics also impact optically selected clusters, but they are subdominant compared with the optical selection bias discussed in this paper.

How do we constrain the projection-induced selection bias using observational data? Spectroscopic redshifts of member galaxies can help us separate projected vs. physically-associated cluster members [30, 33]. Cross-correlating clusters with spectroscopic galaxies can help us calibrate the redshift distribution of cluster members [73, 81]. Ref. [29] also shows how to use photometric galaxies to calibrate projection effects. Combining cluster lensing and cluster–galaxy cross correlations can help us self-calibrate the selection bias, which boosts all cluster-related 2-point statistics [17, 42, 82, 83]. Multiwavelength analyses comparing cluster lensing signals of optically selected clusters and gas-selected clusters (observed in mm-wave or x-ray) can help us calibrate the lensing bias [84]. In sum, we advocate a multiprobe, multiwavelength approach for cluster cosmology. Future modeling strategies should focus on self-consistently modeling the impact of projection on richness, lensing, and gas observables [25, 85, 86].

How do we treat the selection bias in optical cluster cosmology analyses? Recent studies have modeled the selection bias as functions of radius, richness, and redshift [14, 17, 87, 88].

Another promising route is to use N -body simulations to forward-model richness and lensing, an approach that automatically takes into account the selection bias [31, 43, 89]. Since the mass bias depends on the galaxy model, the cluster finding, and the radial range used, we will need a flexible model, which can be achieved with simulation-based forward modeling. We anticipate that the simulation-based forward modeling has the potential to help us realize the constraining power of optical clusters.

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