SHIMURA CURVE ATKIN–LEHNER QUOTIENTS OF GENUS AT MOST TWO

OANA PADURARIU AND FREDERICK SAIA

Abstract. We provide a complete enumeration of all quotients of genus $0, 1$ and 2 of the Shimura curves $X_0^D(N)$ over $\mathbb{Q}$ by non-trivial subgroups of Atkin–Lehner involutions. For all 1270 genus 1 quotients X with N squarefree, we determine the isomorphism class of the Jacobian X. For 146 non-elliptic genus 1 quotients X and for 405 bielliptic genus 2 quotients X, we provide a defining equation for X. A main tool for us is the theory of Čerednik–Drinfeld uniformizations of the curves $X_0^D(N)$, which we implement in wider generality than has previously been done in the literature.

1. Introduction

Since Mazur's work on isogenies of elliptic curves over $\mathbb{Q}$ [Maz78], there has been sustained interest in classifying those modular curves $X_0(N)$ over $\mathbb{Q}$ which possess infinitely many points of specified degree d. The cases of $d = 2$ and $d = 3$ were respectively handled by Bars [Bar99] (combined with work of Ogg [Ogg74]) and by Jeon [Jeo21]. The $d = 4$ case was recently handled independently by Hwang–Jeon [HJ24] and by Derickx–Orlić [DO24].

Whereas the curves $X_0(N)$ parameterize elliptic curves with additional isogeny structure, the Shimura curves $X_0^D(N)$ over $\mathbb{Q}$, with $\gcd(D, N) = 1$ and with $D > 1$ the discriminant of an indefinite quaternion algebra over $\mathbb{Q}$, parameterize abelian surfaces with quaternionic multiplication and additional structure. These Shimura curves have no odd-degree points (see Theorem 2.1), so the first degree to consider is $d = 2$. In this case, an analogue of Bars' result on infinitude of quadratic points was proven for the curves $X_0^D(N)$ with $D > 1$ by the current authors in [PS25a], generalizing a result of Rotger in the $N = 1$ case [Rot02].

Given the motivation of classifying torsion structures of abelian surfaces, the same classification problem for quotients of the Shimura curves $X_0^D(N)$ by subgroups of the full Atkin–Lehner group $W_0(D, N) \leq \mathrm{Aut}(X_0^D(N))$ is equally worthy of attention; these quotients parameterize abelian surfaces with *potential* quaternionic multiplication and additional structure. Additionally, these quotients are themselves significant in studying infinitude of points of fixed degree for the curves $X_0^D(N)$, as they provide subcovers which can act as sources of such points. For example, from [PS25a] we know that each Shimura curve $X_0^D(N)$ with infinitely many quadratic points has an Atkin–Lehner quotient $X_0^D(N)/\langle w_m \rangle$ of genus 0 or 1 with infinitely many rational points.

In this work, we initiate a study of low-degree points on Atkin–Lehner quotients of the Shimura curves $X_0^D(N)$ by focusing on low-genus quotients. Our first main result is a determination of all Atkin–Lehner quotients of genus at most 2.

Theorem 1.1. *There are exactly* 3711 *Shimura curves quotients* $X_0^D(N)/W$*, with* $D > 1$ *and with* $W \leq W_0(D, N)$ *a non-trivial subgroup, of genus at most* 2*. In particular, there are*

- 779 *curves* $X_0^D(N)/W$ *having genus* 0*,*

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- 1352 *curves* $X_0^D(N)/W$ *having genus* 1*, and*
- 1580 *curves* $X_0^D(N)/W$ *having genus* 2*.*

Each quotient of genus 0 is either a pointless conic over $\mathbb{Q}$, or has a rational point and is therefore isomorphic to $\mathbb{P}^1_{\mathbb{Q}}$. Each genus 1 quotient either has a rational point, and is thus an elliptic curve isomorphic to its Jacobian, or is a non-elliptic curve of genus 1. We determine Jacobians in the genus 1 case when N is squarefree, and we determine them up to isogeny when N is not squarefree.

Theorem 1.2. *Let* $X = X_0^D(N)/W$ *be a genus* 1 *curve with* $W \leq W_0(D,N)$ *a non-trivial Atkin–Lehner subgroup.*

(1) If X *is among the* 1270 *such curves with* N *squarefree, then the isomorphism class of* $\mathrm{Jac}(X)$ *is as given in Table 2.*
(2) If X *is among the* 82 *such curves with* N *not squarefree, then the isogeny class of* $\mathrm{Jac}(X)$ *is as given in Table 3.*

From Theorem 1.1 and Theorem 1.2, combined with investigations regarding the existence of rational points on Atkin–Lehner quotients of genera 0 and 1, we reach the following result regarding infinitude of rational points.

Corollary 1.3. *(1) For each of the* 650 *triples* (D,N,W) *listed in Table 4, the curve* $X_0^D(N)/W$ *is isomorphic to* $\mathbb{P}^1_{\mathbb{Q}}$*.*
(2) For each of the 537 *triples* (D,N,W) *listed in Table 5, the curve* $X_0^D(N)/W$ *is an elliptic curve with positive rank.*
(3) If $X_0^D(N)/W$ *has infinitely many rational points and* (D,N,W) *is not in Table 4 or Table 5, then* (D,N,W) *must be among the* 48 *triples listed in Table 6 or among the* 54 *triples listed in Table 7.*

Proof. Part (1) follows from our determination that the listed curves have rational points, which comes in Section 4.2. Part (2) follows from rational points investigations for genus 1 quotients which appear in Section 4.2 and Section 7.2, along with rank computations in Magma for the isogeny classes of the Jacobians of these curves (as given by Theorem 1.2).

It follows from Faltings' Theorem that if $X_0^D(N)/W$ has infinitely many rational points, then it must be isomorphic to $\mathbb{P}^1_{\mathbb{Q}}$ or be an elliptic curve over $\mathbb{Q}$ of positive rank. If this is the case and (D,N,W) does not already appear in one of the tables referenced in parts (1) and (2), then $X_0^D(N)$ must either be

- a genus 0 curve for which we remain unsure of the existence of a rational point, in which case (D,N,W) appears in Table 6, or
- a genus 1 curve with positive rank Jacobian for which we remain unsure of the existence of a rational point, in which case (D,N,W) appears in Table 7. □

We also produce models for hundreds of Atkin–Lehner quotients of Shimura curves. To our knowledge, of the models referenced in the following theorem fewer than 70 have appeared in prior work (including [Kur79, GR04, GR06, GY17, PS23]).

Theorem 1.4. *(1) Each of the curves* $X = X_0^D(N)/W$ *listed in Table 8 is non-elliptic of genus* 1*. For the* 4 *triples* (D,N,W) *in the set*

$$\Big\{ (14, 11, \langle w_2, w_{11}\rangle), (46, 3, \langle w_2, w_3\rangle), (570, 1, \langle w_5, w_{57}\rangle), (570, 1, \langle w_6, w_{190}\rangle) \Big\},$$

the curve X *admits exactly one of the two models given in Table 8 over* $\mathbb{Q}$*. Each of the remaining* 146 *curves* X *in Table 8 admits the given model over* $\mathbb{Q}$*.*

(2) *For the* 405 *curves* $X = X_0^D(N)/W$ *listed in Table 9, we have that* X *is bielliptic of genus* 2 *and admits the given model over* $\mathbb{Q}$*.*

A main tool in all parts of this study, following the initial enumeration of quotients of genus at most 2, is the theory of Čerednik–Drinfeld p-adic uniformizations of the Shimura curves $X_0^D(N)$ for primes $p \mid D$. We implement an algorithm to determine the structure of the dual graph of the model over $\mathbb{Z}_p$, for $p \mid D$, of an Atkin–Lehner quotient $X_0^D(N)/W$ with N squarefree, from which we are able to compute reduction types of such quotients at primes dividing D (see Algorithm 3.1) and to check for the existence of local points at primes dividing D (see Algorithm 3.3). The former referenced algorithm is used in our proof of Theorem 1.2, the latter is used in determining the existence of rational points for Corollary 1.3, and both are integral in our results on equations of Atkin–Lehner quotients.

Explicit computations using the theory of Čerednik–Drinfeld in this context in past works (e.g., [Kur79, Ogg85, GR06]) seem to have been limited either to level $N = 1$ or to quotients by single Atkin–Lehner involutions. It is our hope that our implementation, along with the extensive computations and examples provided in this work, are helpful to readers wishing to learn or use this theory.

The remainder of this work is organized as follows.

- In Section 2, we provide background material on Shimura curves, their Atkin–Lehner quotients, and the theory of Čerednik–Drinfeld uniformizations of these curves.
- We state in Section 3 the algorithms referenced above.
- Section 4 features the proof of Theorem 1.1, as well as a first pass concerning rational points on quotients of genus 0 and 1 in Section 4.2.
- We determine isogeny classes of Jacobians genus 1 in Section 5, and subsequently determine isomorphism classes of these Jacobians when N is squarefree in Section 6. The main results of these two sections combine to give Theorem 1.2.
- Models for non-elliptic genus 1 curves are determined in Section 7. We also briefly return to the question of rational points here (Section 7.2) in order to prove several genus 1 quotients have rational points in a non-constructive manner. We produce equations for bielliptic genus 2 Atkin–Lehner quotients in Section 8. Theorem 1.4 follows from the main results of these two sections, Theorem 7.2 and Theorem 8.3.

All computations described in this paper were performed in Magma [BCP97]. All related code, as well as computed lists, can be found in the Github repository [PS25b].

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2. Background

2.1. **The Shimura curves $X_0^D(N)$.** A **quaternion algebra** over a field F is a central simple algebra over F of dimension 4. For example, we always have the split quaternion algebra $M_2(F)$ over F. For a quaternion algebra B over $\mathbb{Q}$, we call B **indefinite** if it is split over the real numbers, i.e., if

$$B_\infty := B \otimes_{\mathbb{Q}} \mathbb{R} \cong M_2(\mathbb{R}),$$

and otherwise we call B **definite**. A quaternion algebra B over $\mathbb{Q}$ is determined up to isomorphism by its **discriminant** D, which is the product of primes p such that B is ramified (non-split) over $\mathbb{Q}_p$. This discriminant D is the product of an even or odd number of distinct prime numbers according to whether B is indefinite or definite, respectively.

Fix D the discriminant of an indefinite quaternion algebra B over $\mathbb{Q}$ and N a positive integer with $\gcd(D, N) = 1$. We let $\mathcal{O}_N$ be a fixed **Eichler order** in B of level N. That is, $\mathcal{O}_N$ is the intersection of two (not necessarily distinct) maximal orders in B, and can be defined locally at each finite place of $\mathbb{Q}$ as follows:

- If $p \mid D$, then $B_p := B \otimes_{\mathbb{Q}} \mathbb{Q}_p$ is non-split (is a division field) and the localization $\mathcal{O}_{N,p} := \mathcal{O}_N \times_{\mathbb{Z}} \mathbb{Z}_p$ is taken to be the unique maximal order of B_p.
- Otherwise, if $p \nmid D$, we fix an isomorphism $\psi_\ell : B_p \to M_2(\mathbb{Q}_p)$ and we take $\mathcal{O}_{N,p}$ such that

$$\psi_p\left(\mathcal{O}_{N,p})\right) = \left\{ \begin{pmatrix} a & b \\ Nc & d \end{pmatrix} \mid a, b, c, d \in \mathbb{Z}_p \right\}.$$

Fixing an isomorphism $\psi_\infty : B_\infty \to M_2(\mathbb{R})$, we have from this map an action of B_∞ on the double half-plane $\Omega := \mathbb{C} \setminus \mathbb{R}$. We have a Riemann surface $X = \mathcal{O}_N^\times \backslash \Omega$ which is determined up to isomorphism by D and N (in particular, does not depend on our choice of Eichler order), and it is a theorem of Shimura [Shi67, Main Theorem 1] that X has a canonical model defined over $\mathbb{Q}$ which we denote by $X_0^D(N)$. By work of Drinfeld (see [BC91, Buz97]), $X_0^D(N)$ admits a flat proper model over $\mathbb{Z}$ which is smooth over $\mathbb{Z}\left[\frac{1}{DN}\right]$. In particular, $X_0^D(N)$ has good reduction at all primes $p \nmid DN$.

If $D = 1$ then this construction recovers the (non-compact, in our setup) coarse moduli scheme $Y_0(N)$ parameterizing cyclic N-isogenies of elliptic curves. If $D > 1$, then $X_0^D(N)$ is compact and parameterizes abelian surfaces with quaternionic multiplication by the order $\mathcal{O}_N$. We refer to [Sai24, Proposition 2.4] for other moduli descriptions, including one involving isogenies of QM abelian surfaces, which are equivalent up to isomorphism of the associated coarse moduli scheme $X_0^D(N)$. In this work, while we are highly motivated by questions related to torsion structures of abelian surfaces, these moduli interpretations are not important ingredients to our study.

In the compact case, it is a result of Shimura that these curves have no real points:

Theorem 2.1. [Shi75, Theorem 0] *If $D > 1$, then $X_0^D(N)(\mathbb{R}) = \emptyset$.*

We have a natural covering map $\pi_N : X_0^D(N) \to X_0^D(1)$ of degree $\psi(N)$ given by the Dedekind psi function:

$$\psi(N) = N \prod_{\substack{p \mid N \\ \text{prime}}} \left(1 + \frac{1}{p}\right).$$

These covering maps are compatible, in the sense that for $N_1 \mid N_2$ there is a natural map $\pi : X_0^D(N_2) \to X_0^D(N_1)$ of degree $\psi(N_2)/\psi(N_1)$ such that $\pi_{N_1} \circ \pi = \pi_{N_2}$.

We have the following formula for the genus of the curve $X_0^D(N)$ (see, for example, [Voi21, Thm. 39.4.20]).

Proposition 2.2. *For $D > 1$ the discriminant of an indefinite quaternion algebra over $\mathbb{Q}$, for N a positive integer coprime to D and for $k \in \{3,4\}$, define*

$$e_k(D,N) := \prod_{\substack{p \mid D \\ prime}} \left(1 - \left(\frac{-k}{p}\right)\right) \prod_{\substack{\ell \| N \\ prime}} \left(1 + \left(\frac{-k}{\ell}\right)\right) \prod_{\substack{\ell^2 \mid N, \\ \ell \ prime}} \delta_\ell(k),$$

where $\left(\frac{\cdot}{\cdot}\right)$ is the Kronecker quadratic symbol, and

$$\delta_\ell(k) = \begin{cases} 2 & \text{if } \left(\frac{-k}{\ell}\right) = 1, \\ 0 & \text{otherwise.} \end{cases}$$

Then, the genus of $X_0^D(N)$ is given by

$$g(X_0^D(N)) = 1 + \frac{\phi(D)\psi(N)}{12} - \frac{e_4(D,N)}{4} - \frac{e_3(D,N)}{3},$$

where ϕ denotes the Euler totient function.

2.2. **Atkin–Lehner involutions.** Let B be an indefinite quaternion algebra of discriminant D over $\mathbb{Q}$ and let $\mathcal{O}_N$ be an Eichler order of level N coprime to D in B. Consider the subgroup of the normalizer of $\mathcal{O}_N$ in B

$$N_{B^\times_{>0}}(\mathcal{O}_N) := \{u \in B^\times : \operatorname{nrd}(u) > 0 \text{ and } u^{-1}\mathcal{O}_N u = \mathcal{O}_N\}$$

consisting of elements of positive reduced norm. The group of **Atkin–Lehner involutions** of $X_0^D(N)$ is defined as

$$W_0(D,N) := N_{B^\times_{>0}}(\mathcal{O}_N)/\mathbb{Q}^\times \mathcal{O}_N^\times \leq \operatorname{Aut}(X_0^D(N)).$$

Letting $\omega(n)$ denote the number of distinct prime divisors of an integer n, $W_0(D,N)$ is a finite abelian 2-group of the form

$$W_0(D,N) = \{w_m : m \parallel DN\} \cong (\mathbb{Z}/2\mathbb{Z})^{\omega(DN)}.$$

Let us be a bit more concrete about the quaternion arithmetic here in a way that will be useful later, following [Ogg83] which in turn follows Eichler. Because B is indefinite, $\mathcal{O}_N$ has trivial class number and hence each left $\mathcal{O}_N$-ideal $\mathfrak{a}$ is principal; there exists $\alpha \in B^\times$ such that $\mathfrak{a} = \mathcal{O}_N \cdot \alpha$. The group $W_0(D,N)$ is realized as the group of two-sided ideals of $\mathcal{O}_N$ modulo the ideals of the form $x\mathcal{O}_N$ with $x \in \mathbb{Q}^\times$. For a Hall divisor $m \parallel DN$, we obtain the corresponding involution of $X_0^D(N)$ as follows: there exists a two-sided ideal $\mathfrak{b} = \beta\mathcal{O}_N$ with $\mathfrak{b}^2 = m \cdot \mathcal{O}_N$. Letting $\mathfrak{b}_p := \mathfrak{b} \otimes_{\mathbb{Z}} \mathbb{Z}_p$ denote the completion of $\mathfrak{b}$ at p, we have that β is determined locally as follows:

- $\mathfrak{b}_p = \mathcal{O}_{N,p}$ if $p \nmid m$,
- $\mathfrak{b}_p$ is the unique maximal ideal of $\mathcal{O}_{N,p}$ if $p \mid \gcd(m, D)$, and
- $\mathfrak{b}_p$ is associate to the ideal

$$\begin{pmatrix} 0 & 1 \\ p^{\operatorname{ord}_p(N)} & 0 \end{pmatrix} \cdot \psi_p\left(\mathcal{O}_{N,p}\right)$$

  if $p \mid N$.

The element $\beta \in B^\times$ normalizes $\mathcal{O}_N$, yielding the involution w_m of $X_0^D(N)$ over $\mathbb{Q}$.

For a subgroup $W \leq W_0(D,N)$, we call the quotient $X_0^D(N)/W$ an **Atkin–Lehner quotient** of $X_0^D(N)$. We also call the elements of $\operatorname{Aut}\left(X_0^D(N)/W\right)$ induced by the elements of $W_0(D,N)$ Atkin–Lehner involutions.

The fixed points of Atkin–Lehner involutions are determined by Ogg as being complex multiplication (CM) points by certain imaginary quadratic orders.

Proposition 2.3. [Ogg83, p. 283] *Let $m > 1$ with $m \parallel DN$. The fixed points of the Atkin–Lehner involution w_m acting on $X_0^D(N)$ are points with* CM *by the following imaginary quadratic orders:*

$$R = \begin{cases} \mathbb{Z}[i] \textit{ and } \mathbb{Z}[\sqrt{-2}] & \text{if } m = 2, \\ \mathbb{Z}\left[\frac{1+\sqrt{-m}}{2}\right] \textit{ and } \mathbb{Z}[\sqrt{-m}] & \text{if } m \equiv 3 \pmod 4, \\ \mathbb{Z}[\sqrt{-m}] & \text{otherwise.} \end{cases}$$

For each of the orders R listed for m in this proposition, the count of R-CM points fixed by w_m is given explicitly by Ogg as a product

$$h(R) \prod_{\substack{p \mid \frac{DN}{m} \\ \text{prime}}} \nu_p(R, \mathcal{O}_N),$$

where $h(R)$ is the class number of the order R and $\nu_p(R, \mathcal{O}_N)$ is the number of inequivalent optimal embeddings of the localization R_p of R at p into the localization $(\mathcal{O}_N)_p$ of $\mathcal{O}_N$ at p. The latter optimal embedding counts are given by [Ogg83, Theorem 2]. One can compute the genus of any Atkin–Lehner quotient using Proposition 2.3 and Proposition 2.2 with the Riemann–Hurwitz theorem, and such computations are implemented in the file `quot_genus.m` in [PS25b].

The following result of Ribet is an important tool in our study. Ribet's original result in [Rib90] contains a hypothesis that N is squarefree, but this hypothesis can be removed by work of Hijikata–Pizer–Shemanske [HPS89b, HPS89a] and so does not appear here.

Theorem 2.4. [Rib90, BD96, HPS89b, HPS89a] *There is an isogeny defined over $\mathbb{Q}$*

$$\Psi\colon \operatorname{Jac}(X_0(DN))^{D\text{-new}} \longrightarrow \operatorname{Jac}(X_0^D(N))$$

such that, for each Atkin–Lehner involution $w_m(D, N) \in W_0(D, N)$, the corresponding automorphism $\Psi^*(w_m(D, N))$ on $\operatorname{Jac}(X_0(DN))^{D\text{-new}}$ is

$$\Psi^*(w_m(D, N)) = (-1)^{\omega(\gcd(D,m))} w_m(1, DN), \tag{1}$$

where $w_m(1, DN) \in W_0(1, DN)$ denotes the Atkin–Lehner involution corresponding to m on the modular curve $X_0(DN)$.

2.3. **Čerednik–Drinfeld uniformizations.** For the remainder of this section, we fix $D > 1$ the discriminant of an indefinite quaternion algebra B over $\mathbb{Q}$, N a squarefree positive integer with $\gcd(D, N) = 1$, and p a prime divisior of D. We recall the main details of the theory of the p-adic uniformization of $X_0^D(N)$ and its Atkin–Lehner quotients at primes $p \mid D$ due to Čerednik–Drinfeld. The original sources for these results are [Čer76, Dri76], while our main references for this material are [Kur79, Ogg83], [BC91, Chapter 3], and [BD96, §1.7].

For clarity, let us first recall the notion of a graph (with lengths) that we will use, and related constructions, as given in [Kur79, §3].

Definition 2.5. A **graph** X consists of the following data:

- a set $\operatorname{Ver}(X)$ of **vertices** of X,
- a set $\operatorname{Ed}(X)$ of **oriented edges** of X,
- functions $o, t : \operatorname{Ed}(X) \to \operatorname{Ver}(X)$, and
- a function

$$\begin{aligned} \operatorname{Ed}(X) &\longrightarrow \operatorname{Ed}(X) \\ y &\longmapsto \overline{y} \end{aligned}$$

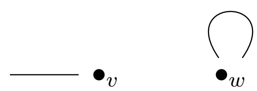
such that $\overline{\overline{y}} = y$ and $o(y) = t(\overline{y})$ for all $y \in \operatorname{Ed}(X)$.

For $y \in \mathrm{Ed}(X)$, we call $o(y)$ and $t(y)$ the **origin** and **terminal** vertices of y, respectively, and we call the oriented edge $\overline{y}$ the **inverse** of y. An **(unoriented) edge** is a set $\{y, \overline{y}\}$ with $y \in \mathrm{Ed}(X)$. If $\overline{y} = y$, we will refer to both y and $\{y\}$ as a **half-edge**.

A **graph with lengths** is a pair (X, f) such that X is as above and f is a function from $\mathrm{Ed}(X)$ to the set of positive integers such that $f(y) = f(\overline{y})$ for each $y \in \mathrm{Ed}(X)$. We call $f(y)$ the **length** of the edge $\{y, \overline{y}\}$.

Note that if y is a half edge, then necessarily $o(y) = t(y)$. In drawing graphs (with lengths), we will distinguish between a half-edge y and an edge z which is not a half edge but satisfies $o(z) = t(z)$ by drawing the former as a straight segment emanating from $o(y)$ and drawing the latter as a loop at $o(z)$, as in Figure 1.

Figure 1. A vertex v which is the origin of a single half-edge, and a vertex w which is the origin of an oriented edge z with $\overline{z} \neq z$.



If X is a graph, then we denote by X^* the graph obtained from X by removing all half-edges. If (X, f) is a graph with lengths, then we similarly get a graph with lengths $(X^*, f|_{\mathrm{Ed}(X^*)})$ via removal of half-edges.

If X is a graph and H is a group acting on X, then we form the **quotient graph** X/H in the usual way via identification of vertices and oriented edges under the action of H. Moreover, if (X, f) is a graph with lengths then we let

$$H_y := \{h \in H \mid h \cdot y = y\}$$

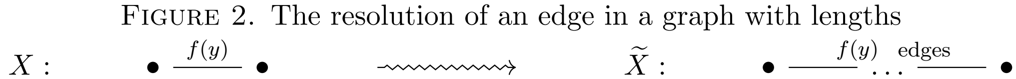
denote the stabilizer in H of a fixed oriented edge y of X and we have the **quotient graph with lengths** $(X/H, f_H)$ via

$$f_H(y) := f(y) \cdot \# H_y.$$

Given a graph X, we construct its **dual graph** $X^\vee$ as follows: the vertex set of $X^\vee$ is in bijection with the set of (unoriented) edges of X, and there will be an edge between two vertices of $X^\vee$ corresponding to edges e_1 and e_2 of X for each vertex in X that both e_1 and e_2 are incident to.

For a graph with lengths (X, f), we construct a graph $\widetilde{X}$, which we call the **resolution of** X, by replacing every edge y of X having length $f(y) > 1$ with a chain of $f(y)$ edges of length 1 (see Figure 2) via introduction of $f(y) - 1$ new vertices.

Figure 2. The resolution of an edge in a graph with lengths



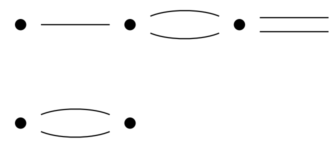
Lastly, for a graph X we construct a graph $X^{\min}$, which we call the **minimization of** X, by taking X^* (via removing all half-edges from X) and then iteratively removing from X^* any edge which is the unique edge incident to a given vertex until no such edges remain. See Figure 3 for an example.

Let $X = X_0^D(N)$, let $\widehat{B}$ and $\widehat{D} = D/p$ denote, respectively, the definite quaternion algebra over $\mathbb{Q}$ which is ramified at the same places as B aside from p and the discriminant of $\widehat{B}$, and let $\widehat{\mathcal{O}}_N$ denote an Eichler order of level N in $\widehat{B}$. Set $h = h\left(\widehat{\mathcal{O}}_N\right)$ as the class number of this order, let $\mathfrak{a}_1, \ldots, \mathfrak{a}_h$ be representative left $\widehat{\mathcal{O}}_N$-ideals of each equivalence

FIGURE 3. The minimization of a graph X

X :

$X^{\min}$:



class, and let $\widehat{\mathcal{O}}_i$ be the right order corresponding to $\mathfrak{a}_i$ for each $1 \le i \le h$. Each $\widehat{\mathcal{O}}_i$ is then an Eichler order of level N in $\widehat{B}$.

We let Δ denote the Bruhat-Tits tree associated to $\mathrm{PGL}_2(\mathbb{Q}_p)$, which in this setting we realize as having vertex set in correspondence with normalized left $\widehat{\mathcal{O}}_N$-ideals up to homothety. If x is a vertex in Δ and $\mathfrak{a}_x$ a corresponding left ideal, we let $\widehat{\mathcal{O}}_x$ denote the right order corresponding to $\mathfrak{a}_x$. We have an oriented edge y from a vertex x_1 to a vertex x_2 if there is a containment $\mathfrak{a}_2 \subseteq \mathfrak{a}_1$ of index p^2, in which case the inverse $\overline{y}$ is the edge corresponding to the containment $p\mathfrak{a}_1 \subseteq \mathfrak{a}_2$. In this way, each vertex has $p+1$ neighbor vertices.

Let

$$\widehat{\mathcal{O}}_N^{(p)} := \widehat{\mathcal{O}}_N \otimes_{\mathbb{Z}} \mathbb{Z}[1/p],$$

and consider the quotient group

$$\Gamma_p := \mathbb{Z}[1/p]^\times \backslash \widehat{\mathcal{O}}_N^{(p)\times}$$

and its index 2 subgroup

$$\Gamma := \{\alpha \in \Gamma_p \mid \mathrm{ord}_p(\mathrm{nrd}(\alpha)) \text{ is even}\}.$$

Both of these groups act on Δ via the action on ideals, and we consider the quotient graphs

$$G_p := \Delta/\Gamma_p \quad \text{and} \quad G := \Delta/\Gamma,$$

with h and $2h$ vertices, respectively. Furthermore, we consider each of G_p and G as graphs with lengths by taking Δ to be a graph with all lengths equal to 1. That is, the length $f(y)$ of an edge y in G_p or in G is the size of its stabilizer in Γ_p or in Γ, respectively. The stabilizer $\widehat{\mathcal{O}}_x^\times/\{\pm 1\}$ of a vertex x in either of theses graphs has size $f(x) \in \{1, 2, 3, 6, 12\}$, and the stabilizer of an edge is cyclic of order at most 3 [Ogg83, p. 201]. For any vertex $x \in G$, an orbit–stabilizer calculation gives

$$\sum_{\substack{\text{edges } y \\ \text{with } o(y)=x}} \frac{f(x)}{f(y)} = p+1.$$

We now describe an action of $W_0(D,N)$, which for expositional purposes we recognize as a product

$$W_0(D,N) = \widehat{W} \times \langle w_p \rangle$$

with $\widehat{W} = \{w_m \in W_0(D,N) \mid p \nmid m\}$, on G. For $w_m \in \widehat{W}$ this is clear – we take w_m to be given by the action of an element of norm m in $\widehat{O}_N$. The involution w_p is then given by the action of any element of Γ_p which is not in Γ. In this way, we recognize the covering $G \to G_p$ as the bipartite covering of graphs with lengths with respect to the action of w_p. This is to say, the $2h$ vertices of G can be partitioned into two sets $V_+ = \{x_{1,+}, \dots, x_{h,+}\}$ and $V_- = \{x_{1,-}, \dots, x_{h,-}\}$, where each of V_+ and V_- give a set of representatives for the h vertices in G_p, such that $w_p(x_{i,+}) = w_p(x_{i,-})$ for each $1 \le i \le h$. We have the following useful observations:

- Each vertex in V_+ is an odd distance from each other vertex in V_+, and is an even distance from each vertex in V_-. It follows that G contains no half-edges.
- If $w_m \in \widehat{W}$, then w_m permutes both V_+ and V_-.
- If $p \mid m$, then w_m sends each vertex in V_+ to a vertex in V_- (and vice-versa).

Let $W \leq W_0(D, N)$ be an Atkin–Lehner subgroup. Then from the action defined above we get an associated graph with lengths[1] $G_W := G/W$. In particular, we have $G = G_{\{\mathrm{Id}\}}$ and $G_p = G_{\langle w_p \rangle}$.

We are now prepared to state the main result of this section, as stated in [Ogg85, p. 202] and [BD96, Theorem 1.3, Corollary 1.4].

Theorem 2.6 (Čerednik–Drinfeld)**.** *Let D be the discriminant of an indefinite quaternion algebra over $\mathbb{Q}$, let N be a squarefree positive integer with $\gcd(D, N) = 1$, and let $p \mid D$ be a prime. Let $W \leq W_0(D, N)$ be an Atkin–Lehner subgroup, and let G_W be as defined above. Then the Atkin–Lehner quotient $X_0^D(N)/W$ over $\mathbb{Z}_p$ is a Mumford curve (c.f. [Mum72]), and the dual graph of its special fiber is given by $\widetilde{G_W^*}$ with w_p inducing the Frobenius automorphism.*

Computations of the graphs G_W are achievable using Magma [BCP97], with functionality for the requisite quaternion arithmetic provided in part by the Brandt Modules package by Kohel. Code to compute the data of the graph G_p and the Atkin–Lehner action on G_p was developed by Stankewicz for [Sta14] and is available at [Sta25]. We extend this implementation to compute the data of G_W for a general $W \leq W_0(D, N)$ in the file `dual_graphs.m` in [PS25b].

3. Reduction types and local obstructions

In this section, we describe our implementation of two algorithms which use as a basis the implementation of the theory in Section 2.3. The first regards reduction types of genus 1 Atkin–Lehner quotients.

Algorithm 3.1 (Kodaira symbols at primes $p \mid D$)**.**

Input: *A tuple (D, N, S) where*

- *D is the discriminant of an indefinite quaternion algebra over $\mathbb{Q}$,*
- *N is a squarefree positive integer with $\gcd(D, N) = 1$, and*
- *S is a set of generators for a subgroup $W \leq W_0(D, N)$ such that the quotient $X_0^D(N)/W$ has genus 1.*

Output: *The set of all Kodaira symbols of $\mathrm{Jac}(X_0^D(N)/W)$ at primes $p \mid D$.*

By Theorem 2.6, we know that the Kodaira symbol $\mathrm{Jac}(X_0^D(N)/W)$ at a prime $p \mid D$ is I_n for some $n \in \mathbb{Z}^+$. For each fixed $p \mid D$, the steps of Algorithm 3.1 to compute the Kodaira symbol at p are as follows:

(1) Using the material of Section 2.3, we compute the edge set E_W of G_W, along with the action of Atkin–Lehner operators on these edges and the stabilizer orders of each edge. If e is an edge in the edge set E of G and

$$[e] = \{w(e) \mid w \in W\}$$

[1] We urge the reader to note that the information of D, N, and p are fixed at the start and are not inherent in the notation G_W for these graphs. The amount of decoration we will add to this notation is enough that we believe this to be best, and we will clearly fix these quantities in all examples in order to avoid confusion.

is its image in E_W, represented as a W-orbit of edges from E, then the size of the stabilizer of $[e]$ is

$$\#\mathrm{Stab}([e]) = \#\mathrm{Stab}(e) \cdot \frac{\#W}{\#[e]}.$$

(2) We remove half-edges from E_W, and we then remove from E_W any edge which is the unique edge incident to either its origin or terminal vertex in order to obtain $G_W^{\min}$. We repeat the latter step until no such edges remain, such that our graph is confirmed to be minimal. Let $\widetilde{E_W}$ denote the resulting edge set.

(3) Following step (2), we need only resolve edges in $\widetilde{E_W}$ to get the dual graph of the minimal regular model of $\mathrm{Jac}(X_0^D(N)/W)$ over $\mathbb{Z}_p$. Otherwise put, our Kodaira symbol at p for this Jacobian is I_n, where

$$n := \sum_{[e] \in \widetilde{E_W}} \#\mathrm{Stab}([e]).$$

Remark 3.2. If the genus of $X = X_0^D(N)/W$ is greater than 1, then the steps outlined above still serve to yield the reduction type of X at any prime $p \mid D$. Of course, the possible reduction types for higher genus are more varied, so we leave the statement of Algorithm 3.1 as it is for simplicity. We will mainly use the above procedure in the genus 1 case, but in Section 8 we will briefly consider reduction types in the genus 2 case.

Our second algorithm regards the existence of local points at primes dividing D. Jordan–Livné [JL85, Theorems 5.1, 5.4, 5.6] proved results on the existence of $\mathbb{Q}_p$ points on $X_0^D(1)$ for $p \mid D$, while Ogg [Ogg85, Théorème] extended these results to Atkin–Lehner quotients $X_0^D(N)/W$ with $\#W \leq 2$. Stankewicz [Sta14] also proved quite general results on local points on twists of the Shimura curves $X_0^D(N)$.

In the works of Ogg and of Jordan–Livne, the authors crucially use Hensel's lemma applied to the situation at hand (see [JL85, Lemma 1.1]: for a Mumford curve X over $\mathbb{Z}_p$, this states that X has a point over $\mathbb{Q}_p$ if and only if the resolution of the special fiber of X has a smooth point over $\mathbb{F}_p$. We translate this equivalence in our situation using Theorem 2.6: fixing a prime $p \mid D$, the quotient $X_0^D(N)/W$ has a point over $\mathbb{Q}_p$ if and only if $\widetilde{G_W^{\min}}$ has a vertex x so that

- $w_p(x) = x$ (such that x corresponds to a component of the special fiber defined over $\mathbb{F}_p$), and so that
- there are strictly less than $p+1$ oriented edges y with $o(y) = x$ and with $w_p(y) = \overline{y}$ (such that the component corresponding to x contains a smooth point).

Using this criterion for local points with our previously described computations of the graphs $\widetilde{G_W}^{\min}$, we have an algorithmic version of the results of Jordan–Livne and Ogg generalized to arbitrary Atkin–Lehner quotients of $X_0^D(N)/W$ with N squarefree.

Algorithm 3.3 (Local points at primes $p \mid D$)**.**

Input: *A tuple (D, N, S, p) where*

- *D is the discriminant of an indefinite quaternion algebra over $\mathbb{Q}$,*
- *N is a squarefree positive integer with $\gcd(D, N) = 1$,*
- *S is a set of generators for a subgroup $W \leq W_0(D, N)$, and*
- *$p \mid D$ is a prime.*

Output: *A boolean `true` if $\left(X_0^D(N)/W\right)(\mathbb{Q}_p) \neq \emptyset$ and `false` otherwise.*

4. Atkin–Lehner quotients of genus at most 2

4.1. **Proof of Theorem 1.1.** We determine here all Atkin–Lehner quotients $X_0^D(N)/W$, with $\gcd(D,N)=1$ and $D>1$, having genus at most 1.

To begin, we recall two results that will allow us restrict attention to a finite list of pairs (D,N). First is a lower bound on the geometric gonality $\gamma_{\mathbb{C}}(X_0^D(N))$ of $X_0^D(N)$, which is a special case of a more general theorem of Abramovich (where we have improved the constant appearing using the best known result on Selberg's eigenvalue conjecture [Kim03, p. 176]).

Theorem 4.1. [Abr96, Theorem 1.1] *For the Shimura curve $X_0^D(N)$, we have*

$$\gamma_{\mathbb{C}}(X_0^D(N)) \geq \frac{975}{8192}\big(g(X_0^D(N))-1\big).$$

Next, we have an explicit bound on the genus of $X_0^D(N)$ in terms of the product DN.

Lemma 4.2. [Sai24, Lemma 10.6] *Let γ denote the Euler–Mascheroni constant. For $D>1$ an indefinite rational quaternion discriminant and N a positive integer coprime to D, we have*

$$g(X_0^D(N)) > 1 + \frac{DN}{12}\left(\frac{1}{e^\gamma \log\log(DN) + \frac{3}{\log\log 6}}\right) - \frac{7\sqrt{DN}}{3}.$$

Let

$$X_0^D(N)^* := X_0^D(N)/W_0(D,N)$$

denote the quotient of $X_0^D(N)$ by the full Atkin–Lehner group. Let $d(m)$ denote the number of divisors of a positive integer m. The natural map

$$X_0^D(N) \to X_0^D(N)^*$$

over $\mathbb{Q}$ has degree $2^{\omega(DN)}$, and thus we have, for every algebraic extension $F/\mathbb{Q}$,

$$\begin{aligned}\gamma_F(X_0^D(N)) &\leq 2^{\omega(DN)}\gamma_F(X_0^D(N)^*)\\ &\leq d(DN)\gamma_F(X_0^D(N)^*)\\ &\leq 2\sqrt{DN}\gamma_F(X_0^D(N)^*).\end{aligned}$$

Theorem 4.1 then provides

$$\frac{975}{16384\sqrt{DN}}\big(g(X_0^D(N))-1\big) \leq \gamma_{\mathbb{C}}(X_0^D(N)^*) \leq \gamma_F(X_0^D(N)^*), \tag{2}$$

from which Lemma 4.2 yields

$$\frac{975}{16384}\left(\frac{\sqrt{DN}}{12}\left(\frac{1}{e^\gamma \log\log(DN) + \frac{3}{\log\log 6}}\right) - \frac{7}{3}\right) \leq \gamma_F(X_0^D(N)^*). \tag{3}$$

Suppose that we have some $W \leq W_0(D,N)$ such that $g\left(X_0^D(N)/W\right) \leq 2$. We then have $g(X_0^D(N)^*) \leq 2$ and $\gamma_{\overline{\mathbb{Q}}}(X_0^D(N)^*) \leq 2$, and so Equation (3) implies $DN \leq 19226700$. Computing the genus of $X_0^D(N)^*$ for all pairs (D,N) with $DN \leq 19226700$, we determine that among the curves $X_0^D(N)^*$ there are exactly 271 of genus 0, exactly 276 curves of genus 1, and exactly 289 of genus 2.

Next, we consider general quotients. If $X_0^D(N)^*$ has genus at most 1, i.e., if (D,N) is among the 836 pairs we have just computed, then $\omega(DN) \leq 5$. With a bit of care in our enumeration of Atkin–Lehner subgroups, we compute the genus of $X_0^D(N)/W$ for every triple (D,N,W) with $X_0^D(N)^*$ having genus at most 1 and with $W \leq W_0(D,N)$ in less than an hour of computational time.

Theorem 1.1 follows from the direct computations described above, with corresponding code found in the file `genus_le2_quotient_checks.m`.

4.2. **Rational points.** For each Atkin–Lehner quotient $X = X_0^D(N)/W$ of genus 0 (respectively, of genus 1), we would next like to determine whether X a rational point and hence is isomorphic to the projective line $\mathbb{P}^1_{\mathbb{Q}}$ (respectively, is an elliptic curve over $\mathbb{Q}$) rather than being a pointless conic (respectively, a non-elliptic genus 1 curve) over $\mathbb{Q}$.

To show that X does have a rational point, our first strategy (when N is squarefree) is to show that it has a rational CM point. We does this by enumerating the set S of quadratic CM points on $X_0^D(N)$, which can be done using the work of [GR06, §5] or of [Sai24], and checking for the existence of a non-trivial involution $w_m \in W$ such that there is a point in S whose image on the quotient $X_0^D(N)/\langle w_m \rangle$, which covers X, is a rational point. We do the latter using [GR06, Corollary 5.14], which has the required hypothesis that N is squarefree.

For X of genus 1, we can also check whether there exists an index 2 subgroup $W' \leq W$ such that $X_0^D(N)/W'$ has genus 2, which implies that $X_0^D(N)$ has a rational point by a result of Kuhn [Kuh88, Corollary p. 45]. Of course, for X having genus 0 or 1, we can then also check for a genus 1 cover of X which is in turn covered by a genus 2, guaranteeing a rational point on X.

To prove that $X(\mathbb{Q}) = \emptyset$, we first check for local obstructions. If N is squarefree and there exists a prime $p \mid D$ such that $X(\mathbb{Q}_p) = \emptyset$, then we determine this using Algorithm 3.3. If $\#W = 2$, then we also check whether $X(\mathbb{R}) = \emptyset$ using Ogg's criterion for real points [Ogg83, Proposition 1].

If X has genus 0, then by the theorem of Hasse–Minkowski we have $X(\mathbb{Q}) \neq \emptyset$ if and only if X has points over $\mathbb{R}$ and over $\mathbb{Q}_p$ for every p. If D is even, $N = 1$, and X passed the local obstruction checks described above, then we know it has points over $\mathbb{Q}_p$ for all primes p; for $p \nmid D$, the reduction of X over $\mathbb{F}_p$ is a smooth conic, hence isomorphic to $\mathbb{P}^1_{\mathbb{F}_p}$, and then Hensel's lemma implies $X(\mathbb{Q}_p) \neq \emptyset$ since $p \neq 2$. In this setup, if we further have some non-trivial involution $w_m \in W$ such that $X_0^D(N)/\langle w_m \rangle$ has a real point by Ogg's result [Ogg83, Proposition 1], then we know $X(\mathbb{Q}) \neq \emptyset$.

If all of the above fails, then we resort to prior results. For the full quotients $X_0^D(N)^*$ of genus 0, [NR15, Proposition 4.2] states that $X_0^D(N)^*$ has a rational CM point. We can also use explicit equations in cases where $X_0^D(N)$ is hyperelliptic or of genus 0 or 1, coming from the works [GR06, GY17, PS23].

Beyond the above strategies, we have one more technique for proving that genus 1 curves have rational points. This requires the material of Section 7, and so we save discussion of this strategy for Section 7.2.

Of course, Hasse Principle violations can occur when X has genus 1. The techniques we use provide no means to detect this unless we already have an explicit equation coming from prior work.

Remark 4.3. We list in the final section of this work all genus 0 quotients which we prove have rational points (Table 4), all genus 0 quotients for which we remain unsure of the existence of rational points (Table 6), and genus 1 quotients with positive rank Jacobian for which we either prove the existence of a rational point (Table 5 or remain unsure of the existence of a rational point (Table 7). For brevity, we do not list *all* quotients of genus at most 2, and for example do not list those curves of genus 0 for which we prove there are no rational points. Complete lists, however, can be found in [PS25b].

Remark 4.4. In performing rational points checks, we noted that is a typo in [GR06, Theorem 3.4] for the equation of the involution w_{10} on the curve $X_0^{10}(7)$. The given equation does not define an automorphism on the provided model, and the correct equation is

$$w_{10}(x,y) = \left(\frac{2x+1}{x-2}, \frac{5y}{(x-2)^2}\right).$$

The first given involution for this curve is also mistakenly called w_{15}, whereas $15 \nmid 70$ so this is a clear typo. Said involution is w_5.

5. Isogeny classes of Jacobians of genus 1 Atkin–Lehner quotients

Let $W \leq W_0(D,N)$ be a non-trivial subgroup such that $X = X_0^D(N)/W$ has genus 1, the Jacobian $J = \mathrm{Jac}(X)$ is an elliptic curve over $\mathbb{Q}$. Using Ribet's isogeny Theorem 2.4 and an explicit decomposition of $\mathrm{Jac}(X_0(DN))^{D\text{-new}}$ (as implemented in Magma), one can in theory determine the isogeny class of J.

However, if the genus of $X_0(DN)$ is large, i.e., if the dimension of $\mathrm{Jac}(X_0(DN))$ is large, the computation of the D-new part and its decomposition can make this method infeasible to complete in a reasonable time. Instead, we can compute using modular forms directly by way of modular symbols functionality implemented in Magma. Specifically, we take the Atkin–Lehner decomposition of the new space M of the cuspidal subspace of modular symbols of level DN, weight 2, and sign -1, and for each component V we check compatibility of the sign pattern of V with W based on the Hecke action described in Theorem 2.4. If we find a symbol V which is compatible with W, then because $X_0^D(N)/W$ has genus 1 it must be the case that V is the only compatible component, that V is a one-dimensional space of symbols, and that $X_0^D(N)/W$ is isogenous to the associated elliptic curve.

The method as described can yield false negatives, though, in the following sense: each elliptic curve over $\mathbb{Q}$ which is isogenous to $X_0^D(N)/W$ may have conductor properly dividing DN. It may be the case that $X_0^D(N)/W$ has good reduction at some prime dividing DN, i.e., that there is a loss of at least one prime of bad reduction upon taking the quotient of $X_0^D(N)$ by W, or N may not be squarefree and we lose a factor whose square divides N in the conductor. This occurs for 348 genus 1 quotients $X_0^D(N)/W$.

We know from Theorem 2.4, though, that all of the symbols attached to our fixed quotient are D-new. Hence, any conductor change only involves divisors of N. In this case, one can also see this from direct computation of Kodaira symbols at all primes $p \mid D$ for all genus 1 quotients $X_0^D(N)/W$ with N squarefree using Algorithm 3.1; no such quotient has Jacobian with good reduction at a prime $p \mid D$. In particular, if N is prime and we observe that there is a loss of prime of bad reduction in taking the quotient by W (by search in the new space of modular symbols of level DN as described above), then we know the correct modular symbol must be new in level D.

This observation helps considerably in reducing computing time in situations where this phenomena occurs, as the following example illustrates. This example also serves to show our method to determine the correct one-dimensional space of symbols in this new space of level D in this situation.

Example 5.1. Consider the genus 1 curve $X = X_0^{210}(11)/\langle w_2, w_5, w_7, w_{33}\rangle$. The curve $X_0(210{\cdot}11) = X_0(561)$ has genus 561, and so it comes as no surprise that the computation of the decomposition of the abelian subvariety of its Jacobian cut out by the action of W via Ribet's isogeny is intensive; the computation did not complete within three days on our machine.

We observe by direct computation that there are no components compatible with the sign pattern given by the action of W in the new space of the cuspidal subspace of modular symbols of level 561, weight 2, and sign -1. We also observe from direct computation using Algorithm 3.1 that $J = \mathrm{Jac}(X)$ does not have good reduction at any primes dividing 210; the symbols at $2, 3, 5$, and 7, respectively are I_4, I_2, I_2, and I_4. Hence, as 11 is prime, we *must* find that the correct modular symbol and modular form is new in level 210.

In searching for a symbol that is compatible with the action of W on this new space of level D, one must be careful as the sign pattern imposed by this action at each prime is not necessarily the same as it was in level DN. We avoid this difficulty as follows: we simply enumerate *all* one-dimensional components in this space, and correspondingly take all curves in isogeny classes over $\mathbb{Q}$ of conductor D, and we then check Kodaira symbols against those computed for J.

In our example, there are 5 such one-dimensional spaces of symbols which are new in level 210 and this 37 isomorphism classes of elliptic curves with conductor 210 to check – those in isogeny classes with Cremona labels 210a, 210b, 210c, 210d, and 210e. The only curve among these classes with the symbols I_4, I_2, I_2, and I_4 at $2, 3, 5$, and 7, respectively, is that of Cremona reference 210c2, with equation

$$y^2 + xy + y = x^3 + x^2 - 70x - 205,$$

and so J is isomorphic (not just isogenous, as we aim to show in this section) over $\mathbb{Q}$ to this curve.

Pleasingly, we find that in all examples where the conductor of J is not DN and where the brute force method using an explicit Jacobian decomposition does not succeed in a reasonable time, we indeed have that N is prime and that we are able to uniquely determine the isomorphism class using Kodaira symbol comparisons as in Example 5.1. The described computations thus provide the following proposition.

Proposition 5.2. *There are exactly* 1270 *curves* $X = X_0^D(N)/W$ *of genus* 1 *with* $W \leq W_0(D, N)$ *non-trivial and with* N *squarefree. Each of these curves is listed in Table 2, and in each case* $\mathrm{Jac}(X)$ *is in the same isogeny class over* $\mathbb{Q}$ *as the curve with the listed Cremona reference.*

For the 82 *quotients* $X = X_0^D(N)/W$ *of genus* 1 *with* $W \leq W_0(D, N)$ *nontrivial and with* N *not squarefree,* $\mathrm{Jac}(X)$ *is in the isogeny class with Cremona reference given in Table 3.*

6. Isomorphism classes of elliptic curve Jacobians

Let $X = X_0^D(N)/W$ be a genus 1 Atkin–Lehner quotient with $W \leq W_0(D, N)$ non-trivial. We know that the elliptic curve $\mathrm{Jac}(X)$ has at worst multiplicative reduction at each $p \mid D$. The following result of Dokchitser–Dokchitser describes the possible behaviors of the Kodaira symbol of $\mathrm{Jac}(X)$ at p under prime-degree isogenies.

Theorem 6.1. [DD15, Theorem 5.4 (1),(2),(4)] *Let* $\varphi : E \to E'$ *be an isogeny of elliptic curves over* $\mathbb{Q}_p$ *of prime degree* ℓ.

(1) If E *has potentially good reduction and* $p \neq \ell$*, then the Kodaira types of* E *and* E' *are the same.*
(2) If E *has potentially good ordinary reduction and* $p = \ell$*, then the Kodaira types of* E *and* E' *are the same.*
(3) If E *has multiplicative reduction, with Kodaira type* I_n *for* E*, then* E' *has Kodaira type either* I_{pn} *or* $I_{n/p}$ *corresponding to* $v(j') = pv(j)$ *and* $pv(j') = v(j)$*, respectively.*

From Theorem 6.1, we note the following: if we know the Kodaira symbols of X at all primes $p \mid D$ and we know the isogeny class over $\mathbb{Q}$ of $\mathrm{Jac}(X)$ (as determined in Section 5), then from this information we can determine the isomorphism class of $\mathrm{Jac}(X)$ *unless* there exist members E, E' of the isogeny class which are isogenous by an isogeny of degree d with $\gcd(d, D) = 1$.

Theorem 6.2. *Let $X = X_0^D(N)/W$ be a genus 1 Atkin–Lehner quotient with $W \leq W_0(D, N)$ non-trivial and N squarefree. Then $\mathrm{Jac}(X)$ is isomorphic over $\mathbb{Q}$ to the elliptic curve with Cremona reference given in Table 2.*

Proof. Let $\mathcal{I}$ denote the isogeny class over $\mathbb{Q}$ of $J = \mathrm{Jac}(X)$ as determined in Proposition 5.2. Because N is squarefree, we are able to compute the Kodaira symbols of X at all primes $p \mid D$ using Algorithm 3.1.

If there is a unique element E of $\mathcal{I}$ whose Kodaira symbols at all primes $p \mid D$ match those of J, then J is isomorphic to E and we are done. This is the case for all genus one quotients with N squarefree except for the following 5 triples (D, N, W):

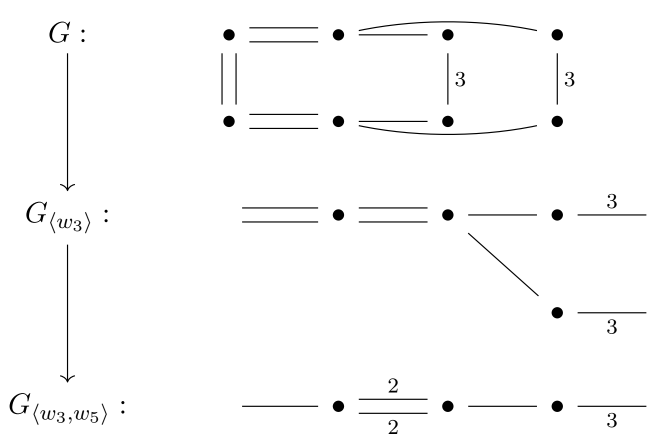
$$(6, 17, \langle w_2, w_3 \rangle)\,, \qquad (15, 2, \langle w_{30} \rangle)\,, \qquad (21, 2, \langle w_{21} \rangle)\,,$$
$$(21, 5, \langle w_3, w_7 \rangle)\,, \qquad \text{and} \qquad (33, 2, \langle w_6, w_{22} \rangle)\,.$$

In each of these 5 listed cases, the Kodaira symbol checks narrow us down to two candidate isomorphism classes (which are necessarily isogenous via a cyclic isogeny of degree coprime to D, as per the discussion preceding this theorem). For the first 3 triples we have that $X_0^D(N)$ is geometrically hyperelliptic, and so we have equations via [GY17, PS23].

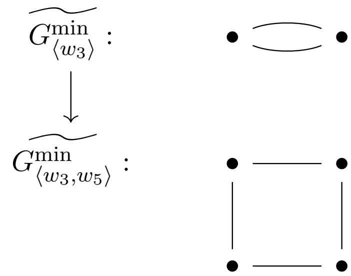
Let $X = X_0^{21}(5)/\langle w_3, w_7 \rangle$. The Kodaira symbol checks for X narrow us down to its Jacobian J being isomorphic to the curve with Cremona reference 105a1 or to that with reference 105a3. The cover $Y = X_0^{21}(5)/\langle w_{21} \rangle$ of X has genus 1 and we find from the methods in Section 4.2 that is has no rational points. We will compute an equation for this curve in Section 7 (see Table 8), using which we determine that J has associated Cremona reference 105a1.

For the elliptic curve $X_0^{33}(2)/\langle w_6, w_{22} \rangle$, we narrow the possibilities down to Cremona references 66b1 and 66b3 via Kodaira symbol checks. We prove that the former reference is correct at the end of this work using material from Section 8; see Remark 8.6. $\square$

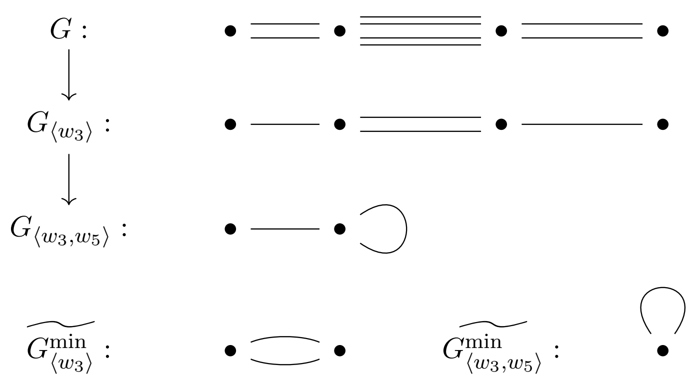
Example 6.3. Consider the genus 1 curves $X = X_0^{15}(7)/\langle w_3 \rangle$ and $Y = X_0^{15}(7)/\langle w_3, w_5 \rangle$. Fixing $D = 15$, $N = 7$, and $p = 3$, we compute the graphs G_W for $W \in \{\{\mathrm{id}\}, \langle w_3 \rangle, \langle w_3, w_5 \rangle\}$:



The dual graphs to the minimal regular models of X and Y over $\mathbb{Z}_3$ are then as follows:



from which we see that X and Y have Kodaira symbols I_2 and I_4 at $p = 3$, respectively. For $p = 5$, we do the same:



finding that X and Y have Kodaira symbols I_2 and I_1 at $p = 5$, respectively. From Proposition 5.2, we know that $\mathrm{Jac}(X)$ and $\mathrm{Jac}(Y)$ are both in the isogeny class with Cremona reference 105a, which has 4 elements. From the above symbols computations, we determine that $\mathrm{Jac}(X)$ has reference 105a2 while $\mathrm{Jac}(Y)$ has reference 105a4 (in fact, the symbols at $p = 3$ suffice in this example). From the work described in Section 4.2, we know that X has no rational points as it lacks points over $\mathbb{Q}_5$, and we will determine an equation for X in Section 7 (see Table 8). The curve Y is seen to have a rational point as its cover $X_0^{15}(7)/\langle w_{15}\rangle$ has a rational -7-CM point, so $Y \cong \mathrm{Jac}(Y)$ over $\mathbb{Q}$.

Example 6.4. The curve $X = X_0^6(17)/\langle w_2, w_3\rangle$ is an elliptic curve over $\mathbb{Q}$, and we compute the isomorphism class to be that of Cremona reference 102b in Proposition 5.2. Comparison of the computed Kodaiara symbols of X with those of all members of this isogeny class does not determine its isomorphism class; both the curve with Cremona reference 102b5 and that with reference 102b6, which are isogenous by a 4-isogeny over $\mathbb{Q}$, have the correct symbols of I_1 and I_2 at the primes 2 and 3, respectively. Since the curve $X_0^6(17)$ is hyperelliptic, though, one finds in [GY17] an equation for $X_0^6(17)$ and in [PS23] an equation for X, from which we know X is isomorphic to the curve with Cremona reference 102b6.

Remark 6.5. In Section 10, we draw for all 129 genus 1 Atkin–Lehner quotients of the form $\mathrm{X} = X_0^D(N)/\langle w_m\rangle$, with $w_m \in W_0(D, N)$ non-trivial, the associated graph $G_{\langle w_m\rangle}$ and the Kodaira symbol of $\mathrm{Jac}(X)$ at each prime $p \mid D$. It is our hope that this wealth of explicit examples will be useful to readers.

Remark 6.6. In [GY17], the equations provided for the Atkin–Lehner involutions w_{22} and w_{33} (and hence for w_2 and w_3 and for w_{66} and w_{11} as well) on the curve $X_0^6(11)$ are swapped. One can see the equations are incorrect as stated by taking the quotients by these involutions, as done in [PS23], and seeing that the isomorphism classes of their

Jacobians are swapped. Comparison with [NR15] confirms that the model in [GY17] for the curve $X_0^6(11)$ is correct; it is just that the equations for these involutions are swapped. Similarly, we note the following:

- The equations in [GY17] for the involutions w_{10} and w_{13} (and hence also for w_5 and w_{26} on the curve $X_0^{10}(13)$ are swapped.
- The equations in [GY17] for the involutions w_{35} and w_{10} on the curve $X_0^{14}(5)$ are swapped. In this case, the error results in involutions of genus 1 being swapped with involutions of genus 2, and this was partially noted in [PS25a].
- The involutions w_2 and w_{26} on the curve $X_0^{39}(2)$ are swapped. In this case the respective quotients are genus 4, while the quotients by the subgroups $\langle w_2, w_3 \rangle$ and $\langle w_3, w_{26} \rangle$ have genus 2. We noted this error while checking our algorithms from Section 3 against genus 2 curves contained in [GY17], but we mention it here in order to collect these items in a single remark.
- We also notice that there is an error in the equations for Atkin–Lehner involutions in [GY17] for the curve $X_0^{15}(4)$. It seems likely that the equations for the involutions in the pair $\{w_{12}, w_{60}\}$ are swapped with the equations for those in $\{w_4, w_{20}\}$. One can see there is an error from the fact that the genera of the involutions in the first pair should be 2, while the genera of those in the second pair should be 3; these genera are swapped according to the models given in [GY17]. It is not clear to us which involution is swapped for which; point counts are not helpful as the quotients by the two involutions in each pair have isogenous jacobians, and because 4 is not squarefree our algorithmic tools from this paper are not of help.

7. Non–elliptic genus one quotients

7.1. **Equations of non-elliptic genus one quotients.** Let N be squarefree and let $W \leq W_0(D, N)$ be a non-trivial Atkin–Lehner subgroup such that the quotient $X = X_0^D(N)/W$ is a non-elliptic curve of genus 1. That is, X has genus 1 and $X(\mathbb{Q}) = \emptyset$ by the results of Section 4.2.

In this section, we seek to determine an equation for X. Our main strategy follows a method from in [GR06], in which the authors determine equations for non–elliptic genus 1 Shimura curves $X_0^D(N)$ and $X_0^D(1)/\langle w_m \rangle$ with $w_m \in W_0(D, N)$ a non-trivial involution. We recall from their work our initial approach for determining candidate equations among twist families. We then state our main result (which recovers equations for the 17 curves $X_0^D(1)/\langle w_m \rangle$ handled in [GR06]).

Proposition 7.1. [GR06, §2.1] *Let X be a nice curve of genus 1 over a field F with char$(F) \neq 2$ satisfying $X(F) = \emptyset$. Suppose that we have the following information:*

(1) an involution $w \in Aut(X)$ such that $X/\langle w \rangle \cong \mathbb{P}^1_F$, with corresponding covering map $\pi : X \to X/\langle w \rangle$,

(2) an element $d \in F^\times \setminus F^{\times 2}$ such that

$$\pi(X(F(\sqrt{d}))) \cap (X/\langle w \rangle)(F) \neq \emptyset ,$$

(3) and a short Weierstrass equation $y^2 = x^3 + Ax + B$ for the elliptic curve $E = \mathrm{Jac}(X)$, with $A, B \in F$.

Denote by E_d the twist of E by d:

$$E_d : y^2 = x^3 + Ad^2x + Bd^3,$$

and let

$$\{\infty, (a_1, b_1), \ldots, (a_r, b_r)\} \subseteq E_d(F)$$

be a complete, non-redundant set of representatives of the classes of the weak Mordell–Weil group $E_d(F)/2E_d(F)$. For $1 \le i \le r$, let

$$f_i(x) := x^4 - 6a_i x^2 + 8b_i x - 3a_i^2 - 4Ad^2 \ , \quad 1 \le i \le r \ .$$

Then C is isomorphic over F to exactly one of the curves among those with equations $y^2 = df_i(x)$ for $1 \le i \le r$.

Theorem 7.2. *Each of the curves $X = X_0^D(N)/W$ listed in Table 8 is non-elliptic of genus 1. For the 4 triples (D, N, W) in the set*

$$\Big\{ (14, 11, \langle w_2, w_{11} \rangle) \, , (46, 3, \langle w_2, w_3 \rangle) \, , \\ (570, 1, \langle w_5, w_{57} \rangle) \, , (570, 1, \langle w_6, w_{190} \rangle) \Big\},$$

the curve X admits exactly one of the two models given in Table 8 over $\mathbb{Q}$. Each of the remaining 146 curves X in Table 8 admits the given model over $\mathbb{Q}$.

Proof. The fact that all of these genus 1 curves lack rational points follows from the techniques mentioned in Section 4.2. For each curve $X = X_0^D(N)/W$ in Table 8, there exists a quadratic CM point x on $X_0^D(N)$ which, using [GR06, Corollary 5.14], we are able to show has rational image on a genus zero quotient of X by an Atkin–Lehner involution w_{m_0}. Letting $K = \mathbb{Q}(x) = \mathbb{Q}(\sqrt{d_0})$, with d_0 negative and squarefree, we have that K is also necessarily the residue field of the image of x on X (as X has no rational points).

Therefore, we can take $d = d_0$ in our application of Proposition 7.1 for X, whereas the required equation for $E = \mathrm{Jac}(X)$ is given by Theorem 6.2. We compute in Magma representatives for $E_D(K)/2E_D(K)$, and for each non-trivial element we get a non-trivial twist C of X. Let $\mathcal{C}_X$ denote the set of these twists.

We next exclude from $\mathcal{C}_X$ any element C which either

- disagrees with local points information for X at finite primes $p \mid D$, computed using Algorithm 3.3, or
- does not agree with the information of whether X has points over $\mathbb{R}$, determined using Ogg's result [Ogg83, Proposition 1].

Code for all of these checks can be found in the file

`non_elliptic_genus_1_eqn_computations.m`

in [PS25b]. Using just this information, we narrow $\mathcal{C}_X$ down to either a single (necessarily correct) model or two candidate models.

In the latter case, which occurs for 53 curves, we attempt one final check, similar to the main check used in [GR06], on the two remaining candidate equations. For this, fix $W' \le W_0(D, N)$ with $[W' : W] = 2$ such that $X' = X_0^D(N)/W'$ is isomorphic to $\mathbb{P}^1_{\mathbb{Q}}$ and has a rational CM point determined as discussed above, such that the cover $\pi : X \to X'$ satisfies the required hypotheses used in our application of Proposition 7.1 to obtain our candidate equations. Let $R \subseteq X(\overline{\mathbb{Q}})$ denote the set of ramification points of π. If $y^2 = f(x)$ is a hyperelliptic model for X obtained via Proposition 7.1, then the splitting field of f must be isomorphic to the field L_π defined as the compositum of the residue fields of all points in R.

Working in our level of generality, we do not always have complete information of the residue fields of points in R; the result [GR06, Corollary 5.13] proven and used by González–Rotger is specific to quotients of $X_0^D(N)$ by a single Atkin–Lehner involution. We know the following, though: the elements of R are exactly the images under the natural map of fixed points on $X_0^D(N)$ of elements of the set $W' \setminus W$. Each such fixed point is given by Proposition 2.3 as a point with CM by an order $\mathfrak{o}$ in an imaginary quadratic field K, whose residue field on $X_0^D(N)$ over K is the ring class field corresponding to $\mathfrak{o}$ [Shi67, Main Theorem II]. Let $\widetilde{R}$ denote the set of points on $X_0^D(N)$ fixed by some element of $W' \setminus W$, and let $\widetilde{L}_\pi$ denote the compositum of all ring class fields of the imaginary quadratic orders corresponding to points in $\widetilde{R}$.

It follows from above that L_π is a subfield of $\widetilde{L_\pi}$. Therefore, if $y^2 = f_1(x)$ and $y^2 = f_2(x)$ are our two candidate equations for X and the splitting field of f_i is not contained in $\widetilde{L_\pi}$ for an index $i \in \{1,2\}$, we can discard the corresponding model from $\mathcal{C}_X$. This final check succeeds for all remaining curves aside from the six in the set included in the statement of this theorem. For the first two curves in this list, both f_1 and f_2 have splitting field contained in $\widetilde{L_\pi}$; this final check is too coarse to succeed. For the remaining four curves, we were not able to complete this required subfield check in Magma. Code for this final strategy can be found in the file `non_elliptic_fixed_pt_fld_checks.m` in [PS25b]. ☐

Remark 7.3. The checks regarding the splitting field of the hyperelliptic equation and the field L_π generated by the ramifcation points of the hyperelliptic quotient map π described in the proof of Theorem 7.2 is one of the more computationally demanding components of this work. For the curves to which we apply this strategy, these checks took roughly 70 hours of total computational time on our machine.

Example 7.4. Consider the genus 1 curve $X = X_0^{21}(5)/\langle w_3, w_5\rangle$. This curve has Kodaira symbols I_1 and I_2 the the primes 3 and 7, respectively. It lacks $\mathbb{Q}_7$-points, and is therefore not an elliptic curve. On the other hand, it has a $\mathbb{Q}_3$-point and is seen to have a real point due to its cover $X_0^{21}(5)/\langle w_5\rangle$ having a real point by [Ogg83, Proposition 1].

The genus 0 quotient $X^* = X_0^{21}(5)/\langle w_3, w_5, w_7\rangle$ of X is isomorphic to $\mathbb{P}^1_{\mathbb{Q}}$, given that it has a rational -4-CM point. We know from Theorem 6.2 that $E = \mathrm{Jac}(X)$ is isomorphic to the elliptic curve with Cremona reference 21a5, whose twist by -4 is isomorphic to

$$E_{-4} : y^2 = x^3 - 16257456x + 25230566016.$$

We compute in Magma that $E_{-4}(\mathbb{Q}) \cong \mathbb{Z} \times \mathbb{Z}/4\mathbb{Z}$, and that a set of representatives for non-identity elements of $E_{-4}(\mathbb{Q})/2E_{-4}(\mathbb{Q})$ is as follows:

$$\left\{(2364, 3024), \left(\frac{58236}{25}, -\frac{15984}{125}\right), (3624, -117936)\right\}$$

Applying Proposition 7.1, we get one candidate hyperelliptic model for X for each element of the above set:

$$\begin{aligned}
C_1 &: y^2 = -4x^4 + 56736x^2 - 96768x - 193057344\ , \\
C_2 &: y^2 = -4x^4 + \frac{1397664}{25}x^2 + \frac{511488}{125}x - \frac{121877379648}{625}\ , \\
C_3 &: y^2 = -4x^4 + 86976x^2 + 3773952x - 102518784\ .
\end{aligned}$$

The curve C_2 is locally solvable at $p = 7$, and so cannot be isomorphic to X. Both C_1 and C_3 have real points, and so we consider the ramification points of the natural degree 2 map $X \to X^*$. These are the images on X of all fixed points on $X_0^{21}(5)$ of the involutions $w_7, w_{21}, w_{35}, w_{105}$. Only w_{105} has fixed points, namely the single -420-CM point on $X_0^{21}(5)$. Letting $K = \mathbb{Q}(\sqrt{-105})$, the residue field of this point over $\mathbb{Q}$ is a totally complex index 2 subfield of the ring class field H corresponding to the maximal order in K, and its image on $X_0^{21}(5)/\langle w_3\rangle$ has degree 4. The splitting field of the defining polynomial in x for C_1 is not contained in this residue field (it is not even contained in H), hence X is isomorphic over $\mathbb{Q}$ to C_3.

7.2. **A return to rational points.** We now briefly return to the investigation of the existence of rational points on genus 1 quotients started in Section 4.2. We propose a final strategy, using the material of the prior section, to prove a genus 1 Atkin–Lehner quotient is an elliptic curve with no explicit knowledge of its rational points.

Let $X = X_0^D(N)/W$ be a genus 1 Atkin–Lehner quotient, let $w \in \mathrm{Aut}(X)$ be an Atkin–Lehner involution, and let $x \in X_0^D(N)$ be a CM point whose image on $X/\langle w \rangle$ is rational. Supposing that X itself has no rational points, we can apply Proposition 7.1 to compute a set $\mathcal{C}_X$ of candidate models for X. We can then discard candidates using the same methods used in the proof of Theorem 7.2, by testing the existence of points over $\mathbb{Q}_p$ for $p \mid D$, the existence of real points, and the agreement of our model with the residue fields of fixed points of w. If these methods leave $\mathcal{C}_W$ empty, then we have proved by way of contradiction that X must have a rational point and be an elliptic curve over $\mathbb{Q}$. This method succeeds for 28 genus 1 curves for which we remained unsure of the existence of rational points on X following the methods of Section 4.2, and these curves are listed in the file

```
genus_1_AL_quotients_rat_pts_by_non_elliptic_test_final.m
```

in [PS25b].

In attempting the above method, we reduce to exactly one candidate non-elliptic equation, which we are not able to eliminate by our techniques, for 25 curves. This leads to the following proposition.

Proposition 7.5. *For each of the* 25 *genus* 1 *curve* $X = X_0^D(N)/W$ *with* (D, N, W) *listed in Table 1, we have that either* X *has a rational point (and is thus isomorphic to its Jacobian, whose isomorphism class is give in Table 2), or* $X(\mathbb{Q}) = \emptyset$ *and* X *admits the model* $y^2 = f(x)$ *over* $\mathbb{Q}$ *for the given polynomial* f*.*

Proof. For each triple (D, N, W) appearing in this table, we are able to find a CM point on $X_0^D(N)$ with the necessary setup to apply Proposition 7.1. We then generate a list of candidate equations, one of which must be an equation for $X_0^D(N)/W$ if this curve lacks a rational point. Using the techniques described in the proof of Theorem 7.2, we are able to exclude all but one candidate model, and hence $X_0^D(N)/W$ is either an elliptic curve or admits this model. $\square$

Table 1: Possible non-elliptic models of the form $y^2 = f(x)$ for 25 genus 1 curves $X = X_0^D(N)/W$ for which we remain unsure of whether X has a rational point. For each X, the hyperelliptic involution with respect to the given model (if correct) is the image of the Fricke involution w_{DN} on X.

D	N	W	f
6	35	$\langle w_{10}, w_{42} \rangle$	$-19x^4 - 1003770x^2 - 3480192x - 13271278707$
6	85	$\langle w_2, w_{15}, w_{51} \rangle$	$-4x^4 + 1733355/8x^2 - 434388339/16x + 974320350867/1024$
6	115	$\langle w_5, w_6, w_{46} \rangle$	$-19x^4 + 123462x^2 + 71114112x + 5986363725$
6	161	$\langle w_2, w_{21}, w_{69} \rangle$	$-19x^4 + 1059174x^2 - 17778528x - 16947937395$
10	21	$\langle w_5, w_{21} \rangle$	$-3x^4 - 25434x^2 - 51667875$
10	33	$\langle w_2, w_{55} \rangle$	$-8x^4 + 37296x^2 - 1270080x + 27705240$
10	39	$\langle w_3, w_5, w_{13} \rangle$	$-3x^4 + 34182x^2 + 139968x - 86869827$
10	51	$\langle w_2, w_{15}, w_{51} \rangle$	$-8x^4 + 116460x^2 - 7862184x + 303684147/2$
10	61	$\langle w_{10}, w_{122} \rangle$	$-3x^4 + 486x^2 - 233280x - 4918563$
10	93	$\langle w_3, w_{10}, w_{62} \rangle$	$-3x^4 + 30294x^2 - 1166400x - 2994003$

14	57	$\langle w_2, w_{21}, w_{57}\rangle$	$-8x^4 - 27828x^2 - 327240x + 31947939/2$
15	14	$\langle w_2, w_5, w_7\rangle$	$-7x^4 + 72324x^2 - 177811200$
21	26	$\langle w_2, w_{21}, w_{39}\rangle$	$-4x^4 + 60768x^2 + 3483648x + 40927680$
34	5	$\langle w_{10}, w_{34}\rangle$	$-11x^4 + 76230x^2 - 2299968x + 26629317$
34	7	$\langle w_2, w_{17}\rangle$	$-3x^4 - 4698x^2 + 18144x - 2002563$
74	5	$\langle w_{10}, w_{74}\rangle$	$-19x^4 - 32490x^2 - 11852352x + 448352253$
119	2	$\langle w_7, w_{17}\rangle$	$-7x^4 + 122094x^2 + 7547904x + 108672921$
210	1	$\langle w_7, w_{15}\rangle$	$-43x^4 - 7695882x^2 - 4070670336x - 492140268747$
210	19	$\langle w_6, w_7, w_{10}, w_{19}\rangle$	$-67x^4 - 23674986x^2 - 7795776960x - 290611947987$
330	1	$\langle w_2, w_{33}\rangle$	$-3x^4 + 1529604x^2 - 191556845568$
330	1	$\langle w_3, w_{10}\rangle$	$-3x^4 - 23004x^2 + 83980800$
462	1	$\langle w_{11}, w_{14}\rangle$	$-4x^4 - 605376x^2 - 22889889792$
798	1	$\langle w_2, w_3, w_{19}\rangle$	$-4x^4 + 1784736/25x^2 + 27772416/125x - 186001386048/625$
1230	1	$\langle w_3, w_{10}, w_{82}\rangle$	$-3x^4 + 486x^2 - 17003520x - 153051363$
1722	1	$\langle w_6, w_{14}, w_{41}\rangle$	$-67x^4 + 98497638x^2 - 65484526464x + 12323344962141$

8. Genus two bielliptic quotients

If X is a bielliptic curve of genus 2, then each bielliptic quotient has a rational point and is thus an elliptic curve over $\mathbb{Q}$ [Kuh88, Corollary, p.45]. Any genus 2 bielliptic curve has exactly 2 non-hyperelliptic (and thus necessarily bielliptic) involutions, which one can see from the possible automorphism groups in [SV04, Theorem 2]. Concretely: if w is a bielliptic involution on X and ι denote the hyperelliptic involution on X, then $\iota \circ w$ is the other bielliptic involution.

Among the 1580 Atkin–Lehner quotients $X_0^D(N)/W$ of genus 2, we enumerate those with at least one bielliptic involution which is an Atkin–Lehner involution. Note that if X is bielliptic via an Atkin–Lehner involution w and $[W_0(D,N) : W] \geq 4$, then both bielliptic involutions on X are Atkin–Lehner. If $[W_0(D,N) : W] = 2$, though, then the hyperelliptic involution ι is non–Atkin–Lehner, as is the bielliptic involution $\iota \circ w$. The latter situation happens for each curve encountered in [GR04], as in each case here $N = 1$ with $\omega(D) = 2$ and the authors study the genus 2 bielliptic quotients $X_0^D(1)/\langle w_m\rangle$ with $m > 1$.

Example 8.1. Consider the genus 2 curve $X = X_0^6(133)/\langle w_7, w_{19}, w_{57}\rangle$. The only non-trivial Atkin–Lehner quotient $X_0^6(133)^*$ has genus 1. Therefore, the hyperelliptic involution ι and the bielliptic involution $\iota \circ w_2$ are both non-Atkin–Lehner.

We compute directly that there are exactly 388 bielliptic genus 2 curves $X_0^D(N)/W$ with two bielliptic Atkin–Lehner quotients, and there are exactly 251 such curves with exactly one bielliptic Atkin–Lehner involution. We seek to generalize the results of González–Rotger [GR04] to arbitrary level and quotients by computing equations for all of these curves. This includes recovering equations for the 10 curves $X_0^D(1)/\langle w_m\rangle$ handled in [GR04]. We use the following proposition from their work, which allows us to compute a finite set of candidate equations for a given bielliptic genus 2 quotient X, to this aim.

Proposition 8.2. [GR04, Proposition 2.1] *Let C be a genus two curve defined over a field k of characteristic not 2 or 3, and let ι be its hyperelliptic involution. Assume that*

$\mathrm{Aut}_k(C)$ *contains a subgroup* $\langle u_1, u_2\rangle \cong (\mathbb{Z}/2\mathbb{Z})^2$ *with* $u_2 = \iota \circ u_1$ *and denote by* C_i *the elliptic quotient* $C/\langle u_i\rangle$ *for* $i \in \{1, 2\}$*. If the two elliptic curves*

$$E_1 : Y^2 = X^3 + A_1X + B_1$$
$$E_2 : Y^2 = X^3 + A_2X + B_2$$

are isomorphic over k *to* C_1 *and* C_2*, respectively, then there exists a solution* $(a, b) \in k^\times \times k$ *to the system of equations*

$$27a^3B_2 = 2A_1^3 + 27B_1^2 + 9A_1B_1b + 2A_1^2b^2 - B_1b^3$$
$$9a^2A_2 = -3A_1^2 + 9B_1b + A_1b^2.$$

such that, letting

$$c := \frac{3A_1 + b^2}{3a} \qquad \text{and}$$
$$d := \frac{27B_1 + 9A_1b + b^3}{27a^2},$$

we have that C *admits the hyperelliptic equation* $y^2 = ax^6 + bx^4 + cx^2 + d$ *with* u_1 *given by* $(x, y) \mapsto (-x, y)$*.*

We now use Proposition 8.2 to prove our main result. A notable distinction in the proof of our result, compared to that of [GR04], is that for the 10 quotients they study the referenced proposition provides a single, necessarily correct, candidate model. In our case, we generally arrive at multiple candidate equations and we need to use arithmetic information to try to determine the correct model.

Theorem 8.3. *(1) There are exactly* 639 *genus* 2 *curves* $X = X_0^D(N)/W$*, with* $W \le W_0(D, N)$ *nontrivial, so that* X *has a bielliptic Atkin–Lehner involution.*

(2) For the 405 *curves* $X = X_0^D(N)/W$ *listed in Table 9, we have that* X *is bielliptic of genus* 2 *and admits the given equation over* $\mathbb{Q}$*.*

(3) If $X = X_0^D(N)/W$*, with* $W \le W_0(D, N)$ *nontrivial, is bielliptic of genus* 2 *and does not have a bielliptic Atkin–Lehner involution, then* X *must be among the* 469 *quotients listed in Table 10.*

Proof. We already described above the enumeration of all bielliptic genus 2 Atkin–Lehner quotients with at least one bielliptic Atkin–Lehner involution, giving part (1), and so we proceed with part (2). Let $X = X_0^D(N)/W$ be such a quotient, and let E_1 be a bielliptic Atkin–Lehner quotient of X. If the other bielliptic involution on X is Atkin–Lehner, then let E_2 be the other bielliptic quotient. Otherwise, let E_2 be any member of the isogeny class of the isogeny factor in $\mathrm{Jac}(X)$ not corresponding to E_1, which we compute using Theorem 2.4.

For $i \in \{1, 2\}$, let $\mathcal{I}_i$ be the singleton set containing the isomorphism class of E_i if E_i is a bielliptic Atkin–Lehner quotient of X and this isomorphism class was computed in Section 6. Otherwise, let $\mathcal{I}_i$ denote the set of all members of the isogeny class of E_i over $\mathbb{Q}$ (as computed in Section 5 if E_i is a bielliptic Atkin–Lehner quotient of X).

For each pair $(E_1', E_2') \in \mathcal{I}_1 \times \mathcal{I}_2$, we attempt to compute all candidate equations for a bielliptic cover of E_1' and E_2' using Proposition 8.2. This works in all cases except for the three genus 2 curves

$$X_0^{390}(11)/\langle w_2, w_5, w_{13}, w_{33}\rangle, \qquad X_0^{714}(5)/\langle w_2, w_5, w_{17}, w_{21}\rangle,$$
$$\text{and} \qquad X_0^{798}(5)/\langle w_2, w_3, w_{19}, w_{35}\rangle.$$

For each of these 3 curves, there is one non-Atkin–Lehner bielliptic quotient, and the Jacobian isogeny factor corresponding to this quotient has conductor properly dividing DN. We lack knowledge of the correct conductor, and the naive method using Ribet's isogeny with a decomposition of the Jacobian of the D-new part of $X_0(DN)$ seems infeasible to compute in a reasonable time due to the genus of $X_0(N)$ being rather large – more specifically, the genera are $993, 849$, and 945.

Aside from the above three curves, we succeed in computing candidate equations in each case and we let $\mathcal{H}_X$ denote the set of all such candidate equations over all pairs in $\mathcal{I}_1 \times \mathcal{I}_2$ up to isomorphism over $\mathbb{Q}$ of the hyperelliptic curve they define. Then, by design, $\mathcal{H}_X$ contains exactly one model for X.

If $\#\mathcal{H}_X = 1$, then we have determined the equation for X. Otherwise, we attempt to exclude elements of $\mathcal{H}_X$ as possible candidate equations using two strategies. If we know that X has real points using Ogg's criterion then we can exclude any models which lack real points. Our main tool, though, is as follows: we compute the number of edges in the dual graph of the minimal regular model of X over $\mathbb{Z}_p$ at all odd primes $p \mid D$ using the same method of Algorithm 3.1,[2] and we compare these counts to those of each hyperelliptic curve H with a model in $\mathcal{H}_X$. This is done in Magma using Tim Dokchitser's package `redlib` [Dok25], which is based on the works [Dok21, Mus24] as well as [NU73] in the genus 2 case. If there is at least one prime $p \mid D$ at which there is a discrepancy, then we discard the model for H from $\mathcal{H}_X$. The code for these computations are contained in the files `genus_2_bielliptic_eqns.m` and `narrowing_bielliptic_eqns.m` in [PS25b].

Once more, if we are left with a single candidate equation in $\mathcal{H}_X$ then that is the equation for X and we list the curve in Table 9 as stated in part (2).

For the 941 genus 2 Atkin–Lehner quotients which are not bielliptic via an Atkin–Lehner involution, we prove that 472 are not bielliptic by computing that $\mathrm{Jac}(X)$ is simple over $\mathbb{Q}$. The relevant code for these computations is included in the file

`computing_genus_2_bielliptics.m`

in [PS25b], and the 469 genus 2 quotients which are not handled by this check (as referenced in the statement of part (3) of this theorem) are listed in the file

`genus_2_remaining_non_AL_bielliptic_candidates.m`

in [PS25b] as well as in Table 10 as stated in part (3). □

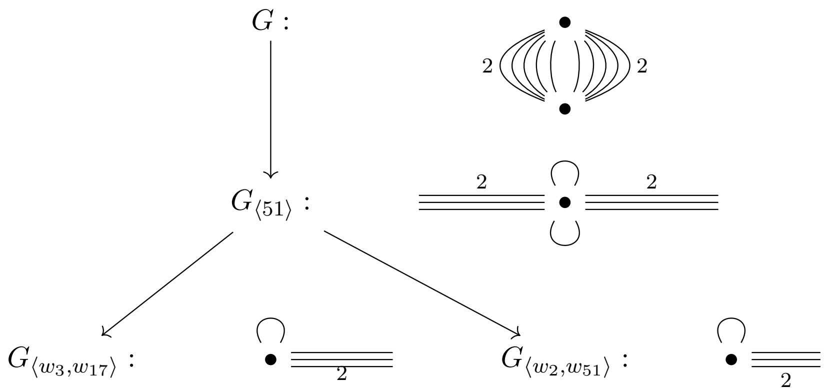
Example 8.4. Consider the genus 2 curve $X := X_0^{51}(2)/\langle w_{51} \rangle$, with bielliptic Atkin–Lehner quotients $E_1 := X/\langle w_3 \rangle$ and $E_2 := X/\langle w_2 \rangle$. From our work in Section 6, we know that E_1 is isomorphic over $\mathbb{Q}$ to the elliptic curve with Cremona label 102b1 and that E_2 is isomorphic to that with Cremona label 102a1. Using Proposition 8.2, we find that X is isomorphic over $\mathbb{Q}$ to one of the following two non-isomorphic candidate hyperelliptic curves (each coming from a rational point on the zero-dimensional scheme with equations in terms of the coefficients of short Weierstrass equations for E_1 and E_2

[2] Of course, this is coarser information than the reduction types themselves. In our case, we check that it is enough to distinguish between any candidates for which there is at least one odd prime $p \mid D$ at which their reduction types differ except for the curve $X_0^{14}(17)/\langle w_7, w_{34} \rangle$. In this case, it helps us exclude one candidate equation, but we are still left with two with the same reduction types at $p = 7$ and $p = 17$. Note that we ignore reductions at 2 when $2 \mid DN$ only because computations at 2 are not implemented in the referenced package; we are able to compute dual graphs at 2 for Atkin–Lehner quotients.

as given in the referenced proposition):

$$H_1 : y^2 = -216x^6 + 225x^4 + 126x^2 + 9$$
$$H_2 : y^2 = \frac{6912}{17}x^6 + \frac{585}{17}x^4 - \frac{1827}{17}x^2 + \frac{288}{17}\ .$$

At $p = 17$, we have the following graphs G_W corresponding to each $W \le W_0(51, 2)$ in $\{\{\mathrm{Id}\}, \langle w_{51}\rangle, \langle w_3, w_{17}\rangle, \langle w_2, w_{51}\rangle\}$. From each G_W, we can minimize and resolve to get the dual graph of the minimal regular model of $X_0^D(N)/W$ over $\mathbb{Z}_p$:



Using the library [Dok25], we find that H_1 has reduction type $I_{1,1,0}$ while H_2 has reduction type $I_1 - I_1 - 1$ using the nomenclature of Namikawa–Ueno's classification [NU73, p. 179]. The former matches the minimization of $G_{\langle 51\rangle}$, and so $X \cong H_1$.

Remark 8.5. There are exactly 231 bielliptic genus 2 curves X with at least one bielliptic Atkin–Lehner quotient for which we do not determine the isomorphism class in Theorem 8.3 and which are not among the three curves mentioned in the proof of this theorem. For each of these, we do compute a finite list of candidate models for X. For 147 such curves, we narrow the determination down to just two isomorphism classes. We do not list those curves and their candidate models in this paper for sake of brevity, but they can be found in the file `genus_2_bielliptics_eqn_not_determined.m` in [PS25b].

Remark 8.6. There is in fact one more strategy we attempted towards Theorem 8.3 to exclude candidate equations. Let $X = X_0^D(N)/W_X$ and $Y = X_0^D(N)/W_Y$ be two bielliptic genus 2 curves for which we have finite lists of candidate models and which share a common elliptic curve quotient. For a given candidate equation $h \in \mathcal{H}_X$ for X, we can let H_X denote the corresponding hyperelliptic curve for X. Using the command `Degree2Subcovers` in Magma, we get models for the two bielliptic quotients of H_X. If there exists no candidate model H_Y for Y for which at least one of the bielliptic quotients of H_X is isomorphic to a bielliptic quotient of H_Y, then $H_X \ncong X$ and h can be discarded.

While this strategy helps narrow down some candidates, it does not uniquely determine the isomorphism class of any remaining curves. However, a related argument lets us determine the isomorphism class of the Jacobian of the only genus 1 quotient with N squarefree we did not already handle in Section 6, namely the curve $Z = X_0^{33}(2)/\langle w_6, w_{22}\rangle$. The genus 2 curves $X = X_0^{33}(2)/\langle w_{22}\rangle$ and $Y = X_0^{33}(2)/\langle w_{33}\rangle$ are both degree 2 covers of Z, and we have two candidate models each for X and Y following the strategies of the proof of Theorem 8.3. We know from our prior work that Z has Cremona reference either 66b1 or 66b3, and 66b1 is the only reference among this pair that appears among those of the bielliptic quotients of the candidate models for Y. We then know the isomorphism class of Z, though 66b1 appears as a bielliptic quotient of both candidate models of X and of Z and therefore we don't learn the isomorphism class of either of these genus 2 curves.

Remark 8.7. One would of course like to obtain a stronger version of part (3) of Theorem 8.3, determining which, if any, of the referenced Atkin–Lehner quotients are bielliptic by a non-Atkin–Lehner bielliptic involution. This seems to be a significant problem on its own, requiring further tools, and so we do not approach it further in this work. One may, for example, hope to use results on Atkin–Lehner quotients having no non-Atkin–Lehner involutions to extinguish this possibility for certain pairs, but the only result we are aware of on this ([MPSS25, Theorem 1.3]) unfortunately does not apply to *any* of the remaining genus 2 candidates from part (3) of Theorem 8.3.

9. Tables

Table 2: All 1270 genus 1 curves $X = X_0^D(N)/W$ with $W \leq W_0(D,N)$ nontrivial and N squarefree. For each X, we give the isomorphism class of the elliptic curve $\mathrm{Jac}(X)$ via its Cremona label, ref.

D	N	W	ref	D	N	W	ref
6	5	$\langle w_3\rangle$	30a8	6	5	$\langle w_5\rangle$	30a7
6	5	$\langle w_{15}\rangle$	30a3	6	5	$\langle w_3, w_5\rangle$	30a6
6	7	$\langle w_2\rangle$	42a5	6	7	$\langle w_7\rangle$	42a2
6	7	$\langle w_{14}\rangle$	42a6	6	7	$\langle w_2, w_7\rangle$	42a3
6	11	$\langle w_6\rangle$	66c3	6	11	$\langle w_{22}\rangle$	66b2
6	11	$\langle w_{33}\rangle$	66a3	6	11	$\langle w_2, w_3\rangle$	66c4
6	11	$\langle w_2, w_{11}\rangle$	66b4	6	11	$\langle w_3, w_{11}\rangle$	66a4
6	13	$\langle w_6\rangle$	78a4	6	13	$\langle w_{26}\rangle$	78a3
6	13	$\langle w_{39}\rangle$	78a1	6	13	$\langle w_6, w_{26}\rangle$	78a2
6	17	$\langle w_2\rangle$	102b3	6	17	$\langle w_{51}\rangle$	102c3
6	17	$\langle w_{102}\rangle$	102a1	6	17	$\langle w_2, w_3\rangle$	102b6
6	17	$\langle w_3, w_{17}\rangle$	102c4	6	17	$\langle w_6, w_{17}\rangle$	102a2
6	19	$\langle w_3\rangle$	114a3	6	19	$\langle w_{19}\rangle$	114c2
6	19	$\langle w_{57}\rangle$	114b1	6	19	$\langle w_2, w_3\rangle$	114a4
6	19	$\langle w_2, w_{19}\rangle$	114c4	6	19	$\langle w_6, w_{38}\rangle$	114b2
6	23	$\langle w_{46}\rangle$	138c2	6	23	$\langle w_{69}\rangle$	138b3
6	23	$\langle w_{138}\rangle$	138a1	6	23	$\langle w_2, w_{23}\rangle$	138c3
6	23	$\langle w_3, w_{23}\rangle$	138b4	6	23	$\langle w_6, w_{23}\rangle$	138a2
6	29	$\langle w_2, w_3\rangle$	174b2	6	29	$\langle w_2, w_{29}\rangle$	174c1
6	29	$\langle w_6, w_{58}\rangle$	174e1	6	31	$\langle w_2, w_3\rangle$	186b2
6	31	$\langle w_3, w_{31}\rangle$	186c1	6	31	$\langle w_6, w_{62}\rangle$	186a1
6	35	$\langle w_2, w_{105}\rangle$	42a1	6	35	$\langle w_3, w_{70}\rangle$	30a1
6	35	$\langle w_5, w_{21}\rangle$	30a3	6	35	$\langle w_6, w_{10}\rangle$	210a3
6	35	$\langle w_6, w_{14}\rangle$	210e2	6	35	$\langle w_6, w_{35}\rangle$	210d1
6	35	$\langle w_7, w_{10}\rangle$	42a2	6	35	$\langle w_{10}, w_{42}\rangle$	210b1
6	35	$\langle w_{21}, w_{30}\rangle$	210c1	6	35	$\langle w_2, w_3, w_5\rangle$	210a6
6	35	$\langle w_2, w_3, w_7\rangle$	210e3	6	35	$\langle w_2, w_5, w_7\rangle$	42a3
6	35	$\langle w_2, w_7, w_{15}\rangle$	42a2	6	35	$\langle w_2, w_{15}, w_{21}\rangle$	210c2
6	35	$\langle w_3, w_5, w_7\rangle$	30a6	6	35	$\langle w_3, w_5, w_{14}\rangle$	30a2

6	35	$\langle w_3, w_{10}, w_{14}\rangle$	210b2	6	35	$\langle w_5, w_6, w_7\rangle$	210d2
6	37	$\langle w_2, w_3\rangle$	222a2	6	37	$\langle w_2, w_{37}\rangle$	222b1
6	37	$\langle w_3, w_{37}\rangle$	222d1	6	41	$\langle w_{246}\rangle$	246d1
6	41	$\langle w_2, w_{41}\rangle$	246a1	6	41	$\langle w_6, w_{41}\rangle$	246d2
6	41	$\langle w_6, w_{82}\rangle$	246g1	6	43	$\langle w_{129}\rangle$	258b1
6	43	$\langle w_3, w_{86}\rangle$	258c1	6	43	$\langle w_6, w_{43}\rangle$	258a1
6	43	$\langle w_6, w_{86}\rangle$	258b2	6	47	$\langle w_{94}\rangle$	282a2
6	47	$\langle w_2, w_{47}\rangle$	282a3	6	47	$\langle w_2, w_{141}\rangle$	282b1
6	53	$\langle w_2, w_{53}\rangle$	318a1	6	53	$\langle w_2, w_{159}\rangle$	318d1
6	53	$\langle w_6, w_{53}\rangle$	318c1	6	53	$\langle w_6, w_{106}\rangle$	318e1
6	55	$\langle w_5, w_{66}\rangle$	30a1	6	55	$\langle w_6, w_{11}\rangle$	330b2
6	55	$\langle w_{30}, w_{55}\rangle$	330c1	6	55	$\langle w_2, w_3, w_5\rangle$	66c3
6	55	$\langle w_2, w_3, w_{11}\rangle$	330b3	6	55	$\langle w_2, w_3, w_{55}\rangle$	66c1
6	55	$\langle w_2, w_{11}, w_{15}\rangle$	66b1	6	55	$\langle w_2, w_{15}, w_{33}\rangle$	330c2
6	55	$\langle w_3, w_5, w_{22}\rangle$	30a2	6	55	$\langle w_3, w_{10}, w_{11}\rangle$	66a1
6	55	$\langle w_5, w_6, w_{22}\rangle$	330a1	6	55	$\langle w_6, w_{10}, w_{22}\rangle$	330e1
6	59	$\langle w_2, w_{177}\rangle$	354f1	6	59	$\langle w_6, w_{59}\rangle$	354c1
6	59	$\langle w_6, w_{118}\rangle$	354d1	6	61	$\langle w_2, w_{61}\rangle$	366d1
6	61	$\langle w_2, w_{183}\rangle$	366g1	6	61	$\langle w_3, w_{61}\rangle$	366c1
6	61	$\langle w_3, w_{122}\rangle$	366f1	6	65	$\langle w_3, w_{130}\rangle$	30a1
6	65	$\langle w_{10}, w_{39}\rangle$	78a1	6	65	$\langle w_{13}, w_{30}\rangle$	390a1
6	65	$\langle w_2, w_3, w_5\rangle$	390c3	6	65	$\langle w_2, w_5, w_{13}\rangle$	390e1
6	65	$\langle w_2, w_{15}, w_{39}\rangle$	390b1	6	65	$\langle w_3, w_5, w_{26}\rangle$	30a2
6	65	$\langle w_3, w_{10}, w_{26}\rangle$	390d1	6	65	$\langle w_5, w_6, w_{13}\rangle$	390a2
6	65	$\langle w_5, w_6, w_{26}\rangle$	78a1	6	65	$\langle w_6, w_{10}, w_{13}\rangle$	390f1
6	65	$\langle w_6, w_{10}, w_{26}\rangle$	78a2	6	67	$\langle w_3, w_{67}\rangle$	402b1
6	67	$\langle w_3, w_{134}\rangle$	402d1	6	67	$\langle w_6, w_{67}\rangle$	402a1
6	71	$\langle w_{426}\rangle$	426b1	6	71	$\langle w_6, w_{71}\rangle$	426b2
6	73	$\langle w_2, w_{219}\rangle$	438f1	6	73	$\langle w_3, w_{73}\rangle$	438e1
6	73	$\langle w_6, w_{73}\rangle$	438c1	6	77	$\langle w_2, w_3, w_{11}\rangle$	462g3
6	77	$\langle w_2, w_3, w_{77}\rangle$	66c1	6	77	$\langle w_2, w_7, w_{33}\rangle$	42a1
6	77	$\langle w_3, w_{11}, w_{14}\rangle$	66a1	6	77	$\langle w_6, w_7, w_{11}\rangle$	462a1
6	77	$\langle w_6, w_7, w_{22}\rangle$	462b1	6	77	$\langle w_6, w_{14}, w_{22}\rangle$	462c1
6	79	$\langle w_3, w_{158}\rangle$	474b1	6	79	$\langle w_6, w_{79}\rangle$	474a1
6	83	$\langle w_3, w_{166}\rangle$	498b1	6	85	$\langle w_{15}, w_{34}\rangle$	30a1
6	85	$\langle w_2, w_3, w_{17}\rangle$	510g1	6	85	$\langle w_2, w_3, w_{85}\rangle$	102b1
6	85	$\langle w_2, w_5, w_{17}\rangle$	510c1	6	85	$\langle w_2, w_5, w_{51}\rangle$	510d1
6	85	$\langle w_2, w_{15}, w_{51}\rangle$	510e1	6	85	$\langle w_3, w_5, w_{34}\rangle$	30a2
6	85	$\langle w_3, w_{10}, w_{17}\rangle$	102c1	6	85	$\langle w_3, w_{10}, w_{34}\rangle$	510b1
6	85	$\langle w_5, w_6, w_{17}\rangle$	102a1	6	85	$\langle w_5, w_6, w_{34}\rangle$	510a1
6	89	$\langle w_2, w_{267}\rangle$	534a1	6	91	$\langle w_2, w_{273}\rangle$	42a1
6	91	$\langle w_3, w_{182}\rangle$	546c1	6	91	$\langle w_6, w_{91}\rangle$	78a1
6	91	$\langle w_2, w_7, w_{39}\rangle$	42a2	6	91	$\langle w_3, w_7, w_{13}\rangle$	546b1
6	91	$\langle w_3, w_7, w_{26}\rangle$	546c2	6	91	$\langle w_3, w_{14}, w_{26}\rangle$	546d1
6	91	$\langle w_6, w_{14}, w_{26}\rangle$	78a2	6	95	$\langle w_{10}, w_{114}\rangle$	570f1
6	95	$\langle w_2, w_3, w_{95}\rangle$	114a1	6	95	$\langle w_2, w_{15}, w_{19}\rangle$	114c1
6	95	$\langle w_3, w_5, w_{38}\rangle$	30a1	6	95	$\langle w_3, w_{10}, w_{19}\rangle$	570e1

6	95	$\langle w_3, w_{10}, w_{38}\rangle$	570f2	6	95	$\langle w_5, w_6, w_{19}\rangle$	570a1
6	97	$\langle w_2, w_{291}\rangle$	582c1	6	97	$\langle w_6, w_{97}\rangle$	582a1
6	101	$\langle w_2, w_3, w_{101}\rangle$	606e1	6	103	$\langle w_2, w_3, w_{103}\rangle$	618f1
6	107	$\langle w_2, w_{321}\rangle$	642c1	6	107	$\langle w_6, w_{214}\rangle$	642a1
6	109	$\langle w_2, w_{327}\rangle$	654b1	6	109	$\langle w_3, w_{218}\rangle$	654a1
6	113	$\langle w_2, w_{339}\rangle$	678c1	6	113	$\langle w_3, w_{226}\rangle$	678b1
6	115	$\langle w_2, w_5, w_{69}\rangle$	690h1	6	115	$\langle w_2, w_{15}, w_{23}\rangle$	138c1
6	115	$\langle w_3, w_{10}, w_{23}\rangle$	138b1	6	115	$\langle w_3, w_{10}, w_{46}\rangle$	690f1
6	115	$\langle w_5, w_6, w_{46}\rangle$	690b1	6	119	$\langle w_{21}, w_{34}\rangle$	714d1
6	119	$\langle w_2, w_3, w_{119}\rangle$	102b1	6	119	$\langle w_3, w_{14}, w_{17}\rangle$	102c1
6	119	$\langle w_6, w_7, w_{34}\rangle$	714b1	6	119	$\langle w_6, w_{14}, w_{34}\rangle$	714d2
6	127	$\langle w_2, w_3, w_{127}\rangle$	762e1	6	131	$\langle w_2, w_3, w_{131}\rangle$	786l1
6	133	$\langle w_2, w_3, w_7, w_{19}\rangle$	798h1	6	137	$\langle w_3, w_{274}\rangle$	822d1
6	139	$\langle w_2, w_3, w_{139}\rangle$	834g1	6	143	$\langle w_2, w_3, w_{143}\rangle$	66c1
6	143	$\langle w_2, w_{13}, w_{33}\rangle$	858g1	6	143	$\langle w_6, w_{11}, w_{26}\rangle$	78a1
6	143	$\langle w_6, w_{13}, w_{22}\rangle$	858a1	6	145	$\langle w_2, w_3, w_{145}\rangle$	174b1
6	145	$\langle w_2, w_5, w_{87}\rangle$	870e1	6	145	$\langle w_2, w_{15}, w_{87}\rangle$	870g1
6	145	$\langle w_3, w_{10}, w_{58}\rangle$	870d1	6	149	$\langle w_2, w_3, w_{149}\rangle$	894g1
6	151	$\langle w_2, w_3, w_{151}\rangle$	906h1	6	155	$\langle w_2, w_3, w_{155}\rangle$	186b1
6	155	$\langle w_2, w_5, w_{93}\rangle$	930m1	6	155	$\langle w_3, w_5, w_{62}\rangle$	30a1
6	155	$\langle w_5, w_6, w_{31}\rangle$	930a1	6	155	$\langle w_6, w_{10}, w_{31}\rangle$	930c1
6	161	$\langle w_2, w_7, w_{69}\rangle$	42a1	6	161	$\langle w_2, w_{21}, w_{69}\rangle$	966h1
6	161	$\langle w_3, w_7, w_{46}\rangle$	966e1	6	161	$\langle w_3, w_{14}, w_{23}\rangle$	138b1
6	163	$\langle w_2, w_3, w_{163}\rangle$	978g1	6	167	$\langle w_2, w_3, w_{167}\rangle$	1002e1
6	179	$\langle w_2, w_3, w_{179}\rangle$	1074h1	6	181	$\langle w_2, w_3, w_{181}\rangle$	1086d1
6	185	$\langle w_2, w_3, w_5, w_{37}\rangle$	1110m1	6	187	$\langle w_2, w_3, w_{11}, w_{17}\rangle$	1122i1
6	191	$\langle w_2, w_3, w_{191}\rangle$	1146b1	6	193	$\langle w_2, w_3, w_{193}\rangle$	1158g1
6	203	$\langle w_2, w_3, w_{203}\rangle$	174b1	6	203	$\langle w_3, w_7, w_{58}\rangle$	1218c1
6	203	$\langle w_6, w_{14}, w_{29}\rangle$	1218b1	6	205	$\langle w_2, w_3, w_5, w_{41}\rangle$	1230h1
6	209	$\langle w_2, w_3, w_{11}, w_{19}\rangle$	1254i1	6	211	$\langle w_2, w_3, w_{211}\rangle$	1266d1
6	215	$\langle w_2, w_3, w_5, w_{43}\rangle$	1290m1	6	217	$\langle w_2, w_3, w_7, w_{31}\rangle$	1302m1
6	221	$\langle w_2, w_3, w_{13}, w_{17}\rangle$	1326e1	6	227	$\langle w_2, w_3, w_{227}\rangle$	1362d1
6	235	$\langle w_2, w_{15}, w_{47}\rangle$	282a1	6	235	$\langle w_6, w_{10}, w_{94}\rangle$	1410d1
6	241	$\langle w_2, w_3, w_{241}\rangle$	1446f1	6	247	$\langle w_2, w_3, w_{13}, w_{19}\rangle$	1482i1
6	253	$\langle w_2, w_3, w_{11}, w_{23}\rangle$	1518p1	6	259	$\langle w_2, w_3, w_7, w_{37}\rangle$	1554k1
6	265	$\langle w_2, w_3, w_5, w_{53}\rangle$	1590s1	6	287	$\langle w_2, w_3, w_7, w_{41}\rangle$	1722n1
6	295	$\langle w_2, w_3, w_5, w_{59}\rangle$	1770g1	6	301	$\langle w_2, w_3, w_7, w_{43}\rangle$	1806l1
6	305	$\langle w_2, w_3, w_5, w_{61}\rangle$	1830j1	6	323	$\langle w_2, w_3, w_{17}, w_{19}\rangle$	1938h1
6	385	$\langle w_2, w_3, w_5, w_7, w_{11}\rangle$	2310q1	6	455	$\langle w_2, w_3, w_5, w_7, w_{13}\rangle$	2730y1
10	3	$\langle w_6\rangle$	30a5	10	3	$\langle w_{10}\rangle$	30a4
10	3	$\langle w_{15}\rangle$	30a1	10	3	$\langle w_6, w_{10}\rangle$	30a2
10	7	$\langle w_2\rangle$	70a3	10	7	$\langle w_7\rangle$	70a1
10	7	$\langle w_{14}\rangle$	70a4	10	7	$\langle w_2, w_7\rangle$	70a2
10	11	$\langle w_2, w_5\rangle$	110a2	10	11	$\langle w_2, w_{11}\rangle$	110b2
10	11	$\langle w_{10}, w_{22}\rangle$	110c1	10	13	$\langle w_{10}\rangle$	130b2
10	13	$\langle w_{13}\rangle$	130c1	10	13	$\langle w_{130}\rangle$	130a1
10	13	$\langle w_2, w_5\rangle$	130b4	10	13	$\langle w_2, w_{13}\rangle$	130c2

10	13	$\langle w_5, w_{26}\rangle$	130a2	10	17	$\langle w_{170}\rangle$	170a1
10	17	$\langle w_2, w_{17}\rangle$	170c2	10	17	$\langle w_5, w_{17}\rangle$	170d2
10	17	$\langle w_5, w_{34}\rangle$	170a2	10	19	$\langle w_2, w_5\rangle$	190c2
10	19	$\langle w_2, w_{95}\rangle$	190a1	10	19	$\langle w_{10}, w_{19}\rangle$	190b1
10	21	$\langle w_2, w_{105}\rangle$	70a1	10	21	$\langle w_3, w_5\rangle$	210c1
10	21	$\langle w_3, w_{14}\rangle$	70a1	10	21	$\langle w_3, w_{70}\rangle$	210d1
10	21	$\langle w_5, w_7\rangle$	210e1	10	21	$\langle w_5, w_{21}\rangle$	210b1
10	21	$\langle w_6, w_{35}\rangle$	30a3	10	21	$\langle w_{10}, w_{42}\rangle$	30a1
10	21	$\langle w_{30}, w_{35}\rangle$	210a1	10	21	$\langle w_2, w_3, w_5\rangle$	210c2
10	21	$\langle w_2, w_3, w_7\rangle$	70a2	10	21	$\langle w_2, w_5, w_7\rangle$	210e2
10	21	$\langle w_2, w_7, w_{15}\rangle$	70a2	10	21	$\langle w_2, w_{15}, w_{21}\rangle$	210a2
10	21	$\langle w_3, w_7, w_{10}\rangle$	210d2	10	21	$\langle w_5, w_6, w_{14}\rangle$	210b2
10	21	$\langle w_6, w_7, w_{10}\rangle$	30a2	10	21	$\langle w_6, w_{10}, w_{14}\rangle$	30a6
10	29	$\langle w_{290}\rangle$	290a1	10	29	$\langle w_{10}, w_{29}\rangle$	290a2
10	31	$\langle w_{62}\rangle$	310a1	10	31	$\langle w_2, w_{31}\rangle$	310a2
10	31	$\langle w_2, w_{155}\rangle$	310b1	10	33	$\langle w_2, w_{55}\rangle$	330b1
10	33	$\langle w_3, w_{110}\rangle$	330e1	10	33	$\langle w_6, w_{110}\rangle$	30a3
10	33	$\langle w_2, w_5, w_{11}\rangle$	330b2	10	33	$\langle w_2, w_5, w_{33}\rangle$	110a1
10	33	$\langle w_2, w_{11}, w_{15}\rangle$	110b1	10	33	$\langle w_3, w_5, w_{22}\rangle$	330e2
10	33	$\langle w_6, w_{10}, w_{11}\rangle$	30a6	10	37	$\langle w_5, w_{37}\rangle$	370b1
10	37	$\langle w_{10}, w_{37}\rangle$	370a1	10	37	$\langle w_{10}, w_{74}\rangle$	370c2
10	39	$\langle w_3, w_{65}\rangle$	390f1	10	39	$\langle w_6, w_{65}\rangle$	30a3
10	39	$\langle w_{30}, w_{65}\rangle$	390c1	10	39	$\langle w_2, w_5, w_{39}\rangle$	130b1
10	39	$\langle w_2, w_{13}, w_{15}\rangle$	130c1	10	39	$\langle w_2, w_{15}, w_{39}\rangle$	390c2
10	39	$\langle w_3, w_5, w_{13}\rangle$	390f2	10	39	$\langle w_3, w_5, w_{26}\rangle$	130a1
10	39	$\langle w_3, w_{10}, w_{13}\rangle$	390a1	10	39	$\langle w_6, w_{10}, w_{26}\rangle$	30a6
10	41	$\langle w_2, w_{205}\rangle$	410d1	10	41	$\langle w_5, w_{82}\rangle$	410a1
10	43	$\langle w_2, w_5, w_{43}\rangle$	430d1	10	47	$\langle w_2, w_5, w_{47}\rangle$	470f1
10	51	$\langle w_2, w_3, w_{85}\rangle$	510d1	10	51	$\langle w_2, w_5, w_{17}\rangle$	510g1
10	51	$\langle w_2, w_{15}, w_{17}\rangle$	170c1	10	51	$\langle w_2, w_{15}, w_{51}\rangle$	510f1
10	51	$\langle w_3, w_5, w_{34}\rangle$	170a1	10	51	$\langle w_5, w_6, w_{17}\rangle$	170d1
10	51	$\langle w_6, w_{10}, w_{17}\rangle$	30a3	10	53	$\langle w_2, w_{265}\rangle$	530d1
10	57	$\langle w_2, w_3, w_{19}\rangle$	570g1	10	57	$\langle w_2, w_3, w_{95}\rangle$	190a1
10	57	$\langle w_2, w_5, w_{57}\rangle$	190c1	10	57	$\langle w_3, w_5, w_{38}\rangle$	570c1
10	57	$\langle w_3, w_{10}, w_{38}\rangle$	570b1	10	57	$\langle w_5, w_6, w_{38}\rangle$	570f1
10	57	$\langle w_6, w_{10}, w_{38}\rangle$	30a3	10	59	$\langle w_2, w_5, w_{59}\rangle$	590d1
10	61	$\langle w_5, w_{122}\rangle$	610b1	10	61	$\langle w_{10}, w_{122}\rangle$	610a1
10	69	$\langle w_2, w_3, w_{115}\rangle$	690h1	10	69	$\langle w_2, w_{15}, w_{69}\rangle$	690i1
10	69	$\langle w_3, w_{10}, w_{23}\rangle$	690a1	10	69	$\langle w_5, w_6, w_{46}\rangle$	690f1
10	69	$\langle w_6, w_{10}, w_{23}\rangle$	30a3	10	71	$\langle w_2, w_5, w_{71}\rangle$	710c1
10	73	$\langle w_2, w_5, w_{73}\rangle$	730j1	10	77	$\langle w_2, w_5, w_{77}\rangle$	110a1
10	77	$\langle w_2, w_7, w_{55}\rangle$	70a1	10	77	$\langle w_5, w_7, w_{22}\rangle$	770d1
10	77	$\langle w_{10}, w_{14}, w_{22}\rangle$	110c2	10	79	$\langle w_2, w_5, w_{79}\rangle$	790a1
10	87	$\langle w_2, w_3, w_5, w_{29}\rangle$	870f1	10	89	$\langle w_2, w_5, w_{89}\rangle$	890g1
10	91	$\langle w_2, w_5, w_7, w_{13}\rangle$	910k1	10	93	$\langle w_2, w_{15}, w_{31}\rangle$	310a1
10	93	$\langle w_3, w_{10}, w_{62}\rangle$	930b1	10	93	$\langle w_6, w_{10}, w_{62}\rangle$	30a3
10	109	$\langle w_2, w_5, w_{109}\rangle$	1090g1	10	111	$\langle w_2, w_3, w_5, w_{37}\rangle$	1110j1

10	113	$\langle w_2, w_5, w_{113}\rangle$	1130c1	10	119	$\langle w_2, w_5, w_7, w_{17}\rangle$	1190f1
10	133	$\langle w_2, w_5, w_7, w_{19}\rangle$	1330i1	10	141	$\langle w_2, w_3, w_5, w_{47}\rangle$	470f1
10	159	$\langle w_2, w_3, w_5, w_{53}\rangle$	1590n1	10	161	$\langle w_2, w_5, w_7, w_{23}\rangle$	1610f1
10	231	$\langle w_2, w_3, w_5, w_7, w_{11}\rangle$	2310n1	14	1	$\langle w_7\rangle$	14a1
14	3	$\langle w_2\rangle$	42a3	14	3	$\langle w_{21}\rangle$	14a1
14	3	$\langle w_{42}\rangle$	14a4	14	3	$\langle w_2, w_3\rangle$	42a5
14	3	$\langle w_3, w_7\rangle$	14a2	14	3	$\langle w_6, w_7\rangle$	14a6
14	5	$\langle w_2\rangle$	70a2	14	5	$\langle w_{35}\rangle$	14a3
14	5	$\langle w_{70}\rangle$	14a4	14	5	$\langle w_2, w_5\rangle$	70a3
14	5	$\langle w_5, w_7\rangle$	14a5	14	5	$\langle w_7, w_{10}\rangle$	14a6
14	11	$\langle w_2, w_{11}\rangle$	154b2	14	11	$\langle w_7, w_{11}\rangle$	14a3
14	11	$\langle w_7, w_{22}\rangle$	14a4	14	11	$\langle w_{11}, w_{14}\rangle$	154a1
14	11	$\langle w_{14}, w_{22}\rangle$	154c1	14	13	$\langle w_{182}\rangle$	14a1
14	13	$\langle w_2, w_7\rangle$	182b2	14	13	$\langle w_7, w_{26}\rangle$	14a2
14	13	$\langle w_{14}, w_{26}\rangle$	182c1	14	15	$\langle w_6, w_{14}\rangle$	210c2
14	15	$\langle w_{10}, w_{14}\rangle$	210a1	14	15	$\langle w_{14}, w_{15}\rangle$	210d1
14	15	$\langle w_2, w_3, w_7\rangle$	210c3	14	15	$\langle w_2, w_3, w_{35}\rangle$	42a1
14	15	$\langle w_2, w_5, w_7\rangle$	210a2	14	15	$\langle w_2, w_5, w_{21}\rangle$	70a1
14	15	$\langle w_2, w_{15}, w_{21}\rangle$	210e1	14	15	$\langle w_3, w_5, w_7\rangle$	14a3
14	15	$\langle w_3, w_5, w_{14}\rangle$	210d2	14	15	$\langle w_3, w_7, w_{10}\rangle$	14a4
14	15	$\langle w_5, w_6, w_7\rangle$	14a1	14	17	$\langle w_2, w_{119}\rangle$	238a1
14	17	$\langle w_2, w_7, w_{17}\rangle$	238a2	14	19	$\langle w_{266}\rangle$	14a1
14	19	$\langle w_7, w_{38}\rangle$	14a2	14	23	$\langle w_2, w_{161}\rangle$	322d1
14	29	$\langle w_2, w_{203}\rangle$	406c1	14	29	$\langle w_{14}, w_{29}\rangle$	406a1
14	31	$\langle w_2, w_{217}\rangle$	434d1	14	31	$\langle w_2, w_7, w_{31}\rangle$	434d2
14	33	$\langle w_2, w_3, w_7, w_{11}\rangle$	462e1	14	37	$\langle w_7, w_{74}\rangle$	14a1
14	39	$\langle w_2, w_3, w_{91}\rangle$	42a1	14	39	$\langle w_2, w_7, w_{39}\rangle$	182b1
14	39	$\langle w_3, w_7, w_{26}\rangle$	14a1	14	39	$\langle w_6, w_{13}, w_{14}\rangle$	546b1
14	43	$\langle w_7, w_{86}\rangle$	14a1	14	51	$\langle w_2, w_3, w_7, w_{17}\rangle$	238a1
14	53	$\langle w_2, w_7, w_{53}\rangle$	742g1	14	55	$\langle w_2, w_5, w_7, w_{11}\rangle$	770f1
14	57	$\langle w_2, w_{21}, w_{57}\rangle$	798i1	14	57	$\langle w_3, w_7, w_{38}\rangle$	14a1
14	57	$\langle w_3, w_{14}, w_{19}\rangle$	798a1	14	57	$\langle w_6, w_{14}, w_{19}\rangle$	798b1
14	61	$\langle w_2, w_7, w_{61}\rangle$	854d1	14	65	$\langle w_2, w_5, w_7, w_{13}\rangle$	910h1
14	67	$\langle w_2, w_7, w_{67}\rangle$	938c1	14	69	$\langle w_2, w_3, w_7, w_{23}\rangle$	966g1
14	79	$\langle w_2, w_7, w_{79}\rangle$	1106e1	14	85	$\langle w_2, w_5, w_7, w_{17}\rangle$	238a1
14	93	$\langle w_2, w_3, w_7, w_{31}\rangle$	434d1	14	95	$\langle w_2, w_5, w_7, w_{19}\rangle$	1330h1
15	1	$\langle w_5\rangle$	15a2	15	2	$\langle w_3\rangle$	30a3
15	2	$\langle w_{10}\rangle$	15a3	15	2	$\langle w_{30}\rangle$	15a8
15	2	$\langle w_2, w_3\rangle$	30a6	15	2	$\langle w_2, w_5\rangle$	15a1
15	2	$\langle w_5, w_6\rangle$	15a3	15	7	$\langle w_3\rangle$	105a2
15	7	$\langle w_7\rangle$	15a2	15	7	$\langle w_{105}\rangle$	15a3
15	7	$\langle w_3, w_5\rangle$	105a4	15	7	$\langle w_5, w_7\rangle$	15a6
15	7	$\langle w_5, w_{21}\rangle$	15a1	15	11	$\langle w_{55}\rangle$	15a2
15	11	$\langle w_5, w_{11}\rangle$	15a5	15	11	$\langle w_5, w_{33}\rangle$	15a3
15	13	$\langle w_{195}\rangle$	15a1	15	13	$\langle w_3, w_{13}\rangle$	195c1
15	13	$\langle w_5, w_{39}\rangle$	15a4	15	14	$\langle w_7, w_{10}\rangle$	15a1
15	14	$\langle w_2, w_3, w_7\rangle$	30a6	15	14	$\langle w_2, w_3, w_{35}\rangle$	30a2

15	14	$\langle w_2, w_5, w_7\rangle$	15a2	15	14	$\langle w_2, w_5, w_{21}\rangle$	15a3
15	14	$\langle w_2, w_7, w_{15}\rangle$	210d2	15	14	$\langle w_3, w_5, w_7\rangle$	210e2
15	14	$\langle w_3, w_5, w_{14}\rangle$	105a2	15	14	$\langle w_3, w_{10}, w_{14}\rangle$	210a2
15	14	$\langle w_5, w_6, w_7\rangle$	15a1	15	17	$\langle w_{255}\rangle$	15a1
15	17	$\langle w_5, w_{51}\rangle$	15a4	15	19	$\langle w_3, w_{95}\rangle$	285a2
15	19	$\langle w_5, w_{57}\rangle$	15a3	15	19	$\langle w_{15}, w_{19}\rangle$	285b2
15	22	$\langle w_{10}, w_{22}\rangle$	15a1	15	22	$\langle w_2, w_3, w_{55}\rangle$	30a2
15	22	$\langle w_2, w_5, w_{11}\rangle$	15a2	15	22	$\langle w_2, w_{15}, w_{33}\rangle$	330a2
15	22	$\langle w_3, w_5, w_{11}\rangle$	330b2	15	22	$\langle w_5, w_6, w_{11}\rangle$	15a1
15	23	$\langle w_3, w_5, w_{23}\rangle$	345f1	15	26	$\langle w_2, w_3, w_{65}\rangle$	30a2
15	26	$\langle w_2, w_5, w_{39}\rangle$	15a1	15	26	$\langle w_2, w_{13}, w_{15}\rangle$	390a2
15	26	$\langle w_3, w_{10}, w_{13}\rangle$	195c1	15	26	$\langle w_3, w_{10}, w_{26}\rangle$	390c1
15	26	$\langle w_6, w_{10}, w_{13}\rangle$	390e1	15	29	$\langle w_5, w_{87}\rangle$	15a1
15	29	$\langle w_{15}, w_{87}\rangle$	435b1	15	31	$\langle w_3, w_5, w_{31}\rangle$	465b3
15	34	$\langle w_2, w_3, w_{85}\rangle$	30a2	15	34	$\langle w_2, w_5, w_{51}\rangle$	15a1
15	34	$\langle w_3, w_5, w_{17}\rangle$	510g2	15	34	$\langle w_3, w_{10}, w_{34}\rangle$	510f3
15	34	$\langle w_6, w_{10}, w_{17}\rangle$	510c2	15	38	$\langle w_2, w_3, w_5, w_{19}\rangle$	570e4
15	41	$\langle w_3, w_{205}\rangle$	615b1	15	41	$\langle w_5, w_{123}\rangle$	15a1
15	46	$\langle w_2, w_3, w_5, w_{23}\rangle$	345f1	15	58	$\langle w_2, w_3, w_5, w_{29}\rangle$	870c1
15	61	$\langle w_3, w_5, w_{61}\rangle$	915d2	15	77	$\langle w_3, w_5, w_7, w_{11}\rangle$	1155k1
15	82	$\langle w_2, w_3, w_5, w_{41}\rangle$	1230d1	21	1	$\langle w_3\rangle$	21a6
21	2	$\langle w_2\rangle$	21a1	21	2	$\langle w_{21}\rangle$	42a1
21	2	$\langle w_{42}\rangle$	21a4	21	2	$\langle w_2, w_3\rangle$	21a2
21	2	$\langle w_3, w_{14}\rangle$	21a1	21	2	$\langle w_6, w_{14}\rangle$	42a2
21	5	$\langle w_{15}\rangle$	21a2	21	5	$\langle w_{21}\rangle$	105a2
21	5	$\langle w_{105}\rangle$	21a1	21	5	$\langle w_3, w_5\rangle$	21a5
21	5	$\langle w_3, w_7\rangle$	105a1	21	5	$\langle w_3, w_{35}\rangle$	21a3
21	10	$\langle w_2, w_{15}\rangle$	21a1	21	10	$\langle w_2, w_3, w_5\rangle$	21a2
21	10	$\langle w_2, w_3, w_{35}\rangle$	21a1	21	10	$\langle w_2, w_5, w_{21}\rangle$	210d2
21	10	$\langle w_3, w_5, w_7\rangle$	210a2	21	10	$\langle w_3, w_5, w_{14}\rangle$	21a1
21	10	$\langle w_3, w_7, w_{10}\rangle$	105a2	21	10	$\langle w_5, w_6, w_{14}\rangle$	42a2
21	10	$\langle w_6, w_7, w_{10}\rangle$	210c2	21	10	$\langle w_6, w_{10}, w_{14}\rangle$	42a3
21	11	$\langle w_{231}\rangle$	21a1	21	11	$\langle w_3, w_{77}\rangle$	21a3
21	13	$\langle w_3, w_{91}\rangle$	21a1	21	13	$\langle w_7, w_{39}\rangle$	273a1
21	17	$\langle w_3, w_7, w_{17}\rangle$	357d1	21	19	$\langle w_3, w_{133}\rangle$	21a1
21	19	$\langle w_7, w_{57}\rangle$	399b1	21	19	$\langle w_{19}, w_{21}\rangle$	399a1
21	22	$\langle w_2, w_3, w_{77}\rangle$	21a1	21	22	$\langle w_2, w_7, w_{33}\rangle$	462c4
21	22	$\langle w_2, w_{11}, w_{21}\rangle$	462a2	21	22	$\langle w_3, w_7, w_{11}\rangle$	462g2
21	22	$\langle w_6, w_{11}, w_{14}\rangle$	42a3	21	26	$\langle w_2, w_7, w_{39}\rangle$	273a1
21	26	$\langle w_2, w_{21}, w_{39}\rangle$	546a1	21	26	$\langle w_6, w_{14}, w_{26}\rangle$	42a3
21	31	$\langle w_3, w_7, w_{31}\rangle$	651d1	21	34	$\langle w_2, w_3, w_7, w_{17}\rangle$	357d1
21	37	$\langle w_3, w_7, w_{37}\rangle$	777g1	21	38	$\langle w_2, w_3, w_7, w_{19}\rangle$	798d1
21	55	$\langle w_3, w_5, w_7, w_{11}\rangle$	1155j1	22	3	$\langle w_3\rangle$	66b2
22	3	$\langle w_{11}\rangle$	66c1	22	3	$\langle w_{33}\rangle$	66a1
22	3	$\langle w_2, w_3\rangle$	66b4	22	3	$\langle w_2, w_{11}\rangle$	66c2
22	3	$\langle w_6, w_{22}\rangle$	66a2	22	5	$\langle w_2, w_5\rangle$	110b2
22	5	$\langle w_2, w_{11}\rangle$	110a1	22	5	$\langle w_5, w_{11}\rangle$	110c2

22	7	$\langle w_{14}\rangle$	154b2	22	7	$\langle w_{77}\rangle$	154c1
22	7	$\langle w_{154}\rangle$	154a1	22	7	$\langle w_2, w_7\rangle$	154b3
22	7	$\langle w_7, w_{11}\rangle$	154c2	22	7	$\langle w_7, w_{22}\rangle$	154a2
22	13	$\langle w_2, w_{13}\rangle$	286e1	22	13	$\langle w_2, w_{143}\rangle$	286b1
22	13	$\langle w_{13}, w_{22}\rangle$	286c1	22	13	$\langle w_{22}, w_{26}\rangle$	286a2
22	15	$\langle w_{15}, w_{66}\rangle$	330b1	22	15	$\langle w_2, w_3, w_{55}\rangle$	66b1
22	15	$\langle w_2, w_5, w_{11}\rangle$	66c1	22	15	$\langle w_2, w_5, w_{33}\rangle$	110b1
22	15	$\langle w_2, w_{15}, w_{33}\rangle$	330b2	22	15	$\langle w_3, w_{10}, w_{11}\rangle$	330e1
22	15	$\langle w_5, w_6, w_{11}\rangle$	110c1	22	15	$\langle w_5, w_6, w_{22}\rangle$	66a3
22	17	$\langle w_{374}\rangle$	374a1	22	17	$\langle w_{17}, w_{22}\rangle$	374a2
22	19	$\langle w_2, w_{209}\rangle$	418b1	22	21	$\langle w_2, w_7, w_{33}\rangle$	154b1
22	21	$\langle w_2, w_{21}, w_{33}\rangle$	462g1	22	21	$\langle w_3, w_{11}, w_{14}\rangle$	462c1
22	21	$\langle w_6, w_7, w_{11}\rangle$	154c1	22	21	$\langle w_6, w_{14}, w_{22}\rangle$	66a3
22	23	$\langle w_2, w_{11}, w_{23}\rangle$	506f1	22	31	$\langle w_2, w_{11}, w_{31}\rangle$	682b1
22	35	$\langle w_2, w_7, w_{55}\rangle$	154b1	22	35	$\langle w_5, w_7, w_{22}\rangle$	154a1
22	35	$\langle w_5, w_{11}, w_{14}\rangle$	110c1	22	39	$\langle w_2, w_3, w_{11}, w_{13}\rangle$	858g1
22	51	$\langle w_2, w_3, w_{11}, w_{17}\rangle$	1122f1	22	57	$\langle w_2, w_3, w_{11}, w_{19}\rangle$	1254h1
26	1	$\langle w_2\rangle$	26b2	26	1	$\langle w_{13}\rangle$	26a1
26	3	$\langle w_2, w_3\rangle$	26b2	26	3	$\langle w_2, w_{39}\rangle$	26b1
26	3	$\langle w_6, w_{13}\rangle$	26a3	26	5	$\langle w_{26}\rangle$	130b2
26	5	$\langle w_2, w_{13}\rangle$	130b4	26	5	$\langle w_2, w_{65}\rangle$	26b1
26	5	$\langle w_5, w_{13}\rangle$	26a2	26	7	$\langle w_2, w_{13}\rangle$	182b2
26	7	$\langle w_2, w_{91}\rangle$	26b1	26	7	$\langle w_{13}, w_{14}\rangle$	26a1
26	7	$\langle w_{14}, w_{26}\rangle$	182e1	26	11	$\langle w_2, w_{11}, w_{13}\rangle$	286b1
26	15	$\langle w_2, w_3, w_{65}\rangle$	26b1	26	15	$\langle w_2, w_5, w_{13}\rangle$	390c1
26	15	$\langle w_2, w_{13}, w_{15}\rangle$	130b1	26	15	$\langle w_3, w_5, w_{26}\rangle$	390a1
26	15	$\langle w_3, w_{10}, w_{26}\rangle$	390f1	26	15	$\langle w_5, w_6, w_{13}\rangle$	26a1
26	15	$\langle w_5, w_6, w_{26}\rangle$	390g1	26	17	$\langle w_{26}, w_{34}\rangle$	442c1
26	19	$\langle w_2, w_{13}, w_{19}\rangle$	494d1	26	21	$\langle w_2, w_3, w_{91}\rangle$	26b1
26	21	$\langle w_2, w_{13}, w_{21}\rangle$	182b1	26	21	$\langle w_6, w_7, w_{26}\rangle$	546b1
26	21	$\langle w_6, w_{14}, w_{26}\rangle$	182e1	33	1	$\langle w_{11}\rangle$	33a4
33	2	$\langle w_{66}\rangle$	33a2	33	2	$\langle w_2, w_3\rangle$	66a3
33	2	$\langle w_2, w_{11}\rangle$	33a2	33	2	$\langle w_3, w_{11}\rangle$	66c1
33	2	$\langle w_6, w_{11}\rangle$	33a1	33	2	$\langle w_6, w_{22}\rangle$	66b1
33	5	$\langle w_{55}\rangle$	33a1	33	5	$\langle w_5, w_{11}\rangle$	33a4
33	5	$\langle w_{11}, w_{15}\rangle$	33a1	33	7	$\langle w_{231}\rangle$	33a1
33	7	$\langle w_{11}, w_{21}\rangle$	33a3	33	10	$\langle w_2, w_3, w_{55}\rangle$	66a2
33	10	$\langle w_3, w_5, w_{11}\rangle$	66c2	33	10	$\langle w_3, w_{10}, w_{22}\rangle$	330b2
33	10	$\langle w_5, w_6, w_{11}\rangle$	33a1	33	10	$\langle w_6, w_{10}, w_{22}\rangle$	66b4
33	14	$\langle w_2, w_3, w_{77}\rangle$	66a2	33	14	$\langle w_2, w_7, w_{33}\rangle$	462a1
33	14	$\langle w_3, w_{14}, w_{22}\rangle$	462g1	33	14	$\langle w_6, w_7, w_{22}\rangle$	66b4
33	14	$\langle w_6, w_{11}, w_{14}\rangle$	33a1	33	23	$\langle w_3, w_{11}, w_{23}\rangle$	759b1
34	1	$\langle w_2\rangle$	34a4	34	3	$\langle w_{17}\rangle$	102b1
34	3	$\langle w_2, w_3\rangle$	34a3	34	3	$\langle w_2, w_{17}\rangle$	102b2
34	3	$\langle w_2, w_{51}\rangle$	34a1	34	3	$\langle w_3, w_{34}\rangle$	102a1
34	3	$\langle w_6, w_{34}\rangle$	102c1	34	5	$\langle w_2, w_{85}\rangle$	34a1
34	5	$\langle w_{10}, w_{17}\rangle$	170a1	34	5	$\langle w_{10}, w_{34}\rangle$	170d1

34	7	$\langle w_2, w_{17}\rangle$	238c1	34	7	$\langle w_7, w_{34}\rangle$	238b1
34	7	$\langle w_{14}, w_{34}\rangle$	238e1	34	11	$\langle w_2, w_{187}\rangle$	34a1
34	11	$\langle w_{11}, w_{34}\rangle$	374a1	34	13	$\langle w_2, w_{13}, w_{17}\rangle$	442b1
34	15	$\langle w_2, w_3, w_5, w_{17}\rangle$	510d1	34	21	$\langle w_2, w_3, w_7, w_{17}\rangle$	714f1
34	29	$\langle w_2, w_{17}, w_{29}\rangle$	986f1	34	33	$\langle w_2, w_3, w_{11}, w_{17}\rangle$	1122e1
35	1	$\langle w_7\rangle$	35a1	35	2	$\langle w_{35}\rangle$	70a1
35	2	$\langle w_2, w_7\rangle$	35a2	35	2	$\langle w_7, w_{10}\rangle$	35a3
35	2	$\langle w_{10}, w_{14}\rangle$	70a2	35	3	$\langle w_{35}\rangle$	105a2
35	3	$\langle w_5, w_7\rangle$	105a4	35	3	$\langle w_7, w_{15}\rangle$	35a3
35	6	$\langle w_2, w_3, w_{35}\rangle$	210d2	35	6	$\langle w_2, w_7, w_{15}\rangle$	35a1
35	6	$\langle w_3, w_5, w_7\rangle$	210c2	35	6	$\langle w_3, w_{10}, w_{14}\rangle$	70a2
35	6	$\langle w_5, w_6, w_7\rangle$	105a2	35	6	$\langle w_6, w_{10}, w_{14}\rangle$	70a2
35	11	$\langle w_5, w_7, w_{11}\rangle$	385b2	35	13	$\langle w_7, w_{65}\rangle$	35a1
35	13	$\langle w_{13}, w_{35}\rangle$	455a1	35	26	$\langle w_2, w_5, w_7, w_{13}\rangle$	910d1
38	1	$\langle w_2\rangle$	38b2	38	1	$\langle w_{19}\rangle$	38a1
38	3	$\langle w_{38}\rangle$	114a1	38	3	$\langle w_2, w_{19}\rangle$	114a2
38	3	$\langle w_2, w_{57}\rangle$	38b1	38	3	$\langle w_6, w_{19}\rangle$	38a3
38	5	$\langle w_2, w_5, w_{19}\rangle$	190a1	38	7	$\langle w_2, w_{133}\rangle$	38b1
38	7	$\langle w_{14}, w_{19}\rangle$	38a1	38	11	$\langle w_2, w_{11}, w_{19}\rangle$	418b1
38	13	$\langle w_{13}, w_{38}\rangle$	494a1	38	13	$\langle w_{19}, w_{26}\rangle$	38a1
38	15	$\langle w_2, w_3, w_5, w_{19}\rangle$	190a1	38	17	$\langle w_2, w_{17}, w_{19}\rangle$	646d1
38	21	$\langle w_2, w_3, w_7, w_{19}\rangle$	798g1	38	39	$\langle w_2, w_3, w_{13}, w_{19}\rangle$	1482g1
39	1	$\langle w_{13}\rangle$	39a1	39	2	$\langle w_6, w_{13}\rangle$	39a4
39	5	$\langle w_{13}, w_{15}\rangle$	39a1	39	5	$\langle w_{15}, w_{39}\rangle$	195d1
39	7	$\langle w_7, w_{13}\rangle$	39a1	39	10	$\langle w_2, w_5, w_{39}\rangle$	390a2
39	10	$\langle w_3, w_5, w_{13}\rangle$	390c2	39	10	$\langle w_5, w_6, w_{26}\rangle$	390e2
39	10	$\langle w_6, w_{10}, w_{26}\rangle$	195d1	39	11	$\langle w_3, w_{11}, w_{13}\rangle$	429b1
39	14	$\langle w_2, w_3, w_7, w_{13}\rangle$	546c2	39	19	$\langle w_3, w_{13}, w_{19}\rangle$	741e1
46	1	$\langle w_{23}\rangle$	46a1	46	3	$\langle w_2, w_3\rangle$	138c2
46	3	$\langle w_3, w_{23}\rangle$	46a1	46	3	$\langle w_3, w_{46}\rangle$	138a1
46	3	$\langle w_6, w_{23}\rangle$	46a1	46	3	$\langle w_6, w_{46}\rangle$	138b1
46	5	$\langle w_{230}\rangle$	46a1	46	5	$\langle w_{10}, w_{23}\rangle$	46a2
46	7	$\langle w_2, w_{161}\rangle$	322d1	46	7	$\langle w_2, w_7, w_{23}\rangle$	322d2
46	11	$\langle w_2, w_{11}, w_{23}\rangle$	506e1	46	13	$\langle w_2, w_{13}, w_{23}\rangle$	598d1
46	15	$\langle w_2, w_3, w_5, w_{23}\rangle$	690h1	46	21	$\langle w_2, w_3, w_7, w_{23}\rangle$	322d1
51	1	$\langle w_3\rangle$	51a2	51	2	$\langle w_2, w_{51}\rangle$	102a1
51	2	$\langle w_3, w_{17}\rangle$	102b1	51	2	$\langle w_3, w_{34}\rangle$	51a1
51	5	$\langle w_3, w_{85}\rangle$	51a1	51	10	$\langle w_2, w_5, w_{51}\rangle$	102a2
51	10	$\langle w_5, w_6, w_{34}\rangle$	510c1	51	13	$\langle w_3, w_{13}, w_{17}\rangle$	663c2
55	1	$\langle w_5\rangle$	55a1	55	2	$\langle w_5, w_{11}\rangle$	110a1
55	2	$\langle w_5, w_{22}\rangle$	55a4	55	2	$\langle w_{10}, w_{22}\rangle$	110b1
55	3	$\langle w_3, w_5\rangle$	55a1	55	3	$\langle w_5, w_{33}\rangle$	55a1
55	6	$\langle w_2, w_3, w_5, w_{11}\rangle$	330e2	55	7	$\langle w_5, w_7, w_{11}\rangle$	385a1
57	1	$\langle w_{57}\rangle$	57a1	57	2	$\langle w_2, w_{19}\rangle$	114b1
57	2	$\langle w_2, w_{57}\rangle$	57a1	57	2	$\langle w_3, w_{19}\rangle$	114a1
57	5	$\langle w_{15}, w_{19}\rangle$	285a1	57	5	$\langle w_3, w_5, w_{19}\rangle$	285a2
57	7	$\langle w_{19}, w_{21}\rangle$	399b2	57	10	$\langle w_2, w_3, w_5, w_{19}\rangle$	285a1

57	14	$\langle w_2, w_3, w_7, w_{19}\rangle$	798c2	58	1	$\langle w_2\rangle$	58b2
58	1	$\langle w_{58}\rangle$	58a1	58	3	$\langle w_2, w_{29}\rangle$	174b1
58	3	$\langle w_2, w_{87}\rangle$	58b1	58	3	$\langle w_3, w_{29}\rangle$	174e1
58	3	$\langle w_3, w_{58}\rangle$	58a1	58	5	$\langle w_2, w_{145}\rangle$	58b1
58	7	$\langle w_2, w_7, w_{29}\rangle$	406c1	58	13	$\langle w_2, w_{13}, w_{29}\rangle$	754d1
58	15	$\langle w_2, w_3, w_5, w_{29}\rangle$	870e1	62	1	$\langle w_2\rangle$	62a3
62	3	$\langle w_2, w_{31}\rangle$	186b1	62	3	$\langle w_2, w_{93}\rangle$	62a1
62	3	$\langle w_6, w_{62}\rangle$	186c1	62	5	$\langle w_2, w_5, w_{31}\rangle$	310b1
62	7	$\langle w_7, w_{62}\rangle$	434a1	62	11	$\langle w_2, w_{11}, w_{31}\rangle$	682a1
65	1	$\langle w_{65}\rangle$	65a1	65	2	$\langle w_2, w_5, w_{13}\rangle$	130a1
65	3	$\langle w_3, w_{65}\rangle$	65a2	65	6	$\langle w_2, w_3, w_5, w_{13}\rangle$	130a2
65	7	$\langle w_5, w_7, w_{13}\rangle$	455b4	69	1	$\langle w_3\rangle$	69a2
69	2	$\langle w_2, w_{69}\rangle$	138a1	69	2	$\langle w_3, w_{46}\rangle$	69a1
69	2	$\langle w_6, w_{46}\rangle$	138c1	69	5	$\langle w_3, w_5, w_{23}\rangle$	345b1
74	3	$\langle w_2, w_{37}\rangle$	222a1	74	3	$\langle w_6, w_{74}\rangle$	222d1
74	5	$\langle w_5, w_{74}\rangle$	370a1	74	5	$\langle w_{10}, w_{74}\rangle$	370b1
77	1	$\langle w_{11}\rangle$	77c2	77	1	$\langle w_{77}\rangle$	77a1
77	2	$\langle w_{11}, w_{14}\rangle$	77c1	77	3	$\langle w_3, w_{77}\rangle$	77a1
77	3	$\langle w_{11}, w_{21}\rangle$	77c1	77	6	$\langle w_2, w_3, w_7, w_{11}\rangle$	462c2
82	1	$\langle w_{82}\rangle$	82a1	82	3	$\langle w_3, w_{41}\rangle$	246g1
82	5	$\langle w_2, w_5, w_{41}\rangle$	410d1	82	7	$\langle w_2, w_7, w_{41}\rangle$	574g1
85	1	$\langle w_{17}\rangle$	85a1	85	2	$\langle w_2, w_5, w_{17}\rangle$	170a1
85	3	$\langle w_{15}, w_{17}\rangle$	85a2	85	6	$\langle w_2, w_3, w_5, w_{17}\rangle$	170a2
86	3	$\langle w_3, w_{86}\rangle$	258a1	86	5	$\langle w_2, w_5, w_{43}\rangle$	430c1
87	2	$\langle w_3, w_{29}\rangle$	174b1	87	2	$\langle w_6, w_{58}\rangle$	174c1
91	1	$\langle w_7, w_{13}\rangle$	91b1	91	2	$\langle w_2, w_7, w_{13}\rangle$	91b2
91	5	$\langle w_5, w_7, w_{13}\rangle$	91b2	93	2	$\langle w_2, w_{31}\rangle$	186a1
93	2	$\langle w_3, w_{31}\rangle$	186b1	93	7	$\langle w_3, w_7, w_{31}\rangle$	651c2
94	1	$\langle w_2\rangle$	94a2	94	3	$\langle w_2, w_3, w_{47}\rangle$	282b1
94	5	$\langle w_2, w_5, w_{47}\rangle$	470e1	94	7	$\langle w_2, w_7, w_{47}\rangle$	658d1
95	2	$\langle w_2, w_{95}\rangle$	190b1	95	2	$\langle w_5, w_{19}\rangle$	190c1
95	3	$\langle w_3, w_{95}\rangle$	285b1	95	6	$\langle w_2, w_3, w_5, w_{19}\rangle$	570c1
106	1	$\langle w_{53}\rangle$	106d1	106	1	$\langle w_{106}\rangle$	106b1
106	3	$\langle w_2, w_3, w_{53}\rangle$	318d1	106	5	$\langle w_2, w_5, w_{53}\rangle$	530d1
111	2	$\langle w_3, w_{37}\rangle$	222a1	111	2	$\langle w_6, w_{74}\rangle$	222b1
115	1	$\langle w_{23}\rangle$	115a1	115	2	$\langle w_{10}, w_{23}\rangle$	115a1
118	1	$\langle w_{59}\rangle$	118d1	118	1	$\langle w_{118}\rangle$	118a1
118	3	$\langle w_2, w_3, w_{59}\rangle$	354f1	119	2	$\langle w_2, w_{119}\rangle$	238b2
119	2	$\langle w_7, w_{17}\rangle$	238c2	119	2	$\langle w_{14}, w_{34}\rangle$	238d2
119	3	$\langle w_3, w_7, w_{17}\rangle$	357b1	122	1	$\langle w_{122}\rangle$	122a1
122	3	$\langle w_2, w_3, w_{61}\rangle$	366g1	122	7	$\langle w_2, w_7, w_{61}\rangle$	854c1
123	1	$\langle w_3, w_{41}\rangle$	123a1	123	2	$\langle w_2, w_3, w_{41}\rangle$	123a1
129	1	$\langle w_{129}\rangle$	129a1	129	2	$\langle w_2, w_3, w_{43}\rangle$	258c1
134	3	$\langle w_3, w_{134}\rangle$	402a1	134	3	$\langle w_6, w_{134}\rangle$	402b1
141	1	$\langle w_3, w_{47}\rangle$	141a1	141	2	$\langle w_2, w_3, w_{47}\rangle$	141a1
142	1	$\langle w_2, w_{71}\rangle$	142a1	142	3	$\langle w_2, w_3, w_{71}\rangle$	142a1
143	1	$\langle w_{143}\rangle$	143a1	143	3	$\langle w_3, w_{11}, w_{13}\rangle$	429a2

146	3	$\langle w_2, w_3, w_{73}\rangle$	438f1	146	5	$\langle w_2, w_5, w_{73}\rangle$	730i1
155	1	$\langle w_5, w_{31}\rangle$	155a1	155	2	$\langle w_2, w_5, w_{31}\rangle$	155a1
158	1	$\langle w_2, w_{79}\rangle$	158a1	158	3	$\langle w_2, w_3, w_{79}\rangle$	158a1
159	2	$\langle w_2, w_{159}\rangle$	318c1	159	2	$\langle w_6, w_{106}\rangle$	318a1
161	2	$\langle w_2, w_7, w_{23}\rangle$	322a1	166	1	$\langle w_{166}\rangle$	166a1
178	1	$\langle w_{89}\rangle$	178b1	178	3	$\langle w_2, w_3, w_{89}\rangle$	534a1
183	2	$\langle w_2, w_3, w_{61}\rangle$	366f1	185	1	$\langle w_5, w_{37}\rangle$	185b1
194	3	$\langle w_2, w_3, w_{97}\rangle$	582c1	201	1	$\langle w_3, w_{67}\rangle$	201b1
202	1	$\langle w_{101}\rangle$	202a1	203	1	$\langle w_7, w_{29}\rangle$	203b1
206	3	$\langle w_2, w_3, w_{103}\rangle$	618e1	209	1	$\langle w_{11}, w_{19}\rangle$	209a1
210	1	$\langle w_{30}\rangle$	210e5	210	1	$\langle w_{42}\rangle$	210a6
210	1	$\langle w_{70}\rangle$	210c3	210	1	$\langle w_{105}\rangle$	210b6
210	1	$\langle w_{210}\rangle$	210d2	210	1	$\langle w_2, w_{15}\rangle$	210e3
210	1	$\langle w_2, w_{21}\rangle$	210a8	210	1	$\langle w_2, w_{35}\rangle$	210c6
210	1	$\langle w_3, w_{10}\rangle$	210e8	210	1	$\langle w_3, w_{14}\rangle$	210a7
210	1	$\langle w_3, w_{35}\rangle$	210b8	210	1	$\langle w_5, w_6\rangle$	210e7
210	1	$\langle w_5, w_{14}\rangle$	210c5	210	1	$\langle w_5, w_{21}\rangle$	210b7
210	1	$\langle w_6, w_7\rangle$	210a3	210	1	$\langle w_6, w_{35}\rangle$	210d3
210	1	$\langle w_7, w_{10}\rangle$	210c2	210	1	$\langle w_7, w_{15}\rangle$	210b3
210	1	$\langle w_{10}, w_{21}\rangle$	210d4	210	1	$\langle w_{14}, w_{15}\rangle$	210d1
210	1	$\langle w_2, w_3, w_5\rangle$	210e5	210	1	$\langle w_2, w_3, w_7\rangle$	210a6
210	1	$\langle w_2, w_5, w_7\rangle$	210c3	210	1	$\langle w_3, w_5, w_7\rangle$	210b6
210	1	$\langle w_6, w_{10}, w_{14}\rangle$	210d2	210	11	$\langle w_{14}, w_{22}, w_{30}\rangle$	2310f1
210	11	$\langle w_2, w_3, w_5, w_{77}\rangle$	210e2	210	11	$\langle w_2, w_3, w_7, w_{55}\rangle$	210a2
210	11	$\langle w_2, w_3, w_{11}, w_{35}\rangle$	2310q2	210	11	$\langle w_2, w_5, w_7, w_{33}\rangle$	210c2
210	11	$\langle w_2, w_5, w_{11}, w_{21}\rangle$	2310n2	210	11	$\langle w_2, w_{11}, w_{15}, w_{21}\rangle$	2310m1
210	11	$\langle w_3, w_5, w_7, w_{22}\rangle$	210b2	210	11	$\langle w_3, w_7, w_{10}, w_{22}\rangle$	2310i3
210	11	$\langle w_3, w_{10}, w_{11}, w_{14}\rangle$	2310e1	210	11	$\langle w_3, w_{10}, w_{14}, w_{22}\rangle$	2310f2
210	11	$\langle w_5, w_6, w_7, w_{11}\rangle$	2310d2	210	11	$\langle w_6, w_7, w_{10}, w_{22}\rangle$	2310c2
210	13	$\langle w_2, w_3, w_5, w_{91}\rangle$	210e2	210	13	$\langle w_2, w_3, w_7, w_{65}\rangle$	210a2
210	13	$\langle w_2, w_3, w_{13}, w_{35}\rangle$	2730y2	210	13	$\langle w_2, w_5, w_{13}, w_{21}\rangle$	2730u1
210	13	$\langle w_2, w_7, w_{15}, w_{39}\rangle$	2730t2	210	13	$\langle w_2, w_{15}, w_{21}, w_{39}\rangle$	2730s1
210	13	$\langle w_3, w_5, w_7, w_{26}\rangle$	210b2	210	13	$\langle w_3, w_7, w_{10}, w_{13}\rangle$	2730m2
210	13	$\langle w_5, w_6, w_7, w_{26}\rangle$	2730j2	210	13	$\langle w_5, w_6, w_{13}, w_{14}\rangle$	2730g2
210	13	$\langle w_6, w_7, w_{10}, w_{13}\rangle$	2730d4	210	17	$\langle w_2, w_3, w_5, w_{119}\rangle$	210e2
210	17	$\langle w_3, w_7, w_{10}, w_{34}\rangle$	3570k3	210	17	$\langle w_5, w_6, w_{14}, w_{17}\rangle$	3570d1
210	17	$\langle w_6, w_7, w_{10}, w_{34}\rangle$	3570c1	210	19	$\langle w_2, w_3, w_5, w_{133}\rangle$	210e2
210	19	$\langle w_2, w_3, w_{19}, w_{35}\rangle$	3990w1	210	19	$\langle w_3, w_5, w_7, w_{38}\rangle$	210b2
210	19	$\langle w_3, w_5, w_{14}, w_{19}\rangle$	3990p2	210	19	$\langle w_3, w_7, w_{10}, w_{19}\rangle$	3990m1
210	19	$\langle w_5, w_6, w_{14}, w_{38}\rangle$	3990i1	210	19	$\langle w_6, w_7, w_{10}, w_{19}\rangle$	3990e1
210	23	$\langle w_2, w_3, w_5, w_7, w_{23}\rangle$	4830bk2	210	29	$\langle w_2, w_3, w_5, w_7, w_{29}\rangle$	6090bb1
210	31	$\langle w_2, w_3, w_5, w_7, w_{31}\rangle$	6510bc1	210	37	$\langle w_2, w_3, w_5, w_7, w_{37}\rangle$	7770bd1
214	1	$\langle w_2, w_{107}\rangle$	214a1	215	1	$\langle w_{215}\rangle$	215a1
215	2	$\langle w_2, w_5, w_{43}\rangle$	430b1	218	1	$\langle w_2, w_{109}\rangle$	218a1
219	1	$\langle w_3, w_{73}\rangle$	219b1	226	1	$\langle w_2, w_{113}\rangle$	226a1
235	1	$\langle w_5, w_{47}\rangle$	235a1	237	2	$\langle w_2, w_3, w_{79}\rangle$	474b1
254	1	$\langle w_2, w_{127}\rangle$	254a1	262	1	$\langle w_2, w_{131}\rangle$	262a1

274	1	$\langle w_2, w_{137}\rangle$	274a1	278	1	$\langle w_2, w_{139}\rangle$	278a1
298	1	$\langle w_2, w_{149}\rangle$	298a1	309	1	$\langle w_3, w_{103}\rangle$	309a1
314	1	$\langle w_{314}\rangle$	314a1	326	1	$\langle w_2, w_{163}\rangle$	326b1
327	1	$\langle w_3, w_{109}\rangle$	327a1	330	1	$\langle w_3\rangle$	330b3
330	1	$\langle w_{22}\rangle$	330c3	330	1	$\langle w_{33}\rangle$	330d2
330	1	$\langle w_{165}\rangle$	330a2	330	1	$\langle w_{330}\rangle$	330e2
330	1	$\langle w_2, w_3\rangle$	330b5	330	1	$\langle w_2, w_{11}\rangle$	330c5
330	1	$\langle w_2, w_{33}\rangle$	330d3	330	1	$\langle w_3, w_5\rangle$	330b2
330	1	$\langle w_3, w_{10}\rangle$	330b6	330	1	$\langle w_5, w_{22}\rangle$	330c6
330	1	$\langle w_5, w_{66}\rangle$	330e4	330	1	$\langle w_6, w_{55}\rangle$	330e1
330	1	$\langle w_6, w_{110}\rangle$	330a3	330	1	$\langle w_{10}, w_{22}\rangle$	330c2
330	1	$\langle w_{10}, w_{66}\rangle$	330a4	330	1	$\langle w_{11}, w_{15}\rangle$	330a1
330	1	$\langle w_{11}, w_{30}\rangle$	330e3	330	1	$\langle w_{15}, w_{33}\rangle$	330d1
330	1	$\langle w_{30}, w_{33}\rangle$	330d4	330	1	$\langle w_2, w_3, w_5\rangle$	330b3
330	1	$\langle w_2, w_5, w_{11}\rangle$	330c3	330	1	$\langle w_2, w_{15}, w_{33}\rangle$	330d2
330	1	$\langle w_5, w_6, w_{11}\rangle$	330e2	330	1	$\langle w_6, w_{10}, w_{11}\rangle$	330a2
330	7	$\langle w_3, w_{11}, w_{70}\rangle$	2310f1	330	7	$\langle w_2, w_3, w_5, w_{77}\rangle$	330b2
330	7	$\langle w_2, w_3, w_7, w_{55}\rangle$	2310q2	330	7	$\langle w_2, w_5, w_7, w_{33}\rangle$	2310n2
330	7	$\langle w_2, w_5, w_{11}, w_{21}\rangle$	330c2	330	7	$\langle w_2, w_{15}, w_{21}, w_{33}\rangle$	330d2
330	7	$\langle w_3, w_5, w_{14}, w_{22}\rangle$	2310l1	330	7	$\langle w_3, w_7, w_{10}, w_{11}\rangle$	2310f2
330	7	$\langle w_3, w_7, w_{10}, w_{22}\rangle$	2310e1	330	7	$\langle w_3, w_{10}, w_{11}, w_{14}\rangle$	2310i1
330	7	$\langle w_5, w_6, w_7, w_{11}\rangle$	330e2	330	7	$\langle w_5, w_6, w_{14}, w_{22}\rangle$	2310d2
330	7	$\langle w_6, w_7, w_{10}, w_{22}\rangle$	2310a2	330	13	$\langle w_2, w_3, w_{11}, w_{65}\rangle$	4290ba1
330	13	$\langle w_2, w_5, w_{11}, w_{39}\rangle$	330c2	330	13	$\langle w_3, w_5, w_{11}, w_{26}\rangle$	4290p1
330	13	$\langle w_3, w_{10}, w_{13}, w_{22}\rangle$	4290j1	330	17	$\langle w_2, w_3, w_5, w_{11}, w_{17}\rangle$	5610bl2
330	19	$\langle w_2, w_3, w_5, w_{11}, w_{19}\rangle$	6270r1	335	1	$\langle w_5, w_{67}\rangle$	335a1
346	1	$\langle w_2, w_{173}\rangle$	346b1	362	1	$\langle w_2, w_{181}\rangle$	362b1
381	1	$\langle w_3, w_{127}\rangle$	381a1	390	1	$\langle w_{390}\rangle$	390a2
390	1	$\langle w_6, w_{13}\rangle$	390c3	390	1	$\langle w_6, w_{65}\rangle$	390a4
390	1	$\langle w_6, w_{130}\rangle$	390f1	390	1	$\langle w_{10}, w_{13}\rangle$	390b3
390	1	$\langle w_{10}, w_{39}\rangle$	390a3	390	1	$\langle w_{10}, w_{78}\rangle$	390g2
390	1	$\langle w_{13}, w_{15}\rangle$	390d3	390	1	$\langle w_{15}, w_{26}\rangle$	390a1
390	1	$\langle w_{15}, w_{78}\rangle$	390e1	390	1	$\langle w_2, w_3, w_{13}\rangle$	390c4
390	1	$\langle w_2, w_5, w_{13}\rangle$	390b5	390	1	$\langle w_2, w_{15}, w_{39}\rangle$	390e2
390	1	$\langle w_3, w_5, w_{13}\rangle$	390d4	390	1	$\langle w_3, w_{10}, w_{26}\rangle$	390g3
390	1	$\langle w_5, w_6, w_{26}\rangle$	390f2	390	1	$\langle w_6, w_{10}, w_{26}\rangle$	390a2
390	7	$\langle w_2, w_3, w_7, w_{65}\rangle$	2730y2	390	7	$\langle w_2, w_3, w_{13}, w_{35}\rangle$	390c1
390	7	$\langle w_2, w_5, w_7, w_{39}\rangle$	2730u2	390	7	$\langle w_2, w_7, w_{13}, w_{15}\rangle$	2730s2
390	7	$\langle w_2, w_{13}, w_{15}, w_{21}\rangle$	2730t1	390	7	$\langle w_3, w_5, w_{13}, w_{14}\rangle$	390d1
390	7	$\langle w_5, w_6, w_{13}, w_{14}\rangle$	2730j1	390	11	$\langle w_2, w_3, w_5, w_{11}, w_{13}\rangle$	4290bb2
390	19	$\langle w_2, w_3, w_5, w_{13}, w_{19}\rangle$	7410x1	446	1	$\langle w_2, w_{223}\rangle$	446b1
458	1	$\langle w_2, w_{229}\rangle$	458b1	462	1	$\langle w_{154}\rangle$	462b2
462	1	$\langle w_2, w_{11}\rangle$	462f2	462	1	$\langle w_2, w_{21}\rangle$	462g3
462	1	$\langle w_2, w_{231}\rangle$	462e1	462	1	$\langle w_6, w_{154}\rangle$	462b1
462	1	$\langle w_{11}, w_{14}\rangle$	462b3	462	1	$\langle w_{11}, w_{42}\rangle$	462c2
462	1	$\langle w_{21}, w_{22}\rangle$	462a1	462	1	$\langle w_{21}, w_{66}\rangle$	462b4
462	1	$\langle w_{22}, w_{42}\rangle$	462d1	462	1	$\langle w_2, w_3, w_7\rangle$	462g4

462	1	$\langle w_2, w_3, w_{11}\rangle$	462f3	462	1	$\langle w_2, w_7, w_{33}\rangle$	462e2
462	1	$\langle w_3, w_{14}, w_{22}\rangle$	462d2	462	1	$\langle w_6, w_7, w_{11}\rangle$	462c4
462	1	$\langle w_6, w_{11}, w_{14}\rangle$	462b2	462	1	$\langle w_6, w_{14}, w_{22}\rangle$	462a2
462	5	$\langle w_2, w_3, w_5, w_{77}\rangle$	2310q2	462	5	$\langle w_2, w_3, w_7, w_{55}\rangle$	462g1
462	5	$\langle w_2, w_3, w_{11}, w_{35}\rangle$	462f1	462	5	$\langle w_2, w_3, w_{35}, w_{55}\rangle$	2310v1
462	5	$\langle w_2, w_5, w_7, w_{33}\rangle$	462e1	462	5	$\langle w_2, w_5, w_{21}, w_{33}\rangle$	2310m1
462	5	$\langle w_2, w_{15}, w_{21}, w_{33}\rangle$	2310n3	462	5	$\langle w_3, w_5, w_{11}, w_{14}\rangle$	2310f2
462	5	$\langle w_3, w_7, w_{10}, w_{22}\rangle$	2310l3	462	5	$\langle w_6, w_7, w_{10}, w_{22}\rangle$	2310d2
462	5	$\langle w_6, w_{10}, w_{11}, w_{14}\rangle$	462b2	462	13	$\langle w_2, w_3, w_7, w_{11}, w_{13}\rangle$	6006bf1
510	1	$\langle w_{510}\rangle$	510d2	510	1	$\langle w_2, w_{255}\rangle$	510d1
510	1	$\langle w_3, w_{10}\rangle$	510g3	510	1	$\langle w_3, w_{34}\rangle$	510f2
510	1	$\langle w_3, w_{85}\rangle$	510b1	510	1	$\langle w_{10}, w_{34}\rangle$	510e5
510	1	$\langle w_{10}, w_{102}\rangle$	510a1	510	1	$\langle w_{15}, w_{34}\rangle$	510d3
510	1	$\langle w_{17}, w_{30}\rangle$	510d4	510	1	$\langle w_{30}, w_{85}\rangle$	510c1
510	1	$\langle w_2, w_3, w_5\rangle$	510g4	510	1	$\langle w_2, w_3, w_{17}\rangle$	510f3
510	1	$\langle w_2, w_5, w_{17}\rangle$	510e7	510	1	$\langle w_2, w_{15}, w_{17}\rangle$	510d2
510	1	$\langle w_2, w_{15}, w_{51}\rangle$	510c2	510	1	$\langle w_3, w_5, w_{17}\rangle$	510b2
510	1	$\langle w_6, w_{10}, w_{17}\rangle$	510a2	510	7	$\langle w_2, w_3, w_5, w_{119}\rangle$	510g1
510	7	$\langle w_2, w_7, w_{15}, w_{17}\rangle$	510d2	510	7	$\langle w_3, w_{10}, w_{14}, w_{17}\rangle$	3570k1
510	7	$\langle w_6, w_7, w_{10}, w_{34}\rangle$	3570a1	510	11	$\langle w_2, w_3, w_5, w_{11}, w_{17}\rangle$	5610bk1
546	1	$\langle w_{546}\rangle$	546c2	546	1	$\langle w_3, w_{182}\rangle$	546c3
546	1	$\langle w_{13}, w_{42}\rangle$	546c4	546	1	$\langle w_{14}, w_{39}\rangle$	546c1
546	1	$\langle w_2, w_{21}, w_{39}\rangle$	546e1	546	1	$\langle w_3, w_7, w_{13}\rangle$	546d3
546	1	$\langle w_3, w_{13}, w_{14}\rangle$	546c2	546	1	$\langle w_3, w_{14}, w_{26}\rangle$	546b1
546	1	$\langle w_6, w_{13}, w_{14}\rangle$	546a1	546	5	$\langle w_2, w_3, w_5, w_{91}\rangle$	2730y2
546	5	$\langle w_2, w_5, w_{13}, w_{21}\rangle$	2730s1	546	5	$\langle w_3, w_5, w_7, w_{26}\rangle$	2730m2
546	5	$\langle w_3, w_5, w_{13}, w_{14}\rangle$	546c2	546	5	$\langle w_3, w_7, w_{10}, w_{13}\rangle$	546d2
546	5	$\langle w_5, w_6, w_{14}, w_{26}\rangle$	2730a1	546	5	$\langle w_6, w_7, w_{10}, w_{13}\rangle$	2730j1
546	5	$\langle w_6, w_{10}, w_{14}, w_{26}\rangle$	2730g1	546	11	$\langle w_2, w_3, w_7, w_{11}, w_{13}\rangle$	6006be2
570	1	$\langle w_3, w_{190}\rangle$	570e2	570	1	$\langle w_5, w_{57}\rangle$	570f3
570	1	$\langle w_6, w_{190}\rangle$	570b1	570	1	$\langle w_{10}, w_{57}\rangle$	570a1
570	1	$\langle w_{19}, w_{30}\rangle$	570c1	570	1	$\langle w_{30}, w_{38}\rangle$	570d2
570	1	$\langle w_{30}, w_{57}\rangle$	570g2	570	1	$\langle w_2, w_{15}, w_{57}\rangle$	570g3
570	1	$\langle w_3, w_5, w_{19}\rangle$	570f4	570	1	$\langle w_3, w_5, w_{38}\rangle$	570e4
570	1	$\langle w_3, w_{10}, w_{38}\rangle$	570d4	570	1	$\langle w_5, w_6, w_{19}\rangle$	570c2
570	1	$\langle w_6, w_{10}, w_{19}\rangle$	570b2	570	1	$\langle w_6, w_{10}, w_{38}\rangle$	570a2
570	7	$\langle w_2, w_3, w_7, w_{95}\rangle$	3990w2	570	7	$\langle w_3, w_5, w_{14}, w_{19}\rangle$	570f1
690	1	$\langle w_2, w_{345}\rangle$	690h1	690	1	$\langle w_3, w_{46}\rangle$	690i1
690	1	$\langle w_3, w_{115}\rangle$	690f2	690	1	$\langle w_3, w_{230}\rangle$	690e2
690	1	$\langle w_6, w_{115}\rangle$	690a1	690	1	$\langle w_{10}, w_{138}\rangle$	690b1
690	1	$\langle w_{30}, w_{115}\rangle$	690g2	690	1	$\langle w_2, w_3, w_{23}\rangle$	690i2
690	1	$\langle w_2, w_{15}, w_{23}\rangle$	690h2	690	1	$\langle w_2, w_{15}, w_{69}\rangle$	690g4
690	1	$\langle w_3, w_5, w_{23}\rangle$	690f3	690	1	$\langle w_3, w_{10}, w_{23}\rangle$	690e3
690	1	$\langle w_6, w_{10}, w_{23}\rangle$	690b2	690	1	$\langle w_6, w_{10}, w_{46}\rangle$	690a2
690	7	$\langle w_2, w_3, w_5, w_7, w_{23}\rangle$	4830bj1	714	1	$\langle w_2, w_{357}\rangle$	714f2
714	1	$\langle w_7, w_{102}\rangle$	714d1	714	1	$\langle w_{14}, w_{51}\rangle$	714a1
714	1	$\langle w_2, w_{17}, w_{21}\rangle$	714f3	714	1	$\langle w_2, w_{21}, w_{51}\rangle$	714e1

714	1	$\langle w_6, w_7, w_{17}\rangle$	714d2	714	1	$\langle w_6, w_7, w_{34}\rangle$	714c1
714	1	$\langle w_6, w_{14}, w_{17}\rangle$	714b1	714	1	$\langle w_6, w_{14}, w_{34}\rangle$	714a2
714	5	$\langle w_2, w_3, w_5, w_7, w_{17}\rangle$	3570x2	770	1	$\langle w_2, w_{385}\rangle$	770f1
770	1	$\langle w_{11}, w_{70}\rangle$	770d1	770	1	$\langle w_{22}, w_{35}\rangle$	770e2
770	1	$\langle w_2, w_7, w_{55}\rangle$	770f2	770	1	$\langle w_5, w_7, w_{22}\rangle$	770e4
770	1	$\langle w_5, w_{11}, w_{14}\rangle$	770d2	770	1	$\langle w_5, w_{14}, w_{22}\rangle$	770c1
770	3	$\langle w_2, w_3, w_5, w_{77}\rangle$	2310n2	770	3	$\langle w_2, w_3, w_7, w_{55}\rangle$	770f1
770	3	$\langle w_2, w_5, w_{21}, w_{33}\rangle$	2310v4	770	3	$\langle w_3, w_5, w_{11}, w_{14}\rangle$	770d1
770	3	$\langle w_3, w_7, w_{10}, w_{11}\rangle$	2310c2	770	3	$\langle w_3, w_{10}, w_{14}, w_{22}\rangle$	2310a2
770	3	$\langle w_6, w_7, w_{10}, w_{11}\rangle$	2310i4	770	3	$\langle w_6, w_{10}, w_{14}, w_{22}\rangle$	2310e2
798	1	$\langle w_2, w_{57}\rangle$	798i2	798	1	$\langle w_7, w_{114}\rangle$	798d1
798	1	$\langle w_{42}, w_{114}\rangle$	798b2	798	1	$\langle w_2, w_3, w_{19}\rangle$	798i3
798	1	$\langle w_2, w_3, w_{133}\rangle$	798h1	798	1	$\langle w_2, w_{19}, w_{21}\rangle$	798g1
798	1	$\langle w_3, w_7, w_{38}\rangle$	798d2	798	1	$\langle w_3, w_{14}, w_{19}\rangle$	798c1
798	1	$\langle w_3, w_{14}, w_{38}\rangle$	798b1	798	1	$\langle w_6, w_{14}, w_{38}\rangle$	798a1
798	5	$\langle w_2, w_3, w_5, w_7, w_{19}\rangle$	3990y2	858	1	$\langle w_3, w_{286}\rangle$	858b1
858	1	$\langle w_6, w_{286}\rangle$	858a2	858	1	$\langle w_{33}, w_{78}\rangle$	858e2
858	1	$\langle w_2, w_{11}, w_{39}\rangle$	858g1	858	1	$\langle w_2, w_{13}, w_{33}\rangle$	858f1
858	1	$\langle w_2, w_{33}, w_{39}\rangle$	858e3	858	1	$\langle w_3, w_{13}, w_{22}\rangle$	858b2
858	1	$\langle w_6, w_{11}, w_{26}\rangle$	858a4	858	5	$\langle w_2, w_3, w_5, w_{11}, w_{13}\rangle$	4290ba1
870	1	$\langle w_3, w_{145}\rangle$	870d2	870	1	$\langle w_6, w_{145}\rangle$	870a1
870	1	$\langle w_{10}, w_{58}\rangle$	870g2	870	1	$\langle w_2, w_5, w_{29}\rangle$	870g3
870	1	$\langle w_2, w_5, w_{87}\rangle$	870f1	870	1	$\langle w_2, w_{15}, w_{29}\rangle$	870e1
870	1	$\langle w_3, w_5, w_{29}\rangle$	870d1	870	1	$\langle w_3, w_5, w_{58}\rangle$	870c1
870	1	$\langle w_3, w_{10}, w_{29}\rangle$	870b1	870	1	$\langle w_5, w_6, w_{29}\rangle$	870a2
910	1	$\langle w_{65}, w_{70}\rangle$	910a2	910	1	$\langle w_2, w_5, w_{91}\rangle$	910k1
910	1	$\langle w_2, w_7, w_{65}\rangle$	910h1	910	1	$\langle w_5, w_7, w_{26}\rangle$	910d1
910	1	$\langle w_7, w_{10}, w_{26}\rangle$	910a1	910	3	$\langle w_2, w_3, w_5, w_7, w_{13}\rangle$	2730x2
930	1	$\langle w_2, w_{465}\rangle$	930m1	930	1	$\langle w_3, w_{310}\rangle$	930h1
930	1	$\langle w_{10}, w_{93}\rangle$	930a2	930	1	$\langle w_2, w_{15}, w_{31}\rangle$	930m2
930	1	$\langle w_3, w_5, w_{62}\rangle$	930h2	930	1	$\langle w_5, w_6, w_{62}\rangle$	930c1
930	1	$\langle w_6, w_{10}, w_{31}\rangle$	930b1	930	1	$\langle w_6, w_{10}, w_{62}\rangle$	930a3
966	1	$\langle w_2, w_{483}\rangle$	966g2	966	1	$\langle w_3, w_{322}\rangle$	966e1
966	1	$\langle w_{21}, w_{46}\rangle$	966a1	966	1	$\langle w_2, w_7, w_{23}\rangle$	966h1
966	1	$\langle w_2, w_7, w_{69}\rangle$	966g4	966	1	$\langle w_3, w_{14}, w_{23}\rangle$	966e2
966	1	$\langle w_6, w_7, w_{46}\rangle$	966b1	966	1	$\langle w_6, w_{14}, w_{46}\rangle$	966a2
966	5	$\langle w_2, w_3, w_5, w_7, w_{23}\rangle$	4830bf1	1110	1	$\langle w_{15}, w_{222}\rangle$	1110i1
1110	1	$\langle w_2, w_3, w_{185}\rangle$	1110m1	1110	1	$\langle w_2, w_5, w_{111}\rangle$	1110j1
1110	1	$\langle w_2, w_{15}, w_{111}\rangle$	1110i2	1110	1	$\langle w_3, w_{10}, w_{37}\rangle$	1110g1
1110	1	$\langle w_5, w_6, w_{37}\rangle$	1110d1	1122	1	$\langle w_2, w_3, w_{187}\rangle$	1122i2
1122	1	$\langle w_2, w_{11}, w_{51}\rangle$	1122f2	1122	1	$\langle w_2, w_{17}, w_{33}\rangle$	1122e2
1122	1	$\langle w_3, w_{11}, w_{34}\rangle$	1122d1	1122	1	$\langle w_6, w_{22}, w_{34}\rangle$	1122a1
1155	1	$\langle w_3, w_5, w_7, w_{11}\rangle$	1155n1	1155	2	$\langle w_2, w_3, w_5, w_7, w_{11}\rangle$	1155n1
1190	1	$\langle w_7, w_{170}\rangle$	1190a1	1190	1	$\langle w_2, w_5, w_{119}\rangle$	1190f1
1190	1	$\langle w_5, w_{14}, w_{17}\rangle$	1190b1	1190	1	$\langle w_7, w_{10}, w_{17}\rangle$	1190a2
1190	3	$\langle w_2, w_3, w_5, w_7, w_{17}\rangle$	3570u2	1218	1	$\langle w_3, w_{406}\rangle$	1218c1
1218	1	$\langle w_2, w_{21}, w_{29}\rangle$	1218f1	1218	1	$\langle w_3, w_{14}, w_{29}\rangle$	1218c2

1218	1	$\langle w_6, w_7, w_{58}\rangle$	1218b1	1218	1	$\langle w_6, w_{14}, w_{29}\rangle$	1218a1
1230	1	$\langle w_2, w_3, w_{205}\rangle$	1230h1	1230	1	$\langle w_2, w_{15}, w_{41}\rangle$	1230e1
1230	1	$\langle w_3, w_5, w_{82}\rangle$	1230d1	1230	1	$\langle w_3, w_{10}, w_{41}\rangle$	1230c2
1230	1	$\langle w_3, w_{10}, w_{82}\rangle$	1230b2	1230	1	$\langle w_5, w_6, w_{41}\rangle$	1230a1
1254	1	$\langle w_2, w_3, w_{209}\rangle$	1254i2	1254	1	$\langle w_2, w_{11}, w_{57}\rangle$	1254h2
1254	1	$\langle w_2, w_{19}, w_{33}\rangle$	1254g2	1254	1	$\langle w_2, w_{33}, w_{57}\rangle$	1254f1
1254	1	$\langle w_3, w_{11}, w_{38}\rangle$	1254e2	1254	1	$\langle w_3, w_{19}, w_{22}\rangle$	1254d2
1254	1	$\langle w_6, w_{11}, w_{19}\rangle$	1254c2	1290	1	$\langle w_2, w_3, w_{215}\rangle$	1290m1
1290	1	$\langle w_2, w_5, w_{129}\rangle$	1290l1	1290	1	$\langle w_3, w_5, w_{86}\rangle$	1290f1
1290	1	$\langle w_3, w_{10}, w_{43}\rangle$	1290e2	1302	1	$\langle w_2, w_3, w_{217}\rangle$	1302m1
1302	1	$\langle w_2, w_{21}, w_{93}\rangle$	1302j2	1302	1	$\langle w_3, w_{14}, w_{62}\rangle$	1302d1
1302	1	$\langle w_6, w_7, w_{31}\rangle$	1302c1	1326	1	$\langle w_2, w_3, w_{221}\rangle$	1326e1
1326	1	$\langle w_2, w_{17}, w_{39}\rangle$	1326d1	1326	1	$\langle w_3, w_{17}, w_{26}\rangle$	1326c1
1326	1	$\langle w_6, w_{13}, w_{17}\rangle$	1326b2	1326	1	$\langle w_6, w_{26}, w_{34}\rangle$	1326a1
1330	1	$\langle w_2, w_5, w_7, w_{19}\rangle$	1330j1	1410	1	$\langle w_2, w_{705}\rangle$	1410j1
1410	1	$\langle w_2, w_{15}, w_{47}\rangle$	1410j2	1410	1	$\langle w_3, w_5, w_{94}\rangle$	1410f1
1410	1	$\langle w_3, w_{10}, w_{94}\rangle$	1410e1	1410	1	$\langle w_5, w_6, w_{47}\rangle$	1410d1
1430	1	$\langle w_2, w_5, w_{11}, w_{13}\rangle$	1430k1	1430	3	$\langle w_2, w_3, w_5, w_{11}, w_{13}\rangle$	1430k1
1482	1	$\langle w_2, w_3, w_{13}, w_{19}\rangle$	1482l1	1518	1	$\langle w_2, w_3, w_{11}, w_{23}\rangle$	1518t1
1554	1	$\langle w_2, w_3, w_7, w_{37}\rangle$	1554n1	1590	1	$\langle w_2, w_3, w_{265}\rangle$	1590s1
1590	1	$\langle w_2, w_5, w_{159}\rangle$	1590n1	1590	1	$\langle w_3, w_5, w_{106}\rangle$	1590i1
1590	1	$\langle w_3, w_{10}, w_{106}\rangle$	1590f2	1610	1	$\langle w_2, w_5, w_7, w_{23}\rangle$	1610g1
1722	1	$\langle w_2, w_3, w_{287}\rangle$	1722n1	1722	1	$\langle w_2, w_7, w_{123}\rangle$	1722m1
1722	1	$\langle w_3, w_7, w_{82}\rangle$	1722f1	1722	1	$\langle w_6, w_{14}, w_{41}\rangle$	1722c1
1770	1	$\langle w_2, w_3, w_{295}\rangle$	1770g1	1770	1	$\langle w_2, w_5, w_{177}\rangle$	1770f1
1770	1	$\langle w_3, w_5, w_{118}\rangle$	1770d1	1770	1	$\langle w_6, w_{10}, w_{118}\rangle$	1770a1
1785	1	$\langle w_3, w_5, w_7, w_{17}\rangle$	1785o2	1794	1	$\langle w_3, w_{13}, w_{46}\rangle$	1794e1
1806	1	$\langle w_2, w_3, w_7, w_{43}\rangle$	1806n1	1830	1	$\langle w_2, w_3, w_5, w_{61}\rangle$	1830l1
1914	1	$\langle w_2, w_{11}, w_{87}\rangle$	1914l1	1914	1	$\langle w_2, w_{29}, w_{33}\rangle$	1914k1
1914	1	$\langle w_3, w_{11}, w_{58}\rangle$	1914e2	1914	1	$\langle w_6, w_{11}, w_{29}\rangle$	1914d1
1938	1	$\langle w_2, w_3, w_{323}\rangle$	1938h2	1974	1	$\langle w_2, w_7, w_{141}\rangle$	1974f1
2010	1	$\langle w_2, w_5, w_{201}\rangle$	2010f1	2010	1	$\langle w_3, w_5, w_{134}\rangle$	2010e1
2010	1	$\langle w_6, w_{10}, w_{67}\rangle$	2010a1	2046	1	$\langle w_2, w_3, w_{11}, w_{31}\rangle$	2046j1
2090	1	$\langle w_2, w_5, w_{11}, w_{19}\rangle$	2090o1	2130	1	$\langle w_6, w_{10}, w_{142}\rangle$	2130a1
2170	1	$\langle w_2, w_5, w_7, w_{31}\rangle$	2170q1	2190	1	$\langle w_2, w_3, w_5, w_{73}\rangle$	2190q1
2210	1	$\langle w_2, w_5, w_{13}, w_{17}\rangle$	2210g2	2226	1	$\langle w_2, w_3, w_7, w_{53}\rangle$	2226j1
2262	1	$\langle w_2, w_3, w_{13}, w_{29}\rangle$	2262l1	2370	1	$\langle w_2, w_3, w_5, w_{79}\rangle$	2370o1
2415	1	$\langle w_3, w_5, w_7, w_{23}\rangle$	2415i2	2442	1	$\langle w_2, w_3, w_{11}, w_{37}\rangle$	2442i1
2478	1	$\langle w_2, w_3, w_7, w_{59}\rangle$	2478h1	2490	1	$\langle w_2, w_3, w_5, w_{83}\rangle$	2490k1
2670	1	$\langle w_2, w_3, w_5, w_{89}\rangle$	2670f1	2706	1	$\langle w_2, w_3, w_{11}, w_{41}\rangle$	2706r2
2838	1	$\langle w_2, w_3, w_{11}, w_{43}\rangle$	2838f2	2910	1	$\langle w_2, w_3, w_5, w_{97}\rangle$	2910k1
3030	1	$\langle w_2, w_3, w_5, w_{101}\rangle$	3030u1	3090	1	$\langle w_2, w_3, w_5, w_{103}\rangle$	3090m1

Table 3: All 82 genus 1 curves $X = X_0^D(N)/W$ with $W \leq W_0(D,N)$ nontrivial and N not squarefree. For each X, we give the isogeny class of the elliptic curve $\mathrm{Jac}(X)$ via its Cremona reference, ref.

D	N	W	ref	D	N	W	ref
6	25	$\langle w_{150}\rangle$	30a	6	25	$\langle w_2, w_3\rangle$	150a
6	25	$\langle w_2, w_{25}\rangle$	150c	6	25	$\langle w_3, w_{25}\rangle$	30a
6	25	$\langle w_3, w_{50}\rangle$	30a	6	25	$\langle w_6, w_{50}\rangle$	150b
6	49	$\langle w_2, w_{147}\rangle$	42a	6	49	$\langle w_3, w_{49}\rangle$	294d
6	49	$\langle w_3, w_{98}\rangle$	294g	6	121	$\langle w_2, w_3, w_{121}\rangle$	66c
6	125	$\langle w_2, w_3, w_{125}\rangle$	150a	6	169	$\langle w_2, w_3, w_{169}\rangle$	1014f
6	175	$\langle w_2, w_3, w_7, w_{25}\rangle$	210e	6	245	$\langle w_2, w_3, w_5, w_{49}\rangle$	210a
6	275	$\langle w_2, w_3, w_{11}, w_{25}\rangle$	330b	10	9	$\langle w_{90}\rangle$	30a
10	9	$\langle w_2, w_5\rangle$	90c	10	9	$\langle w_2, w_9\rangle$	90b
10	9	$\langle w_5, w_9\rangle$	90a	10	9	$\langle w_9, w_{10}\rangle$	30a
10	9	$\langle w_{10}, w_{18}\rangle$	30a	10	27	$\langle w_2, w_5, w_{27}\rangle$	90c
10	49	$\langle w_5, w_{98}\rangle$	490d	10	49	$\langle w_{10}, w_{98}\rangle$	490b
10	63	$\langle w_2, w_5, w_7, w_9\rangle$	210e	10	99	$\langle w_2, w_5, w_9, w_{11}\rangle$	330b
14	9	$\langle w_2, w_7\rangle$	126a	14	9	$\langle w_2, w_9\rangle$	42a
14	9	$\langle w_2, w_{63}\rangle$	42a	14	9	$\langle w_7, w_{18}\rangle$	14a
14	9	$\langle w_{14}, w_{18}\rangle$	126b	14	25	$\langle w_7, w_{50}\rangle$	14a
14	25	$\langle w_{14}, w_{25}\rangle$	350c	14	27	$\langle w_2, w_7, w_{27}\rangle$	126a
14	45	$\langle w_2, w_5, w_7, w_9\rangle$	210a	14	75	$\langle w_2, w_3, w_7, w_{25}\rangle$	210c
15	4	$\langle w_3, w_4\rangle$	30a	15	4	$\langle w_3, w_{20}\rangle$	30a
15	4	$\langle w_5, w_{12}\rangle$	15a	15	8	$\langle w_{15}\rangle$	120a
15	8	$\langle w_3, w_5\rangle$	120a	15	8	$\langle w_3, w_{40}\rangle$	30a
15	16	$\langle w_3, w_5, w_{16}\rangle$	120a	15	28	$\langle w_3, w_4, w_5, w_7\rangle$	210e
15	44	$\langle w_3, w_4, w_5, w_{11}\rangle$	330b	15	68	$\langle w_3, w_4, w_5, w_{17}\rangle$	510g
21	4	$\langle w_7\rangle$	84a	21	4	$\langle w_3, w_7\rangle$	84a
21	4	$\langle w_3, w_{28}\rangle$	21a	21	4	$\langle w_4, w_{21}\rangle$	42a
21	8	$\langle w_7, w_{24}\rangle$	84a	21	8	$\langle w_3, w_7, w_8\rangle$	84a
21	16	$\langle w_3, w_7, w_{16}\rangle$	84a	21	20	$\langle w_3, w_4, w_5, w_7\rangle$	210a
21	25	$\langle w_3, w_7, w_{25}\rangle$	105a	21	44	$\langle w_3, w_4, w_7, w_{11}\rangle$	462g
22	9	$\langle w_2, w_9, w_{11}\rangle$	66c	26	9	$\langle w_2, w_{117}\rangle$	26b
26	9	$\langle w_9, w_{26}\rangle$	234c	26	9	$\langle w_{18}, w_{26}\rangle$	234a
26	25	$\langle w_2, w_{13}, w_{25}\rangle$	130b	33	4	$\langle w_3, w_4, w_{11}\rangle$	66c
33	20	$\langle w_3, w_4, w_5, w_{11}\rangle$	66c	34	9	$\langle w_2, w_9, w_{17}\rangle$	102b
35	4	$\langle w_4, w_{35}\rangle$	70a	35	4	$\langle w_5, w_7\rangle$	140a
35	4	$\langle w_7, w_{20}\rangle$	35a	35	9	$\langle w_5, w_7, w_9\rangle$	105a
35	12	$\langle w_3, w_4, w_5, w_7\rangle$	210c	38	9	$\langle w_2, w_9, w_{19}\rangle$	114a
39	4	$\langle w_{39}\rangle$	156b	39	4	$\langle w_3, w_{13}\rangle$	156b
39	8	$\langle w_3, w_8, w_{13}\rangle$	156b	39	20	$\langle w_3, w_4, w_5, w_{13}\rangle$	390c
51	4	$\langle w_3, w_4, w_{17}\rangle$	102b	55	4	$\langle w_4, w_5, w_{11}\rangle$	110a
57	4	$\langle w_3, w_4, w_{19}\rangle$	114a	87	4	$\langle w_3, w_4, w_{29}\rangle$	174b
93	4	$\langle w_3, w_4, w_{31}\rangle$	186b	95	4	$\langle w_4, w_5, w_{19}\rangle$	190c
111	4	$\langle w_3, w_4, w_{37}\rangle$	222a	119	4	$\langle w_4, w_7, w_{17}\rangle$	238c

Table 4: 650 genus 0 curves $X = X_0^D(N)/W$, with $W \leq W_0(D,N)$ nontrivial, for which we prove $X(\mathbb{Q}) \neq \emptyset$.

D	N	W	D	N	W	D	N	W
6	1	$\langle w_2 \rangle$	6	1	$\langle w_3 \rangle$	6	1	$\langle w_6 \rangle$
6	1	$\langle w_2, w_3 \rangle$	6	5	$\langle w_6 \rangle$	6	5	$\langle w_{30} \rangle$
6	5	$\langle w_2, w_3 \rangle$	6	5	$\langle w_2, w_5 \rangle$	6	5	$\langle w_2, w_{15} \rangle$
6	5	$\langle w_3, w_{10} \rangle$	6	5	$\langle w_5, w_6 \rangle$	6	5	$\langle w_6, w_{10} \rangle$
6	5	$\langle w_2, w_3, w_5 \rangle$	6	7	$\langle w_{42} \rangle$	6	7	$\langle w_2, w_{21} \rangle$
6	7	$\langle w_2, w_3 \rangle$	6	7	$\langle w_3, w_7 \rangle$	6	7	$\langle w_3, w_{14} \rangle$
6	7	$\langle w_6, w_7 \rangle$	6	7	$\langle w_6, w_{14} \rangle$	6	7	$\langle w_2, w_3, w_7 \rangle$
6	11	$\langle w_{66} \rangle$	6	11	$\langle w_2, w_{33} \rangle$	6	11	$\langle w_3, w_{22} \rangle$
6	11	$\langle w_6, w_{11} \rangle$	6	11	$\langle w_2, w_3, w_{11} \rangle$	6	13	$\langle w_{78} \rangle$
6	13	$\langle w_3 \rangle$	6	13	$\langle w_2, w_3 \rangle$	6	13	$\langle w_2, w_{13} \rangle$
6	13	$\langle w_2, w_{39} \rangle$	6	13	$\langle w_3, w_{13} \rangle$	6	13	$\langle w_3, w_{26} \rangle$
6	13	$\langle w_6, w_{13} \rangle$	6	13	$\langle w_2, w_3, w_{13} \rangle$	6	17	$\langle w_2, w_{17} \rangle$
6	17	$\langle w_2, w_{51} \rangle$	6	17	$\langle w_3, w_{34} \rangle$	6	17	$\langle w_6, w_{34} \rangle$
6	17	$\langle w_2, w_3, w_{17} \rangle$	6	19	$\langle w_{114} \rangle$	6	19	$\langle w_2, w_{57} \rangle$
6	19	$\langle w_3, w_{38} \rangle$	6	19	$\langle w_6, w_{19} \rangle$	6	19	$\langle w_2, w_3, w_{19} \rangle$
6	23	$\langle w_2, w_{69} \rangle$	6	23	$\langle w_3, w_{46} \rangle$	6	23	$\langle w_2, w_3, w_{23} \rangle$
6	25	$\langle w_2, w_3, w_{25} \rangle$	6	29	$\langle w_{174} \rangle$	6	29	$\langle w_2, w_{87} \rangle$
6	29	$\langle w_3, w_{58} \rangle$	6	29	$\langle w_6, w_{29} \rangle$	6	29	$\langle w_2, w_3, w_{29} \rangle$
6	31	$\langle w_{186} \rangle$	6	31	$\langle w_2, w_{93} \rangle$	6	31	$\langle w_3, w_{62} \rangle$
6	31	$\langle w_6, w_{31} \rangle$	6	31	$\langle w_2, w_3, w_{31} \rangle$	6	35	$\langle w_{10}, w_{21} \rangle$
6	35	$\langle w_2, w_3, w_{35} \rangle$	6	35	$\langle w_2, w_5, w_{21} \rangle$	6	35	$\langle w_3, w_7, w_{10} \rangle$
6	35	$\langle w_6, w_{10}, w_{14} \rangle$	6	35	$\langle w_2, w_3, w_5, w_7 \rangle$	6	37	$\langle w_{222} \rangle$
6	37	$\langle w_2, w_{111} \rangle$	6	37	$\langle w_3, w_{74} \rangle$	6	37	$\langle w_6, w_{37} \rangle$
6	37	$\langle w_2, w_3, w_{37} \rangle$	6	41	$\langle w_2, w_{123} \rangle$	6	41	$\langle w_3, w_{82} \rangle$
6	41	$\langle w_2, w_3, w_{41} \rangle$	6	43	$\langle w_2, w_{129} \rangle$	6	43	$\langle w_2, w_3, w_{43} \rangle$
6	47	$\langle w_3, w_{94} \rangle$	6	47	$\langle w_2, w_3, w_{47} \rangle$	6	49	$\langle w_2, w_3, w_{49} \rangle$
6	53	$\langle w_3, w_{106} \rangle$	6	53	$\langle w_2, w_3, w_{53} \rangle$	6	55	$\langle w_2, w_5, w_{33} \rangle$
6	55	$\langle w_5, w_6, w_{11} \rangle$	6	55	$\langle w_2, w_3, w_5, w_{11} \rangle$	6	59	$\langle w_3, w_{118} \rangle$
6	59	$\langle w_2, w_3, w_{59} \rangle$	6	61	$\langle w_6, w_{61} \rangle$	6	61	$\langle w_2, w_3, w_{61} \rangle$
6	65	$\langle w_2, w_{195} \rangle$	6	65	$\langle w_2, w_3, w_{65} \rangle$	6	65	$\langle w_2, w_5, w_{39} \rangle$
6	65	$\langle w_2, w_{13}, w_{15} \rangle$	6	65	$\langle w_3, w_{10}, w_{13} \rangle$	6	65	$\langle w_2, w_3, w_5, w_{13} \rangle$
6	67	$\langle w_2, w_{201} \rangle$	6	67	$\langle w_2, w_3, w_{67} \rangle$	6	71	$\langle w_2, w_{213} \rangle$
6	71	$\langle w_3, w_{142} \rangle$	6	71	$\langle w_2, w_3, w_{71} \rangle$	6	73	$\langle w_2, w_3, w_{73} \rangle$
6	77	$\langle w_3, w_7, w_{22} \rangle$	6	77	$\langle w_2, w_3, w_7, w_{11} \rangle$	6	79	$\langle w_2, w_{237} \rangle$
6	79	$\langle w_2, w_3, w_{79} \rangle$	6	83	$\langle w_2, w_3, w_{83} \rangle$	6	85	$\langle w_2, w_{15}, w_{17} \rangle$
6	85	$\langle w_6, w_{10}, w_{34} \rangle$	6	85	$\langle w_2, w_3, w_5, w_{17} \rangle$	6	89	$\langle w_3, w_{178} \rangle$
6	89	$\langle w_2, w_3, w_{89} \rangle$	6	91	$\langle w_{13}, w_{42} \rangle$	6	91	$\langle w_2, w_3, w_{91} \rangle$
6	91	$\langle w_2, w_{13}, w_{21} \rangle$	6	91	$\langle w_3, w_{13}, w_{14} \rangle$	6	91	$\langle w_6, w_7, w_{13} \rangle$
6	91	$\langle w_2, w_3, w_7, w_{13} \rangle$	6	95	$\langle w_2, w_5, w_{57} \rangle$	6	95	$\langle w_2, w_3, w_5, w_{19} \rangle$
6	97	$\langle w_2, w_3, w_{97} \rangle$	6	107	$\langle w_2, w_3, w_{107} \rangle$	6	109	$\langle w_6, w_{109} \rangle$
6	109	$\langle w_2, w_3, w_{109} \rangle$	6	113	$\langle w_2, w_3, w_{113} \rangle$	6	115	$\langle w_6, w_{10}, w_{46} \rangle$
6	115	$\langle w_2, w_3, w_5, w_{23} \rangle$	6	119	$\langle w_2, w_{17}, w_{21} \rangle$	6	119	$\langle w_3, w_7, w_{34} \rangle$
6	119	$\langle w_2, w_3, w_7, w_{17} \rangle$	6	137	$\langle w_2, w_3, w_{137} \rangle$	6	143	$\langle w_3, w_{13}, w_{22} \rangle$
6	143	$\langle w_2, w_3, w_{11}, w_{13} \rangle$	6	145	$\langle w_5, w_6, w_{29} \rangle$	6	145	$\langle w_2, w_3, w_5, w_{29} \rangle$

6	155	$\langle w_2, w_3, w_5, w_{31}\rangle$	6	161	$\langle w_2, w_3, w_7, w_{23}\rangle$	6	203	$\langle w_2, w_3, w_7, w_{29}\rangle$
6	235	$\langle w_2, w_3, w_5, w_{47}\rangle$	10	1	$\langle w_2\rangle$	10	1	$\langle w_5\rangle$
10	1	$\langle w_{10}\rangle$	10	1	$\langle w_2, w_5\rangle$	10	3	$\langle w_{30}\rangle$
10	3	$\langle w_2, w_3\rangle$	10	3	$\langle w_2, w_5\rangle$	10	3	$\langle w_2, w_{15}\rangle$
10	3	$\langle w_3, w_5\rangle$	10	3	$\langle w_3, w_{10}\rangle$	10	3	$\langle w_5, w_6\rangle$
10	3	$\langle w_2, w_3, w_5\rangle$	10	7	$\langle w_{70}\rangle$	10	7	$\langle w_2, w_{35}\rangle$
10	7	$\langle w_5, w_{14}\rangle$	10	7	$\langle w_7, w_{10}\rangle$	10	7	$\langle w_2, w_5, w_7\rangle$
10	9	$\langle w_2, w_5, w_9\rangle$	10	11	$\langle w_{110}\rangle$	10	11	$\langle w_2, w_{55}\rangle$
10	11	$\langle w_5, w_{22}\rangle$	10	11	$\langle w_{10}, w_{11}\rangle$	10	11	$\langle w_2, w_5, w_{11}\rangle$
10	13	$\langle w_2, w_{65}\rangle$	10	13	$\langle w_5, w_{13}\rangle$	10	13	$\langle w_{10}, w_{13}\rangle$
10	13	$\langle w_{10}, w_{26}\rangle$	10	13	$\langle w_2, w_5, w_{13}\rangle$	10	17	$\langle w_2, w_{85}\rangle$
10	17	$\langle w_{10}, w_{17}\rangle$	10	17	$\langle w_2, w_5, w_{17}\rangle$	10	19	$\langle w_2, w_{19}\rangle$
10	19	$\langle w_5, w_{38}\rangle$	10	19	$\langle w_{10}, w_{38}\rangle$	10	19	$\langle w_2, w_5, w_{19}\rangle$
10	21	$\langle w_5, w_{42}\rangle$	10	21	$\langle w_2, w_3, w_{35}\rangle$	10	21	$\langle w_2, w_5, w_{21}\rangle$
10	21	$\langle w_3, w_5, w_{14}\rangle$	10	21	$\langle w_5, w_6, w_7\rangle$	10	21	$\langle w_2, w_3, w_5, w_7\rangle$
10	23	$\langle w_{230}\rangle$	10	23	$\langle w_2, w_{115}\rangle$	10	23	$\langle w_5, w_{46}\rangle$
10	23	$\langle w_{10}, w_{23}\rangle$	10	23	$\langle w_2, w_5, w_{23}\rangle$	10	29	$\langle w_2, w_{145}\rangle$
10	29	$\langle w_5, w_{58}\rangle$	10	29	$\langle w_2, w_5, w_{29}\rangle$	10	31	$\langle w_5, w_{62}\rangle$
10	31	$\langle w_2, w_5, w_{31}\rangle$	10	33	$\langle w_2, w_3, w_{55}\rangle$	10	33	$\langle w_3, w_{10}, w_{11}\rangle$
10	33	$\langle w_5, w_6, w_{22}\rangle$	10	33	$\langle w_2, w_3, w_5, w_{11}\rangle$	10	37	$\langle w_2, w_{185}\rangle$
10	37	$\langle w_2, w_5, w_{37}\rangle$	10	39	$\langle w_2, w_3, w_{65}\rangle$	10	39	$\langle w_5, w_6, w_{13}\rangle$
10	39	$\langle w_2, w_3, w_5, w_{13}\rangle$	10	41	$\langle w_2, w_5, w_{41}\rangle$	10	49	$\langle w_2, w_5, w_{49}\rangle$
10	51	$\langle w_3, w_{10}, w_{17}\rangle$	10	51	$\langle w_2, w_3, w_5, w_{17}\rangle$	10	53	$\langle w_{10}, w_{53}\rangle$
10	53	$\langle w_2, w_5, w_{53}\rangle$	10	57	$\langle w_2, w_{15}, w_{19}\rangle$	10	57	$\langle w_2, w_3, w_5, w_{19}\rangle$
10	61	$\langle w_2, w_5, w_{61}\rangle$	10	69	$\langle w_2, w_3, w_5, w_{23}\rangle$	10	77	$\langle w_2, w_5, w_7, w_{11}\rangle$
10	93	$\langle w_2, w_3, w_5, w_{31}\rangle$	14	1	$\langle w_{14}\rangle$	14	1	$\langle w_2, w_7\rangle$
14	3	$\langle w_2, w_7\rangle$	14	3	$\langle w_2, w_{21}\rangle$	14	3	$\langle w_3, w_{14}\rangle$
14	3	$\langle w_6, w_{14}\rangle$	14	3	$\langle w_2, w_3, w_7\rangle$	14	5	$\langle w_{14}\rangle$
14	5	$\langle w_2, w_7\rangle$	14	5	$\langle w_2, w_{35}\rangle$	14	5	$\langle w_5, w_{14}\rangle$
14	5	$\langle w_{10}, w_{14}\rangle$	14	5	$\langle w_2, w_5, w_7\rangle$	14	9	$\langle w_2, w_7, w_9\rangle$
14	11	$\langle w_2, w_{77}\rangle$	14	11	$\langle w_2, w_7, w_{11}\rangle$	14	13	$\langle w_2, w_{91}\rangle$
14	13	$\langle w_{13}, w_{14}\rangle$	14	13	$\langle w_2, w_7, w_{13}\rangle$	14	15	$\langle w_2, w_7, w_{15}\rangle$
14	15	$\langle w_6, w_{10}, w_{14}\rangle$	14	15	$\langle w_2, w_3, w_5, w_7\rangle$	14	19	$\langle w_2, w_{133}\rangle$
14	19	$\langle w_{14}, w_{19}\rangle$	14	19	$\langle w_2, w_7, w_{19}\rangle$	14	23	$\langle w_2, w_7, w_{23}\rangle$
14	25	$\langle w_2, w_7, w_{25}\rangle$	14	29	$\langle w_2, w_7, w_{29}\rangle$	14	37	$\langle w_2, w_7, w_{37}\rangle$
14	39	$\langle w_2, w_3, w_7, w_{13}\rangle$	14	43	$\langle w_2, w_7, w_{43}\rangle$	14	57	$\langle w_2, w_3, w_7, w_{19}\rangle$
15	1	$\langle w_3\rangle$	15	1	$\langle w_{15}\rangle$	15	1	$\langle w_3, w_5\rangle$
15	2	$\langle w_{15}\rangle$	15	2	$\langle w_2, w_{15}\rangle$	15	2	$\langle w_3, w_5\rangle$
15	2	$\langle w_3, w_{10}\rangle$	15	2	$\langle w_6, w_{10}\rangle$	15	2	$\langle w_2, w_3, w_5\rangle$
15	4	$\langle w_3, w_5\rangle$	15	4	$\langle w_4, w_{15}\rangle$	15	4	$\langle w_{12}, w_{15}\rangle$
15	4	$\langle w_3, w_4, w_5\rangle$	15	7	$\langle w_3, w_{35}\rangle$	15	7	$\langle w_7, w_{15}\rangle$
15	7	$\langle w_3, w_5, w_7\rangle$	15	8	$\langle w_3, w_5, w_8\rangle$	15	11	$\langle w_3, w_{55}\rangle$
15	11	$\langle w_3, w_5, w_{11}\rangle$	15	13	$\langle w_3, w_{65}\rangle$	15	13	$\langle w_{13}, w_{15}\rangle$
15	13	$\langle w_3, w_5, w_{13}\rangle$	15	14	$\langle w_3, w_7, w_{10}\rangle$	15	14	$\langle w_2, w_3, w_5, w_7\rangle$
15	17	$\langle w_3, w_{85}\rangle$	15	17	$\langle w_{15}, w_{17}\rangle$	15	17	$\langle w_3, w_5, w_{17}\rangle$
15	19	$\langle w_3, w_5, w_{19}\rangle$	15	22	$\langle w_6, w_{10}, w_{22}\rangle$	15	22	$\langle w_2, w_3, w_5, w_{11}\rangle$
15	26	$\langle w_2, w_3, w_5, w_{13}\rangle$	15	29	$\langle w_3, w_5, w_{29}\rangle$	15	34	$\langle w_2, w_3, w_5, w_{17}\rangle$

15	41	$\langle w_3, w_5, w_{41}\rangle$	21	1	$\langle w_{21}\rangle$	21	1	$\langle w_3, w_7\rangle$
21	2	$\langle w_2, w_7\rangle$	21	2	$\langle w_2, w_{21}\rangle$	21	2	$\langle w_3, w_7\rangle$
21	2	$\langle w_6, w_7\rangle$	21	2	$\langle w_2, w_3, w_7\rangle$	21	4	$\langle w_3, w_4, w_7\rangle$
21	5	$\langle w_5, w_{21}\rangle$	21	5	$\langle w_7, w_{15}\rangle$	21	5	$\langle w_3, w_5, w_7\rangle$
21	10	$\langle w_2, w_7, w_{15}\rangle$	21	10	$\langle w_2, w_3, w_5, w_7\rangle$	21	11	$\langle w_7, w_{33}\rangle$
21	11	$\langle w_{11}, w_{21}\rangle$	21	11	$\langle w_3, w_7, w_{11}\rangle$	21	13	$\langle w_3, w_7, w_{13}\rangle$
21	19	$\langle w_3, w_7, w_{19}\rangle$	21	22	$\langle w_2, w_3, w_7, w_{11}\rangle$	21	26	$\langle w_2, w_3, w_7, w_{13}\rangle$
22	1	$\langle w_2\rangle$	22	1	$\langle w_{11}\rangle$	22	1	$\langle w_{22}\rangle$
22	1	$\langle w_2, w_{11}\rangle$	22	3	$\langle w_{66}\rangle$	22	3	$\langle w_2, w_{33}\rangle$
22	3	$\langle w_3, w_{22}\rangle$	22	3	$\langle w_6, w_{11}\rangle$	22	3	$\langle w_2, w_3, w_{11}\rangle$
22	5	$\langle w_{110}\rangle$	22	5	$\langle w_2, w_{55}\rangle$	22	5	$\langle w_5, w_{22}\rangle$
22	5	$\langle w_{10}, w_{11}\rangle$	22	5	$\langle w_2, w_5, w_{11}\rangle$	22	7	$\langle w_2, w_{77}\rangle$
22	7	$\langle w_{11}, w_{14}\rangle$	22	7	$\langle w_2, w_7, w_{11}\rangle$	22	13	$\langle w_{11}, w_{26}\rangle$
22	13	$\langle w_2, w_{11}, w_{13}\rangle$	22	15	$\langle w_3, w_5, w_{22}\rangle$	22	15	$\langle w_2, w_3, w_5, w_{11}\rangle$
22	17	$\langle w_2, w_{187}\rangle$	22	17	$\langle w_{11}, w_{34}\rangle$	22	17	$\langle w_2, w_{11}, w_{17}\rangle$
22	19	$\langle w_2, w_{11}, w_{19}\rangle$	22	21	$\langle w_2, w_3, w_7, w_{11}\rangle$	22	35	$\langle w_2, w_5, w_7, w_{11}\rangle$
26	1	$\langle w_{26}\rangle$	26	1	$\langle w_2, w_{13}\rangle$	26	3	$\langle w_2, w_{13}\rangle$
26	3	$\langle w_3, w_{26}\rangle$	26	3	$\langle w_6, w_{26}\rangle$	26	3	$\langle w_2, w_3, w_{13}\rangle$
26	5	$\langle w_5, w_{26}\rangle$	26	5	$\langle w_2, w_5, w_{13}\rangle$	26	7	$\langle w_7, w_{26}\rangle$
26	7	$\langle w_2, w_7, w_{13}\rangle$	26	9	$\langle w_2, w_9, w_{13}\rangle$	26	15	$\langle w_6, w_{10}, w_{26}\rangle$
26	15	$\langle w_2, w_3, w_5, w_{13}\rangle$	26	17	$\langle w_2, w_{13}, w_{17}\rangle$	26	21	$\langle w_2, w_3, w_7, w_{13}\rangle$
33	1	$\langle w_{33}\rangle$	33	1	$\langle w_3, w_{11}\rangle$	33	2	$\langle w_2, w_{33}\rangle$
33	2	$\langle w_3, w_{22}\rangle$	33	2	$\langle w_2, w_3, w_{11}\rangle$	33	5	$\langle w_3, w_{55}\rangle$
33	5	$\langle w_3, w_5, w_{11}\rangle$	33	7	$\langle w_3, w_{77}\rangle$	33	7	$\langle w_7, w_{33}\rangle$
33	7	$\langle w_3, w_7, w_{11}\rangle$	33	10	$\langle w_2, w_3, w_5, w_{11}\rangle$	33	14	$\langle w_2, w_3, w_7, w_{11}\rangle$
34	1	$\langle w_{34}\rangle$	34	1	$\langle w_2, w_{17}\rangle$	34	3	$\langle w_6, w_{17}\rangle$
34	3	$\langle w_2, w_3, w_{17}\rangle$	34	5	$\langle w_5, w_{34}\rangle$	34	5	$\langle w_2, w_5, w_{17}\rangle$
34	7	$\langle w_{14}, w_{17}\rangle$	34	7	$\langle w_2, w_7, w_{17}\rangle$	34	11	$\langle w_{17}, w_{22}\rangle$
34	11	$\langle w_2, w_{11}, w_{17}\rangle$	35	1	$\langle w_{35}\rangle$	35	1	$\langle w_5, w_7\rangle$
35	2	$\langle w_2, w_{35}\rangle$	35	2	$\langle w_5, w_7\rangle$	35	2	$\langle w_2, w_5, w_7\rangle$
35	3	$\langle w_3, w_{35}\rangle$	35	3	$\langle w_3, w_5, w_7\rangle$	35	4	$\langle w_4, w_5, w_7\rangle$
35	6	$\langle w_2, w_3, w_5, w_7\rangle$	35	13	$\langle w_5, w_7, w_{13}\rangle$	38	1	$\langle w_{38}\rangle$
38	1	$\langle w_2, w_{19}\rangle$	38	3	$\langle w_3, w_{38}\rangle$	38	3	$\langle w_2, w_3, w_{19}\rangle$
38	7	$\langle w_7, w_{38}\rangle$	38	7	$\langle w_2, w_7, w_{19}\rangle$	38	13	$\langle w_2, w_{13}, w_{19}\rangle$
39	1	$\langle w_{39}\rangle$	39	1	$\langle w_3, w_{13}\rangle$	39	2	$\langle w_{39}\rangle$
39	2	$\langle w_2, w_{39}\rangle$	39	2	$\langle w_3, w_{13}\rangle$	39	2	$\langle w_6, w_{26}\rangle$
39	2	$\langle w_2, w_3, w_{13}\rangle$	39	4	$\langle w_3, w_4, w_{13}\rangle$	39	5	$\langle w_5, w_{39}\rangle$
39	5	$\langle w_3, w_5, w_{13}\rangle$	39	7	$\langle w_3, w_7, w_{13}\rangle$	39	10	$\langle w_2, w_3, w_5, w_{13}\rangle$
46	1	$\langle w_{46}\rangle$	46	1	$\langle w_2, w_{23}\rangle$	46	3	$\langle w_2, w_{69}\rangle$
46	3	$\langle w_2, w_3, w_{23}\rangle$	46	5	$\langle w_2, w_{115}\rangle$	46	5	$\langle w_5, w_{46}\rangle$
46	5	$\langle w_2, w_5, w_{23}\rangle$	51	1	$\langle w_{51}\rangle$	51	1	$\langle w_3, w_{17}\rangle$
51	2	$\langle w_2, w_3, w_{17}\rangle$	51	5	$\langle w_3, w_5, w_{17}\rangle$	51	10	$\langle w_2, w_3, w_5, w_{17}\rangle$
55	1	$\langle w_{55}\rangle$	55	1	$\langle w_5, w_{11}\rangle$	55	2	$\langle w_2, w_5, w_{11}\rangle$
55	3	$\langle w_3, w_5, w_{11}\rangle$	57	1	$\langle w_3, w_{19}\rangle$	57	2	$\langle w_6, w_{19}\rangle$
57	2	$\langle w_2, w_3, w_{19}\rangle$	57	7	$\langle w_3, w_7, w_{19}\rangle$	58	1	$\langle w_{29}\rangle$
58	1	$\langle w_2, w_{29}\rangle$	58	3	$\langle w_6, w_{29}\rangle$	58	3	$\langle w_2, w_3, w_{29}\rangle$
58	5	$\langle w_{10}, w_{29}\rangle$	58	5	$\langle w_2, w_5, w_{29}\rangle$	62	1	$\langle w_{62}\rangle$

62	1	$\langle w_2, w_{31}\rangle$	62	3	$\langle w_3, w_{62}\rangle$	62	3	$\langle w_2, w_3, w_{31}\rangle$
62	7	$\langle w_2, w_7, w_{31}\rangle$	65	1	$\langle w_5, w_{13}\rangle$	65	3	$\langle w_3, w_5, w_{13}\rangle$
69	1	$\langle w_{69}\rangle$	69	1	$\langle w_3, w_{23}\rangle$	69	2	$\langle w_2, w_3, w_{23}\rangle$
74	1	$\langle w_{74}\rangle$	74	1	$\langle w_2, w_{37}\rangle$	74	3	$\langle w_3, w_{74}\rangle$
74	3	$\langle w_2, w_3, w_{37}\rangle$	74	5	$\langle w_2, w_5, w_{37}\rangle$	77	1	$\langle w_7, w_{11}\rangle$
77	2	$\langle w_2, w_7, w_{11}\rangle$	77	3	$\langle w_3, w_7, w_{11}\rangle$	82	1	$\langle w_2, w_{41}\rangle$
82	3	$\langle w_6, w_{41}\rangle$	82	3	$\langle w_2, w_3, w_{41}\rangle$	85	1	$\langle w_5, w_{17}\rangle$
85	3	$\langle w_3, w_5, w_{17}\rangle$	86	1	$\langle w_{86}\rangle$	86	1	$\langle w_2, w_{43}\rangle$
86	3	$\langle w_2, w_3, w_{43}\rangle$	87	1	$\langle w_{87}\rangle$	87	1	$\langle w_3, w_{29}\rangle$
87	2	$\langle w_2, w_3, w_{29}\rangle$	93	1	$\langle w_3, w_{31}\rangle$	93	2	$\langle w_6, w_{31}\rangle$
93	2	$\langle w_2, w_3, w_{31}\rangle$	94	1	$\langle w_{94}\rangle$	94	1	$\langle w_2, w_{47}\rangle$
95	1	$\langle w_{95}\rangle$	95	1	$\langle w_5, w_{19}\rangle$	95	2	$\langle w_2, w_5, w_{19}\rangle$
95	3	$\langle w_3, w_5, w_{19}\rangle$	106	1	$\langle w_2, w_{53}\rangle$	111	1	$\langle w_{111}\rangle$
111	1	$\langle w_3, w_{37}\rangle$	111	2	$\langle w_2, w_3, w_{37}\rangle$	115	1	$\langle w_5, w_{23}\rangle$
115	2	$\langle w_2, w_5, w_{23}\rangle$	118	1	$\langle w_2, w_{59}\rangle$	119	1	$\langle w_{119}\rangle$
119	1	$\langle w_7, w_{17}\rangle$	119	2	$\langle w_2, w_7, w_{17}\rangle$	122	1	$\langle w_2, w_{61}\rangle$
129	1	$\langle w_3, w_{43}\rangle$	134	1	$\langle w_{134}\rangle$	134	1	$\langle w_2, w_{67}\rangle$
134	3	$\langle w_2, w_3, w_{67}\rangle$	143	1	$\langle w_{11}, w_{13}\rangle$	143	2	$\langle w_2, w_{11}, w_{13}\rangle$
146	1	$\langle w_{146}\rangle$	146	1	$\langle w_2, w_{73}\rangle$	159	1	$\langle w_{159}\rangle$
159	1	$\langle w_3, w_{53}\rangle$	159	2	$\langle w_2, w_3, w_{53}\rangle$	161	1	$\langle w_7, w_{23}\rangle$
166	1	$\langle w_2, w_{83}\rangle$	178	1	$\langle w_2, w_{89}\rangle$	183	1	$\langle w_3, w_{61}\rangle$
194	1	$\langle w_{194}\rangle$	194	1	$\langle w_2, w_{97}\rangle$	202	1	$\langle w_2, w_{101}\rangle$
206	1	$\langle w_{206}\rangle$	206	1	$\langle w_2, w_{103}\rangle$	210	1	$\langle w_2, w_{105}\rangle$
210	1	$\langle w_3, w_{70}\rangle$	210	1	$\langle w_5, w_{42}\rangle$	210	1	$\langle w_7, w_{30}\rangle$
210	1	$\langle w_2, w_3, w_{35}\rangle$	210	1	$\langle w_2, w_5, w_{21}\rangle$	210	1	$\langle w_2, w_7, w_{15}\rangle$
210	1	$\langle w_2, w_{15}, w_{21}\rangle$	210	1	$\langle w_3, w_5, w_{14}\rangle$	210	1	$\langle w_3, w_7, w_{10}\rangle$
210	1	$\langle w_3, w_{10}, w_{14}\rangle$	210	1	$\langle w_5, w_6, w_7\rangle$	210	1	$\langle w_5, w_6, w_{14}\rangle$
210	1	$\langle w_6, w_7, w_{10}\rangle$	210	1	$\langle w_2, w_3, w_5, w_7\rangle$	210	11	$\langle w_2, w_7, w_{11}, w_{15}\rangle$
210	11	$\langle w_5, w_6, w_{14}, w_{22}\rangle$	210	11	$\langle w_2, w_3, w_5, w_7, w_{11}\rangle$	210	13	$\langle w_3, w_{10}, w_{14}, w_{26}\rangle$
210	13	$\langle w_2, w_3, w_5, w_7, w_{13}\rangle$	210	17	$\langle w_2, w_{15}, w_{21}, w_{51}\rangle$	210	17	$\langle w_2, w_3, w_5, w_7, w_{17}\rangle$
210	19	$\langle w_2, w_3, w_5, w_7, w_{19}\rangle$	215	1	$\langle w_5, w_{43}\rangle$	237	1	$\langle w_3, w_{79}\rangle$
314	1	$\langle w_2, w_{157}\rangle$	330	1	$\langle w_2, w_{165}\rangle$	330	1	$\langle w_3, w_{110}\rangle$
330	1	$\langle w_{10}, w_{33}\rangle$	330	1	$\langle w_{15}, w_{22}\rangle$	330	1	$\langle w_2, w_3, w_{11}\rangle$
330	1	$\langle w_2, w_3, w_{55}\rangle$	330	1	$\langle w_2, w_5, w_{33}\rangle$	330	1	$\langle w_2, w_{11}, w_{15}\rangle$
330	1	$\langle w_3, w_5, w_{11}\rangle$	330	1	$\langle w_3, w_5, w_{22}\rangle$	330	1	$\langle w_3, w_{10}, w_{11}\rangle$
330	1	$\langle w_3, w_{10}, w_{22}\rangle$	330	1	$\langle w_5, w_6, w_{22}\rangle$	330	1	$\langle w_6, w_{10}, w_{22}\rangle$
330	1	$\langle w_2, w_3, w_5, w_{11}\rangle$	330	7	$\langle w_2, w_3, w_{11}, w_{35}\rangle$	330	7	$\langle w_3, w_5, w_{11}, w_{14}\rangle$
330	7	$\langle w_2, w_3, w_5, w_7, w_{11}\rangle$	330	13	$\langle w_2, w_3, w_5, w_{11}, w_{13}\rangle$	390	1	$\langle w_2, w_{195}\rangle$
390	1	$\langle w_3, w_{130}\rangle$	390	1	$\langle w_5, w_{78}\rangle$	390	1	$\langle w_{13}, w_{30}\rangle$
390	1	$\langle w_2, w_3, w_{65}\rangle$	390	1	$\langle w_2, w_5, w_{39}\rangle$	390	1	$\langle w_2, w_{13}, w_{15}\rangle$
390	1	$\langle w_3, w_5, w_{26}\rangle$	390	1	$\langle w_3, w_{10}, w_{13}\rangle$	390	1	$\langle w_5, w_6, w_{13}\rangle$
390	1	$\langle w_2, w_3, w_5, w_{13}\rangle$	390	7	$\langle w_2, w_3, w_5, w_7, w_{13}\rangle$	462	1	$\langle w_3, w_{154}\rangle$
462	1	$\langle w_2, w_3, w_{77}\rangle$	462	1	$\langle w_2, w_7, w_{11}\rangle$	462	1	$\langle w_2, w_{11}, w_{21}\rangle$
462	1	$\langle w_2, w_{21}, w_{33}\rangle$	462	1	$\langle w_3, w_7, w_{22}\rangle$	462	1	$\langle w_3, w_{11}, w_{14}\rangle$
462	1	$\langle w_6, w_7, w_{22}\rangle$	462	1	$\langle w_2, w_3, w_7, w_{11}\rangle$	462	5	$\langle w_2, w_7, w_{11}, w_{15}\rangle$
462	5	$\langle w_2, w_3, w_5, w_7, w_{11}\rangle$	510	1	$\langle w_3, w_{170}\rangle$	510	1	$\langle w_5, w_{102}\rangle$
510	1	$\langle w_6, w_{85}\rangle$	510	1	$\langle w_{10}, w_{51}\rangle$	510	1	$\langle w_2, w_3, w_{85}\rangle$

510	1	$\langle w_2, w_5, w_{51}\rangle$	510	1	$\langle w_3, w_5, w_{34}\rangle$	510	1	$\langle w_3, w_{10}, w_{17}\rangle$
510	1	$\langle w_5, w_6, w_{17}\rangle$	510	1	$\langle w_6, w_{10}, w_{34}\rangle$	510	1	$\langle w_2, w_3, w_5, w_{17}\rangle$
510	7	$\langle w_2, w_3, w_5, w_7, w_{17}\rangle$	546	1	$\langle w_2, w_{273}\rangle$	546	1	$\langle w_6, w_{91}\rangle$
546	1	$\langle w_7, w_{78}\rangle$	546	1	$\langle w_{21}, w_{26}\rangle$	546	1	$\langle w_2, w_3, w_{91}\rangle$
546	1	$\langle w_2, w_7, w_{39}\rangle$	546	1	$\langle w_2, w_{13}, w_{21}\rangle$	546	1	$\langle w_3, w_7, w_{26}\rangle$
546	1	$\langle w_6, w_7, w_{13}\rangle$	546	1	$\langle w_6, w_{14}, w_{26}\rangle$	546	1	$\langle w_2, w_3, w_7, w_{13}\rangle$
546	5	$\langle w_2, w_3, w_5, w_7, w_{13}\rangle$	570	1	$\langle w_2, w_{285}\rangle$	570	1	$\langle w_2, w_3, w_{95}\rangle$
570	1	$\langle w_2, w_5, w_{57}\rangle$	570	1	$\langle w_2, w_{15}, w_{19}\rangle$	570	1	$\langle w_3, w_{10}, w_{19}\rangle$
570	1	$\langle w_2, w_3, w_5, w_{19}\rangle$	570	7	$\langle w_2, w_3, w_5, w_7, w_{19}\rangle$	690	1	$\langle w_5, w_{138}\rangle$
690	1	$\langle w_2, w_3, w_{115}\rangle$	690	1	$\langle w_2, w_5, w_{69}\rangle$	690	1	$\langle w_3, w_5, w_{46}\rangle$
690	1	$\langle w_5, w_6, w_{23}\rangle$	690	1	$\langle w_2, w_3, w_5, w_{23}\rangle$	714	1	$\langle w_3, w_{238}\rangle$
714	1	$\langle w_2, w_3, w_{119}\rangle$	714	1	$\langle w_2, w_7, w_{51}\rangle$	714	1	$\langle w_3, w_7, w_{34}\rangle$
714	1	$\langle w_3, w_{14}, w_{17}\rangle$	714	1	$\langle w_2, w_3, w_7, w_{17}\rangle$	770	1	$\langle w_{10}, w_{77}\rangle$
770	1	$\langle w_2, w_5, w_{77}\rangle$	770	1	$\langle w_2, w_{11}, w_{35}\rangle$	770	1	$\langle w_7, w_{10}, w_{11}\rangle$
770	1	$\langle w_{10}, w_{14}, w_{22}\rangle$	770	1	$\langle w_2, w_5, w_7, w_{11}\rangle$	770	3	$\langle w_2, w_3, w_5, w_7, w_{11}\rangle$
798	1	$\langle w_2, w_7, w_{57}\rangle$	798	1	$\langle w_6, w_7, w_{19}\rangle$	798	1	$\langle w_2, w_3, w_7, w_{19}\rangle$
858	1	$\langle w_2, w_3, w_{143}\rangle$	858	1	$\langle w_3, w_{11}, w_{26}\rangle$	858	1	$\langle w_2, w_3, w_{11}, w_{13}\rangle$
870	1	$\langle w_2, w_3, w_{145}\rangle$	870	1	$\langle w_6, w_{10}, w_{58}\rangle$	870	1	$\langle w_2, w_3, w_5, w_{29}\rangle$
910	1	$\langle w_5, w_{13}, w_{14}\rangle$	910	1	$\langle w_2, w_5, w_7, w_{13}\rangle$	930	1	$\langle w_2, w_3, w_{155}\rangle$
930	1	$\langle w_2, w_5, w_{93}\rangle$	930	1	$\langle w_3, w_{10}, w_{31}\rangle$	930	1	$\langle w_2, w_3, w_5, w_{31}\rangle$
966	1	$\langle w_2, w_3, w_{161}\rangle$	966	1	$\langle w_2, w_{21}, w_{23}\rangle$	966	1	$\langle w_3, w_7, w_{46}\rangle$
966	1	$\langle w_2, w_3, w_7, w_{23}\rangle$	1110	1	$\langle w_3, w_5, w_{74}\rangle$	1110	1	$\langle w_2, w_3, w_5, w_{37}\rangle$
1122	1	$\langle w_2, w_3, w_{11}, w_{17}\rangle$	1190	1	$\langle w_2, w_7, w_{85}\rangle$	1190	1	$\langle w_5, w_7, w_{34}\rangle$
1190	1	$\langle w_2, w_5, w_7, w_{17}\rangle$	1218	1	$\langle w_2, w_3, w_{203}\rangle$	1218	1	$\langle w_3, w_7, w_{58}\rangle$
1218	1	$\langle w_2, w_3, w_7, w_{29}\rangle$	1230	1	$\langle w_6, w_{10}, w_{82}\rangle$	1230	1	$\langle w_2, w_3, w_5, w_{41}\rangle$
1254	1	$\langle w_2, w_3, w_{11}, w_{19}\rangle$	1290	1	$\langle w_2, w_3, w_5, w_{43}\rangle$	1302	1	$\langle w_2, w_7, w_{93}\rangle$
1302	1	$\langle w_2, w_3, w_7, w_{31}\rangle$	1326	1	$\langle w_3, w_{13}, w_{34}\rangle$	1326	1	$\langle w_2, w_3, w_{13}, w_{17}\rangle$
1410	1	$\langle w_2, w_3, w_{235}\rangle$	1410	1	$\langle w_2, w_5, w_{141}\rangle$	1410	1	$\langle w_2, w_3, w_5, w_{47}\rangle$
1590	1	$\langle w_5, w_6, w_{53}\rangle$	1590	1	$\langle w_2, w_3, w_5, w_{53}\rangle$	1722	1	$\langle w_2, w_3, w_7, w_{41}\rangle$
1770	1	$\langle w_2, w_{15}, w_{59}\rangle$	1770	1	$\langle w_2, w_3, w_5, w_{59}\rangle$	1794	1	$\langle w_2, w_3, w_{13}, w_{23}\rangle$
1914	1	$\langle w_2, w_3, w_{11}, w_{29}\rangle$	1938	1	$\langle w_2, w_3, w_{17}, w_{19}\rangle$	1974	1	$\langle w_2, w_3, w_7, w_{47}\rangle$
2010	1	$\langle w_2, w_3, w_5, w_{67}\rangle$	2130	1	$\langle w_2, w_3, w_5, w_{71}\rangle$			

Table 5: 537 genus 1 curves $X = X_0^D(N)/W$, with $W \leq W_0(D,N)$ nontrivial, for which we prove that X is an elliptic curve ($X(\mathbb{Q}) \neq \emptyset$) and for which X has positive rank over $\mathbb{Q}$.

D	N	W	D	N	W	D	N	W
6	17	$\langle w_{102}\rangle$	6	17	$\langle w_6, w_{17}\rangle$	6	23	$\langle w_{138}\rangle$
6	23	$\langle w_6, w_{23}\rangle$	6	35	$\langle w_6, w_{35}\rangle$	6	35	$\langle w_5, w_6, w_7\rangle$
6	41	$\langle w_{246}\rangle$	6	41	$\langle w_6, w_{41}\rangle$	6	43	$\langle w_3, w_{86}\rangle$
6	43	$\langle w_6, w_{43}\rangle$	6	47	$\langle w_2, w_{141}\rangle$	6	49	$\langle w_3, w_{98}\rangle$
6	53	$\langle w_2, w_{159}\rangle$	6	53	$\langle w_6, w_{53}\rangle$	6	55	$\langle w_6, w_{10}, w_{22}\rangle$
6	59	$\langle w_2, w_{177}\rangle$	6	59	$\langle w_6, w_{59}\rangle$	6	61	$\langle w_2, w_{183}\rangle$
6	61	$\langle w_3, w_{122}\rangle$	6	65	$\langle w_{13}, w_{30}\rangle$	6	65	$\langle w_5, w_6, w_{13}\rangle$
6	67	$\langle w_3, w_{134}\rangle$	6	67	$\langle w_6, w_{67}\rangle$	6	71	$\langle w_{426}\rangle$

6	71	$\langle w_6, w_{71}\rangle$	6	73	$\langle w_2, w_{219}\rangle$	6	73	$\langle w_6, w_{73}\rangle$
6	77	$\langle w_6, w_7, w_{11}\rangle$	6	77	$\langle w_6, w_{14}, w_{22}\rangle$	6	79	$\langle w_3, w_{158}\rangle$
6	79	$\langle w_6, w_{79}\rangle$	6	83	$\langle w_3, w_{166}\rangle$	6	85	$\langle w_2, w_5, w_{51}\rangle$
6	85	$\langle w_5, w_6, w_{17}\rangle$	6	89	$\langle w_2, w_{267}\rangle$	6	91	$\langle w_3, w_{182}\rangle$
6	91	$\langle w_3, w_7, w_{26}\rangle$	6	95	$\langle w_3, w_{10}, w_{19}\rangle$	6	95	$\langle w_5, w_6, w_{19}\rangle$
6	97	$\langle w_2, w_{291}\rangle$	6	97	$\langle w_6, w_{97}\rangle$	6	101	$\langle w_2, w_3, w_{101}\rangle$
6	103	$\langle w_2, w_3, w_{103}\rangle$	6	107	$\langle w_2, w_{321}\rangle$	6	109	$\langle w_2, w_{327}\rangle$
6	109	$\langle w_3, w_{218}\rangle$	6	113	$\langle w_2, w_{339}\rangle$	6	113	$\langle w_3, w_{226}\rangle$
6	115	$\langle w_2, w_5, w_{69}\rangle$	6	119	$\langle w_{21}, w_{34}\rangle$	6	119	$\langle w_6, w_{14}, w_{34}\rangle$
6	127	$\langle w_2, w_3, w_{127}\rangle$	6	131	$\langle w_2, w_3, w_{131}\rangle$	6	133	$\langle w_2, w_3, w_7, w_{19}\rangle$
6	137	$\langle w_3, w_{274}\rangle$	6	139	$\langle w_2, w_3, w_{139}\rangle$	6	143	$\langle w_2, w_{13}, w_{33}\rangle$
6	145	$\langle w_2, w_5, w_{87}\rangle$	6	149	$\langle w_2, w_3, w_{149}\rangle$	6	151	$\langle w_2, w_3, w_{151}\rangle$
6	161	$\langle w_3, w_7, w_{46}\rangle$	6	163	$\langle w_2, w_3, w_{163}\rangle$	6	167	$\langle w_2, w_3, w_{167}\rangle$
6	179	$\langle w_2, w_3, w_{179}\rangle$	6	181	$\langle w_2, w_3, w_{181}\rangle$	6	185	$\langle w_2, w_3, w_5, w_{37}\rangle$
6	187	$\langle w_2, w_3, w_{11}, w_{17}\rangle$	6	191	$\langle w_2, w_3, w_{191}\rangle$	6	193	$\langle w_2, w_3, w_{193}\rangle$
6	205	$\langle w_2, w_3, w_5, w_{41}\rangle$	6	209	$\langle w_2, w_3, w_{11}, w_{19}\rangle$	6	211	$\langle w_2, w_3, w_{211}\rangle$
6	215	$\langle w_2, w_3, w_5, w_{43}\rangle$	6	217	$\langle w_2, w_3, w_7, w_{31}\rangle$	6	221	$\langle w_2, w_3, w_{13}, w_{17}\rangle$
6	227	$\langle w_2, w_3, w_{227}\rangle$	6	235	$\langle w_6, w_{10}, w_{94}\rangle$	6	241	$\langle w_2, w_3, w_{241}\rangle$
6	247	$\langle w_2, w_3, w_{13}, w_{19}\rangle$	6	253	$\langle w_2, w_3, w_{11}, w_{23}\rangle$	6	259	$\langle w_2, w_3, w_7, w_{37}\rangle$
6	265	$\langle w_2, w_3, w_5, w_{53}\rangle$	6	295	$\langle w_2, w_3, w_5, w_{59}\rangle$	6	301	$\langle w_2, w_3, w_7, w_{43}\rangle$
6	305	$\langle w_2, w_3, w_5, w_{61}\rangle$	6	323	$\langle w_2, w_3, w_{17}, w_{19}\rangle$	6	385	$\langle w_2, w_3, w_5, w_7, w_{11}\rangle$
10	13	$\langle w_{130}\rangle$	10	13	$\langle w_5, w_{26}\rangle$	10	17	$\langle w_{170}\rangle$
10	17	$\langle w_5, w_{34}\rangle$	10	19	$\langle w_2, w_{95}\rangle$	10	19	$\langle w_{10}, w_{19}\rangle$
10	21	$\langle w_3, w_{70}\rangle$	10	21	$\langle w_3, w_7, w_{10}\rangle$	10	29	$\langle w_{290}\rangle$
10	29	$\langle w_{10}, w_{29}\rangle$	10	31	$\langle w_2, w_{155}\rangle$	10	33	$\langle w_3, w_{110}\rangle$
10	33	$\langle w_3, w_5, w_{22}\rangle$	10	37	$\langle w_{10}, w_{37}\rangle$	10	39	$\langle w_3, w_5, w_{26}\rangle$
10	39	$\langle w_3, w_{10}, w_{13}\rangle$	10	41	$\langle w_2, w_{205}\rangle$	10	41	$\langle w_5, w_{82}\rangle$
10	43	$\langle w_2, w_5, w_{43}\rangle$	10	47	$\langle w_2, w_5, w_{47}\rangle$	10	51	$\langle w_2, w_3, w_{85}\rangle$
10	51	$\langle w_3, w_5, w_{34}\rangle$	10	53	$\langle w_2, w_{265}\rangle$	10	57	$\langle w_2, w_3, w_{95}\rangle$
10	57	$\langle w_3, w_5, w_{38}\rangle$	10	59	$\langle w_2, w_5, w_{59}\rangle$	10	61	$\langle w_5, w_{122}\rangle$
10	69	$\langle w_2, w_3, w_{115}\rangle$	10	69	$\langle w_3, w_{10}, w_{23}\rangle$	10	71	$\langle w_2, w_5, w_{71}\rangle$
10	73	$\langle w_2, w_5, w_{73}\rangle$	10	77	$\langle w_5, w_7, w_{22}\rangle$	10	79	$\langle w_2, w_5, w_{79}\rangle$
10	87	$\langle w_2, w_3, w_5, w_{29}\rangle$	10	89	$\langle w_2, w_5, w_{89}\rangle$	10	91	$\langle w_2, w_5, w_7, w_{13}\rangle$
10	109	$\langle w_2, w_5, w_{109}\rangle$	10	111	$\langle w_2, w_3, w_5, w_{37}\rangle$	10	113	$\langle w_2, w_5, w_{113}\rangle$
10	119	$\langle w_2, w_5, w_7, w_{17}\rangle$	10	133	$\langle w_2, w_5, w_7, w_{19}\rangle$	14	11	$\langle w_{11}, w_{14}\rangle$
14	15	$\langle w_{14}, w_{15}\rangle$	14	15	$\langle w_3, w_5, w_{14}\rangle$	14	17	$\langle w_2, w_{119}\rangle$
14	17	$\langle w_2, w_7, w_{17}\rangle$	14	23	$\langle w_2, w_{161}\rangle$	14	29	$\langle w_2, w_{203}\rangle$
14	29	$\langle w_{14}, w_{29}\rangle$	14	31	$\langle w_2, w_{217}\rangle$	14	31	$\langle w_2, w_7, w_{31}\rangle$
14	33	$\langle w_2, w_3, w_7, w_{11}\rangle$	14	51	$\langle w_2, w_3, w_7, w_{17}\rangle$	14	53	$\langle w_2, w_7, w_{53}\rangle$
14	55	$\langle w_2, w_5, w_7, w_{11}\rangle$	14	57	$\langle w_3, w_{14}, w_{19}\rangle$	14	61	$\langle w_2, w_7, w_{61}\rangle$
14	65	$\langle w_2, w_5, w_7, w_{13}\rangle$	14	67	$\langle w_2, w_7, w_{67}\rangle$	14	69	$\langle w_2, w_3, w_7, w_{23}\rangle$
14	79	$\langle w_2, w_7, w_{79}\rangle$	14	85	$\langle w_2, w_5, w_7, w_{17}\rangle$	14	93	$\langle w_2, w_3, w_7, w_{31}\rangle$
15	14	$\langle w_2, w_7, w_{15}\rangle$	15	19	$\langle w_3, w_{95}\rangle$	15	19	$\langle w_{15}, w_{19}\rangle$
15	23	$\langle w_3, w_5, w_{23}\rangle$	15	31	$\langle w_3, w_5, w_{31}\rangle$	15	41	$\langle w_3, w_{205}\rangle$
15	46	$\langle w_2, w_3, w_5, w_{23}\rangle$	15	58	$\langle w_2, w_3, w_5, w_{29}\rangle$	15	61	$\langle w_3, w_5, w_{61}\rangle$
15	77	$\langle w_3, w_5, w_7, w_{11}\rangle$	21	10	$\langle w_2, w_5, w_{21}\rangle$	21	13	$\langle w_7, w_{39}\rangle$
21	17	$\langle w_3, w_7, w_{17}\rangle$	21	19	$\langle w_7, w_{57}\rangle$	21	19	$\langle w_{19}, w_{21}\rangle$

21	22	$\langle w_2, w_7, w_{33}\rangle$	21	22	$\langle w_2, w_{11}, w_{21}\rangle$	21	26	$\langle w_2, w_7, w_{39}\rangle$
21	31	$\langle w_3, w_7, w_{31}\rangle$	21	34	$\langle w_2, w_3, w_7, w_{17}\rangle$	21	37	$\langle w_3, w_7, w_{37}\rangle$
21	38	$\langle w_2, w_3, w_7, w_{19}\rangle$	22	7	$\langle w_{154}\rangle$	22	7	$\langle w_7, w_{22}\rangle$
22	13	$\langle w_2, w_{143}\rangle$	22	13	$\langle w_{13}, w_{22}\rangle$	22	15	$\langle w_3, w_{10}, w_{11}\rangle$
22	17	$\langle w_{374}\rangle$	22	17	$\langle w_{17}, w_{22}\rangle$	22	19	$\langle w_2, w_{209}\rangle$
22	21	$\langle w_3, w_{11}, w_{14}\rangle$	22	23	$\langle w_2, w_{11}, w_{23}\rangle$	22	31	$\langle w_2, w_{11}, w_{31}\rangle$
22	39	$\langle w_2, w_3, w_{11}, w_{13}\rangle$	22	57	$\langle w_2, w_3, w_{11}, w_{19}\rangle$	26	11	$\langle w_2, w_{11}, w_{13}\rangle$
26	15	$\langle w_3, w_5, w_{26}\rangle$	26	19	$\langle w_2, w_{13}, w_{19}\rangle$	33	23	$\langle w_3, w_{11}, w_{23}\rangle$
34	3	$\langle w_3, w_{34}\rangle$	34	5	$\langle w_{10}, w_{17}\rangle$	34	7	$\langle w_7, w_{34}\rangle$
34	11	$\langle w_{11}, w_{34}\rangle$	34	13	$\langle w_2, w_{13}, w_{17}\rangle$	34	15	$\langle w_2, w_3, w_5, w_{17}\rangle$
34	21	$\langle w_2, w_3, w_7, w_{17}\rangle$	34	33	$\langle w_2, w_3, w_{11}, w_{17}\rangle$	35	6	$\langle w_2, w_3, w_{35}\rangle$
35	11	$\langle w_5, w_7, w_{11}\rangle$	35	13	$\langle w_{13}, w_{35}\rangle$	38	5	$\langle w_2, w_5, w_{19}\rangle$
38	11	$\langle w_2, w_{11}, w_{19}\rangle$	38	13	$\langle w_{13}, w_{38}\rangle$	38	15	$\langle w_2, w_3, w_5, w_{19}\rangle$
38	17	$\langle w_2, w_{17}, w_{19}\rangle$	38	21	$\langle w_2, w_3, w_7, w_{19}\rangle$	39	11	$\langle w_3, w_{11}, w_{13}\rangle$
39	14	$\langle w_2, w_3, w_7, w_{13}\rangle$	39	19	$\langle w_3, w_{13}, w_{19}\rangle$	46	3	$\langle w_3, w_{46}\rangle$
46	7	$\langle w_2, w_{161}\rangle$	46	7	$\langle w_2, w_7, w_{23}\rangle$	46	11	$\langle w_2, w_{11}, w_{23}\rangle$
46	13	$\langle w_2, w_{13}, w_{23}\rangle$	46	15	$\langle w_2, w_3, w_5, w_{23}\rangle$	46	21	$\langle w_2, w_3, w_7, w_{23}\rangle$
51	2	$\langle w_2, w_{51}\rangle$	51	13	$\langle w_3, w_{13}, w_{17}\rangle$	55	6	$\langle w_2, w_3, w_5, w_{11}\rangle$
55	7	$\langle w_5, w_7, w_{11}\rangle$	57	1	$\langle w_{57}\rangle$	57	2	$\langle w_2, w_{57}\rangle$
57	5	$\langle w_{15}, w_{19}\rangle$	57	5	$\langle w_3, w_5, w_{19}\rangle$	57	7	$\langle w_{19}, w_{21}\rangle$
57	10	$\langle w_2, w_3, w_5, w_{19}\rangle$	57	14	$\langle w_2, w_3, w_7, w_{19}\rangle$	58	1	$\langle w_{58}\rangle$
58	3	$\langle w_3, w_{58}\rangle$	58	7	$\langle w_2, w_7, w_{29}\rangle$	58	13	$\langle w_2, w_{13}, w_{29}\rangle$
58	15	$\langle w_2, w_3, w_5, w_{29}\rangle$	62	5	$\langle w_2, w_5, w_{31}\rangle$	62	7	$\langle w_7, w_{62}\rangle$
62	11	$\langle w_2, w_{11}, w_{31}\rangle$	65	1	$\langle w_{65}\rangle$	65	2	$\langle w_2, w_5, w_{13}\rangle$
65	3	$\langle w_3, w_{65}\rangle$	65	6	$\langle w_2, w_3, w_5, w_{13}\rangle$	65	7	$\langle w_5, w_7, w_{13}\rangle$
69	2	$\langle w_2, w_{69}\rangle$	69	5	$\langle w_3, w_5, w_{23}\rangle$	74	5	$\langle w_5, w_{74}\rangle$
77	1	$\langle w_{77}\rangle$	77	3	$\langle w_3, w_{77}\rangle$	82	1	$\langle w_{82}\rangle$
82	5	$\langle w_2, w_5, w_{41}\rangle$	82	7	$\langle w_2, w_7, w_{41}\rangle$	85	2	$\langle w_2, w_5, w_{17}\rangle$
86	3	$\langle w_3, w_{86}\rangle$	86	5	$\langle w_2, w_5, w_{43}\rangle$	91	1	$\langle w_7, w_{13}\rangle$
91	2	$\langle w_2, w_7, w_{13}\rangle$	91	5	$\langle w_5, w_7, w_{13}\rangle$	93	7	$\langle w_3, w_7, w_{31}\rangle$
94	3	$\langle w_2, w_3, w_{47}\rangle$	94	5	$\langle w_2, w_5, w_{47}\rangle$	94	7	$\langle w_2, w_7, w_{47}\rangle$
95	2	$\langle w_2, w_{95}\rangle$	106	1	$\langle w_{106}\rangle$	106	3	$\langle w_2, w_3, w_{53}\rangle$
106	5	$\langle w_2, w_5, w_{53}\rangle$	118	1	$\langle w_{118}\rangle$	118	3	$\langle w_2, w_3, w_{59}\rangle$
119	2	$\langle w_2, w_{119}\rangle$	119	3	$\langle w_3, w_7, w_{17}\rangle$	122	1	$\langle w_{122}\rangle$
122	3	$\langle w_2, w_3, w_{61}\rangle$	123	1	$\langle w_3, w_{41}\rangle$	123	2	$\langle w_2, w_3, w_{41}\rangle$
129	1	$\langle w_{129}\rangle$	129	2	$\langle w_2, w_3, w_{43}\rangle$	141	1	$\langle w_3, w_{47}\rangle$
141	2	$\langle w_2, w_3, w_{47}\rangle$	142	1	$\langle w_2, w_{71}\rangle$	142	3	$\langle w_2, w_3, w_{71}\rangle$
143	1	$\langle w_{143}\rangle$	143	3	$\langle w_3, w_{11}, w_{13}\rangle$	146	3	$\langle w_2, w_3, w_{73}\rangle$
146	5	$\langle w_2, w_5, w_{73}\rangle$	155	1	$\langle w_5, w_{31}\rangle$	155	2	$\langle w_2, w_5, w_{31}\rangle$
158	1	$\langle w_2, w_{79}\rangle$	158	3	$\langle w_2, w_3, w_{79}\rangle$	161	2	$\langle w_2, w_7, w_{23}\rangle$
166	1	$\langle w_{166}\rangle$	178	3	$\langle w_2, w_3, w_{89}\rangle$	183	2	$\langle w_2, w_3, w_{61}\rangle$
185	1	$\langle w_5, w_{37}\rangle$	194	3	$\langle w_2, w_3, w_{97}\rangle$	201	1	$\langle w_3, w_{67}\rangle$
203	1	$\langle w_7, w_{29}\rangle$	206	3	$\langle w_2, w_3, w_{103}\rangle$	209	1	$\langle w_{11}, w_{19}\rangle$
210	1	$\langle w_{210}\rangle$	210	1	$\langle w_6, w_{35}\rangle$	210	1	$\langle w_{10}, w_{21}\rangle$
210	1	$\langle w_{14}, w_{15}\rangle$	210	1	$\langle w_6, w_{10}, w_{14}\rangle$	210	11	$\langle w_{14}, w_{22}, w_{30}\rangle$
210	11	$\langle w_2, w_3, w_{11}, w_{35}\rangle$	210	11	$\langle w_2, w_5, w_{11}, w_{21}\rangle$	210	11	$\langle w_3, w_{10}, w_{14}, w_{22}\rangle$
210	11	$\langle w_5, w_6, w_7, w_{11}\rangle$	210	11	$\langle w_6, w_7, w_{10}, w_{22}\rangle$	210	13	$\langle w_2, w_3, w_{13}, w_{35}\rangle$

210	13	$\langle w_2, w_5, w_{13}, w_{21} \rangle$	210	13	$\langle w_2, w_{15}, w_{21}, w_{39} \rangle$	210	13	$\langle w_3, w_7, w_{10}, w_{13} \rangle$
210	17	$\langle w_6, w_7, w_{10}, w_{34} \rangle$	210	19	$\langle w_2, w_3, w_{19}, w_{35} \rangle$	210	19	$\langle w_3, w_5, w_{14}, w_{19} \rangle$
210	19	$\langle w_3, w_7, w_{10}, w_{19} \rangle$	210	19	$\langle w_5, w_6, w_{14}, w_{38} \rangle$	210	23	$\langle w_2, w_3, w_5, w_7, w_{23} \rangle$
210	29	$\langle w_2, w_3, w_5, w_7, w_{29} \rangle$	210	31	$\langle w_2, w_3, w_5, w_7, w_{31} \rangle$	210	37	$\langle w_2, w_3, w_5, w_7, w_{37} \rangle$
214	1	$\langle w_2, w_{107} \rangle$	215	1	$\langle w_{215} \rangle$	218	1	$\langle w_2, w_{109} \rangle$
219	1	$\langle w_3, w_{73} \rangle$	226	1	$\langle w_2, w_{113} \rangle$	235	1	$\langle w_5, w_{47} \rangle$
237	2	$\langle w_2, w_3, w_{79} \rangle$	254	1	$\langle w_2, w_{127} \rangle$	262	1	$\langle w_2, w_{131} \rangle$
274	1	$\langle w_2, w_{137} \rangle$	278	1	$\langle w_2, w_{139} \rangle$	298	1	$\langle w_2, w_{149} \rangle$
309	1	$\langle w_3, w_{103} \rangle$	314	1	$\langle w_{314} \rangle$	326	1	$\langle w_2, w_{163} \rangle$
327	1	$\langle w_3, w_{109} \rangle$	330	1	$\langle w_{330} \rangle$	330	1	$\langle w_5, w_{66} \rangle$
330	1	$\langle w_6, w_{55} \rangle$	330	1	$\langle w_{11}, w_{30} \rangle$	330	1	$\langle w_5, w_6, w_{11} \rangle$
330	7	$\langle w_3, w_{11}, w_{70} \rangle$	330	7	$\langle w_2, w_3, w_7, w_{55} \rangle$	330	7	$\langle w_2, w_5, w_7, w_{33} \rangle$
330	7	$\langle w_3, w_7, w_{10}, w_{11} \rangle$	330	7	$\langle w_5, w_6, w_7, w_{11} \rangle$	330	7	$\langle w_5, w_6, w_{14}, w_{22} \rangle$
330	7	$\langle w_6, w_7, w_{10}, w_{22} \rangle$	330	13	$\langle w_2, w_3, w_{11}, w_{65} \rangle$	330	13	$\langle w_3, w_5, w_{11}, w_{26} \rangle$
330	17	$\langle w_2, w_3, w_5, w_{11}, w_{17} \rangle$	330	19	$\langle w_2, w_3, w_5, w_{11}, w_{19} \rangle$	335	1	$\langle w_5, w_{67} \rangle$
346	1	$\langle w_2, w_{173} \rangle$	362	1	$\langle w_2, w_{181} \rangle$	381	1	$\langle w_3, w_{127} \rangle$
390	1	$\langle w_{390} \rangle$	390	1	$\langle w_6, w_{65} \rangle$	390	1	$\langle w_{10}, w_{39} \rangle$
390	1	$\langle w_{15}, w_{26} \rangle$	390	1	$\langle w_6, w_{10}, w_{26} \rangle$	390	11	$\langle w_2, w_3, w_5, w_{11}, w_{13} \rangle$
390	19	$\langle w_2, w_3, w_5, w_{13}, w_{19} \rangle$	446	1	$\langle w_2, w_{223} \rangle$	458	1	$\langle w_2, w_{229} \rangle$
462	1	$\langle w_2, w_{231} \rangle$	462	1	$\langle w_{11}, w_{42} \rangle$	462	1	$\langle w_{21}, w_{22} \rangle$
462	1	$\langle w_2, w_7, w_{33} \rangle$	462	1	$\langle w_6, w_7, w_{11} \rangle$	462	1	$\langle w_6, w_{14}, w_{22} \rangle$
462	5	$\langle w_2, w_3, w_5, w_{77} \rangle$	462	5	$\langle w_2, w_5, w_7, w_{33} \rangle$	462	5	$\langle w_2, w_{15}, w_{21}, w_{33} \rangle$
462	5	$\langle w_3, w_5, w_{11}, w_{14} \rangle$	462	5	$\langle w_6, w_7, w_{10}, w_{22} \rangle$	462	13	$\langle w_2, w_3, w_7, w_{11}, w_{13} \rangle$
510	1	$\langle w_{510} \rangle$	510	1	$\langle w_2, w_{255} \rangle$	510	1	$\langle w_{15}, w_{34} \rangle$
510	1	$\langle w_{17}, w_{30} \rangle$	510	1	$\langle w_2, w_{15}, w_{17} \rangle$	510	7	$\langle w_2, w_7, w_{15}, w_{17} \rangle$
510	7	$\langle w_6, w_7, w_{10}, w_{34} \rangle$	546	1	$\langle w_{546} \rangle$	546	1	$\langle w_3, w_{182} \rangle$
546	1	$\langle w_{13}, w_{42} \rangle$	546	1	$\langle w_{14}, w_{39} \rangle$	546	1	$\langle w_3, w_{13}, w_{14} \rangle$
570	1	$\langle w_3, w_{190} \rangle$	570	1	$\langle w_{10}, w_{57} \rangle$	570	1	$\langle w_{19}, w_{30} \rangle$
570	1	$\langle w_3, w_5, w_{38} \rangle$	570	1	$\langle w_5, w_6, w_{19} \rangle$	570	1	$\langle w_6, w_{10}, w_{38} \rangle$
690	1	$\langle w_2, w_{345} \rangle$	690	1	$\langle w_3, w_{230} \rangle$	690	1	$\langle w_6, w_{115} \rangle$
690	1	$\langle w_2, w_{15}, w_{23} \rangle$	690	1	$\langle w_3, w_{10}, w_{23} \rangle$	690	1	$\langle w_6, w_{10}, w_{46} \rangle$
690	7	$\langle w_2, w_3, w_5, w_7, w_{23} \rangle$	714	1	$\langle w_2, w_{357} \rangle$	714	1	$\langle w_7, w_{102} \rangle$
714	1	$\langle w_{14}, w_{51} \rangle$	714	1	$\langle w_2, w_{17}, w_{21} \rangle$	714	1	$\langle w_6, w_7, w_{17} \rangle$
714	1	$\langle w_6, w_{14}, w_{34} \rangle$	714	5	$\langle w_2, w_3, w_5, w_7, w_{17} \rangle$	770	1	$\langle w_2, w_{385} \rangle$
770	1	$\langle w_{11}, w_{70} \rangle$	770	1	$\langle w_{22}, w_{35} \rangle$	770	1	$\langle w_2, w_7, w_{55} \rangle$
770	1	$\langle w_5, w_7, w_{22} \rangle$	770	1	$\langle w_5, w_{11}, w_{14} \rangle$	798	1	$\langle w_7, w_{114} \rangle$
798	1	$\langle w_2, w_3, w_{133} \rangle$	798	1	$\langle w_2, w_{19}, w_{21} \rangle$	798	1	$\langle w_3, w_7, w_{38} \rangle$
798	1	$\langle w_3, w_{14}, w_{19} \rangle$	798	1	$\langle w_6, w_{14}, w_{38} \rangle$	798	5	$\langle w_2, w_3, w_5, w_7, w_{19} \rangle$
858	1	$\langle w_3, w_{286} \rangle$	858	1	$\langle w_2, w_{11}, w_{39} \rangle$	858	1	$\langle w_2, w_{13}, w_{33} \rangle$
858	1	$\langle w_3, w_{13}, w_{22} \rangle$	870	1	$\langle w_6, w_{145} \rangle$	870	1	$\langle w_2, w_5, w_{87} \rangle$
870	1	$\langle w_2, w_{15}, w_{29} \rangle$	870	1	$\langle w_3, w_5, w_{58} \rangle$	870	1	$\langle w_3, w_{10}, w_{29} \rangle$
870	1	$\langle w_5, w_6, w_{29} \rangle$	910	1	$\langle w_2, w_5, w_{91} \rangle$	910	1	$\langle w_2, w_7, w_{65} \rangle$
910	1	$\langle w_5, w_7, w_{26} \rangle$	910	3	$\langle w_2, w_3, w_5, w_7, w_{13} \rangle$	930	1	$\langle w_2, w_{465} \rangle$
930	1	$\langle w_3, w_{310} \rangle$	930	1	$\langle w_{10}, w_{93} \rangle$	930	1	$\langle w_2, w_{15}, w_{31} \rangle$
930	1	$\langle w_3, w_5, w_{62} \rangle$	930	1	$\langle w_6, w_{10}, w_{62} \rangle$	966	1	$\langle w_2, w_{483} \rangle$
966	1	$\langle w_3, w_{322} \rangle$	966	1	$\langle w_{21}, w_{46} \rangle$	966	1	$\langle w_2, w_7, w_{69} \rangle$
966	1	$\langle w_3, w_{14}, w_{23} \rangle$	966	1	$\langle w_6, w_{14}, w_{46} \rangle$	966	5	$\langle w_2, w_3, w_5, w_7, w_{23} \rangle$

1110	1	$\langle w_2, w_3, w_{185}\rangle$	1110	1	$\langle w_2, w_5, w_{111}\rangle$	1110	1	$\langle w_3, w_{10}, w_{37}\rangle$
1110	1	$\langle w_5, w_6, w_{37}\rangle$	1122	1	$\langle w_2, w_3, w_{187}\rangle$	1122	1	$\langle w_2, w_{11}, w_{51}\rangle$
1122	1	$\langle w_2, w_{17}, w_{33}\rangle$	1122	1	$\langle w_3, w_{11}, w_{34}\rangle$	1122	1	$\langle w_6, w_{22}, w_{34}\rangle$
1155	1	$\langle w_3, w_5, w_7, w_{11}\rangle$	1190	1	$\langle w_7, w_{170}\rangle$	1190	1	$\langle w_2, w_5, w_{119}\rangle$
1190	1	$\langle w_5, w_{14}, w_{17}\rangle$	1190	1	$\langle w_7, w_{10}, w_{17}\rangle$	1218	1	$\langle w_3, w_{406}\rangle$
1218	1	$\langle w_2, w_{21}, w_{29}\rangle$	1218	1	$\langle w_3, w_{14}, w_{29}\rangle$	1230	1	$\langle w_2, w_3, w_{205}\rangle$
1230	1	$\langle w_2, w_{15}, w_{41}\rangle$	1230	1	$\langle w_3, w_5, w_{82}\rangle$	1230	1	$\langle w_3, w_{10}, w_{41}\rangle$
1230	1	$\langle w_5, w_6, w_{41}\rangle$	1254	1	$\langle w_2, w_3, w_{209}\rangle$	1254	1	$\langle w_2, w_{11}, w_{57}\rangle$
1254	1	$\langle w_2, w_{19}, w_{33}\rangle$	1254	1	$\langle w_3, w_{11}, w_{38}\rangle$	1254	1	$\langle w_3, w_{19}, w_{22}\rangle$
1254	1	$\langle w_6, w_{11}, w_{19}\rangle$	1290	1	$\langle w_2, w_3, w_{215}\rangle$	1290	1	$\langle w_2, w_5, w_{129}\rangle$
1290	1	$\langle w_3, w_5, w_{86}\rangle$	1290	1	$\langle w_3, w_{10}, w_{43}\rangle$	1302	1	$\langle w_2, w_3, w_{217}\rangle$
1302	1	$\langle w_6, w_7, w_{31}\rangle$	1326	1	$\langle w_2, w_3, w_{221}\rangle$	1326	1	$\langle w_2, w_{17}, w_{39}\rangle$
1326	1	$\langle w_3, w_{17}, w_{26}\rangle$	1326	1	$\langle w_6, w_{13}, w_{17}\rangle$	1326	1	$\langle w_6, w_{26}, w_{34}\rangle$
1330	1	$\langle w_2, w_5, w_7, w_{19}\rangle$	1410	1	$\langle w_2, w_{705}\rangle$	1410	1	$\langle w_2, w_{15}, w_{47}\rangle$
1410	1	$\langle w_3, w_5, w_{94}\rangle$	1410	1	$\langle w_5, w_6, w_{47}\rangle$	1430	1	$\langle w_2, w_5, w_{11}, w_{13}\rangle$
1482	1	$\langle w_2, w_3, w_{13}, w_{19}\rangle$	1518	1	$\langle w_2, w_3, w_{11}, w_{23}\rangle$	1554	1	$\langle w_2, w_3, w_7, w_{37}\rangle$
1590	1	$\langle w_2, w_3, w_{265}\rangle$	1590	1	$\langle w_2, w_5, w_{159}\rangle$	1590	1	$\langle w_3, w_5, w_{106}\rangle$
1610	1	$\langle w_2, w_5, w_7, w_{23}\rangle$	1722	1	$\langle w_2, w_3, w_{287}\rangle$	1722	1	$\langle w_2, w_7, w_{123}\rangle$
1722	1	$\langle w_3, w_7, w_{82}\rangle$	1770	1	$\langle w_2, w_3, w_{295}\rangle$	1770	1	$\langle w_2, w_5, w_{177}\rangle$
1770	1	$\langle w_3, w_5, w_{118}\rangle$	1770	1	$\langle w_6, w_{10}, w_{118}\rangle$	1785	1	$\langle w_3, w_5, w_7, w_{17}\rangle$
1794	1	$\langle w_3, w_{13}, w_{46}\rangle$	1806	1	$\langle w_2, w_3, w_7, w_{43}\rangle$	1830	1	$\langle w_2, w_3, w_5, w_{61}\rangle$
1914	1	$\langle w_2, w_{11}, w_{87}\rangle$	1914	1	$\langle w_2, w_{29}, w_{33}\rangle$	1914	1	$\langle w_3, w_{11}, w_{58}\rangle$
1914	1	$\langle w_6, w_{11}, w_{29}\rangle$	1938	1	$\langle w_2, w_3, w_{323}\rangle$	1974	1	$\langle w_2, w_7, w_{141}\rangle$
2010	1	$\langle w_2, w_5, w_{201}\rangle$	2010	1	$\langle w_3, w_5, w_{134}\rangle$	2046	1	$\langle w_2, w_3, w_{11}, w_{31}\rangle$
2090	1	$\langle w_2, w_5, w_{11}, w_{19}\rangle$	2130	1	$\langle w_6, w_{10}, w_{142}\rangle$	2170	1	$\langle w_2, w_5, w_7, w_{31}\rangle$
2190	1	$\langle w_2, w_3, w_5, w_{73}\rangle$	2210	1	$\langle w_2, w_5, w_{13}, w_{17}\rangle$	2226	1	$\langle w_2, w_3, w_7, w_{53}\rangle$
2262	1	$\langle w_2, w_3, w_{13}, w_{29}\rangle$	2370	1	$\langle w_2, w_3, w_5, w_{79}\rangle$	2415	1	$\langle w_3, w_5, w_7, w_{23}\rangle$
2442	1	$\langle w_2, w_3, w_{11}, w_{37}\rangle$	2478	1	$\langle w_2, w_3, w_7, w_{59}\rangle$	2490	1	$\langle w_2, w_3, w_5, w_{83}\rangle$
2670	1	$\langle w_2, w_3, w_5, w_{89}\rangle$	2706	1	$\langle w_2, w_3, w_{11}, w_{41}\rangle$	2838	1	$\langle w_2, w_3, w_{11}, w_{43}\rangle$
2910	1	$\langle w_2, w_3, w_5, w_{97}\rangle$	3030	1	$\langle w_2, w_3, w_5, w_{101}\rangle$	3090	1	$\langle w_2, w_3, w_5, w_{103}\rangle$

Table 6: All 48 genus 0 curves $X = X_0^D(N)/W$, with $W \leq W_0(D,N)$ nontrivial, for which we remain unsure of whether $X(\mathbb{Q}) = \emptyset$.

D	N	W	D	N	W	D	N	W
6	25	$\langle w_2, w_{75}\rangle$	6	25	$\langle w_6, w_{25}\rangle$	6	35	$\langle w_5, w_6, w_{14}\rangle$
6	35	$\langle w_6, w_7, w_{10}\rangle$	6	49	$\langle w_6, w_{49}\rangle$	6	55	$\langle w_3, w_{10}, w_{22}\rangle$
6	55	$\langle w_6, w_{10}, w_{11}\rangle$	6	77	$\langle w_2, w_{21}, w_{33}\rangle$	6	77	$\langle w_6, w_{11}, w_{14}\rangle$
10	7	$\langle w_2, w_5\rangle$	10	7	$\langle w_5, w_7\rangle$	10	7	$\langle w_{10}, w_{14}\rangle$
10	9	$\langle w_2, w_{45}\rangle$	10	9	$\langle w_5, w_{18}\rangle$	10	21	$\langle w_3, w_5, w_7\rangle$
10	21	$\langle w_3, w_{10}, w_{14}\rangle$	10	33	$\langle w_2, w_{15}, w_{33}\rangle$	10	69	$\langle w_3, w_5, w_{46}\rangle$
10	77	$\langle w_7, w_{10}, w_{11}\rangle$	14	9	$\langle w_9, w_{14}\rangle$	14	15	$\langle w_3, w_{10}, w_{14}\rangle$
14	15	$\langle w_5, w_6, w_{14}\rangle$	14	39	$\langle w_3, w_{13}, w_{14}\rangle$	15	8	$\langle w_8, w_{15}\rangle$
15	8	$\langle w_{15}, w_{24}\rangle$	15	14	$\langle w_6, w_7, w_{10}\rangle$	15	22	$\langle w_3, w_{10}, w_{22}\rangle$
15	34	$\langle w_2, w_{15}, w_{17}\rangle$	21	4	$\langle w_4, w_7\rangle$	21	4	$\langle w_7, w_{12}\rangle$

21	10	$\langle w_2, w_{15}, w_{21}\rangle$	21	22	$\langle w_6, w_7, w_{22}\rangle$	22	15	$\langle w_6, w_{10}, w_{11}\rangle$
22	21	$\langle w_3, w_{14}, w_{22}\rangle$	26	5	$\langle w_{10}, w_{26}\rangle$	26	21	$\langle w_3, w_7, w_{26}\rangle$
33	10	$\langle w_2, w_{15}, w_{33}\rangle$	34	3	$\langle w_3, w_{17}\rangle$	35	6	$\langle w_2, w_{15}, w_{21}\rangle$
39	4	$\langle w_4, w_{39}\rangle$	39	4	$\langle w_{12}, w_{39}\rangle$	51	2	$\langle w_6, w_{34}\rangle$
51	5	$\langle w_5, w_{51}\rangle$	55	2	$\langle w_2, w_{55}\rangle$	87	2	$\langle w_2, w_{87}\rangle$
95	2	$\langle w_{10}, w_{38}\rangle$	111	2	$\langle w_2, w_{111}\rangle$	390	7	$\langle w_3, w_7, w_{10}, w_{13}\rangle$

Table 7: All 54 genus 1 curves $X = X_0^D(N)/W$, with $W \leq W_0(D,N)$ nontrivial, for which we remain unsure of whether $X(\mathbb{Q}) = \emptyset$ and for which $\mathrm{Jac}(X)$ has positive rank over $\mathbb{Q}$.

D	N	W	D	N	W	D	N	W
6	155	$\langle w_2, w_5, w_{93}\rangle$	6	155	$\langle w_5, w_6, w_{31}\rangle$	6	169	$\langle w_2, w_3, w_{169}\rangle$
6	203	$\langle w_3, w_7, w_{58}\rangle$	6	287	$\langle w_2, w_3, w_7, w_{41}\rangle$	6	455	$\langle w_2, w_3, w_5, w_7, w_{13}\rangle$
10	49	$\langle w_5, w_{98}\rangle$	10	141	$\langle w_2, w_3, w_5, w_{47}\rangle$	10	159	$\langle w_2, w_3, w_5, w_{53}\rangle$
10	161	$\langle w_2, w_5, w_7, w_{23}\rangle$	10	231	$\langle w_2, w_3, w_5, w_7, w_{11}\rangle$	14	25	$\langle w_{14}, w_{25}\rangle$
14	95	$\langle w_2, w_5, w_7, w_{19}\rangle$	15	26	$\langle w_2, w_{13}, w_{15}\rangle$	15	38	$\langle w_2, w_3, w_5, w_{19}\rangle$
15	82	$\langle w_2, w_3, w_5, w_{41}\rangle$	21	55	$\langle w_3, w_5, w_7, w_{11}\rangle$	22	35	$\langle w_5, w_7, w_{22}\rangle$
22	51	$\langle w_2, w_3, w_{11}, w_{17}\rangle$	26	9	$\langle w_9, w_{26}\rangle$	33	14	$\langle w_2, w_7, w_{33}\rangle$
34	29	$\langle w_2, w_{17}, w_{29}\rangle$	35	26	$\langle w_2, w_5, w_7, w_{13}\rangle$	38	39	$\langle w_2, w_3, w_{13}, w_{19}\rangle$
39	10	$\langle w_2, w_5, w_{39}\rangle$	51	10	$\langle w_2, w_5, w_{51}\rangle$	77	6	$\langle w_2, w_3, w_7, w_{11}\rangle$
85	6	$\langle w_2, w_3, w_5, w_{17}\rangle$	95	3	$\langle w_3, w_{95}\rangle$	95	6	$\langle w_2, w_3, w_5, w_{19}\rangle$
122	7	$\langle w_2, w_7, w_{61}\rangle$	134	3	$\langle w_3, w_{134}\rangle$	159	2	$\langle w_2, w_{159}\rangle$
215	2	$\langle w_2, w_5, w_{43}\rangle$	390	7	$\langle w_2, w_3, w_7, w_{65}\rangle$	390	7	$\langle w_2, w_5, w_7, w_{39}\rangle$
390	7	$\langle w_2, w_7, w_{13}, w_{15}\rangle$	510	11	$\langle w_2, w_3, w_5, w_{11}, w_{17}\rangle$	546	5	$\langle w_2, w_3, w_5, w_{91}\rangle$
546	5	$\langle w_2, w_5, w_{13}, w_{21}\rangle$	546	5	$\langle w_3, w_5, w_7, w_{26}\rangle$	546	5	$\langle w_3, w_5, w_{13}, w_{14}\rangle$
546	5	$\langle w_5, w_6, w_{14}, w_{26}\rangle$	546	11	$\langle w_2, w_3, w_7, w_{11}, w_{13}\rangle$	570	7	$\langle w_2, w_3, w_7, w_{95}\rangle$
770	3	$\langle w_2, w_3, w_5, w_{77}\rangle$	770	3	$\langle w_2, w_3, w_7, w_{55}\rangle$	770	3	$\langle w_3, w_5, w_{11}, w_{14}\rangle$
770	3	$\langle w_3, w_7, w_{10}, w_{11}\rangle$	770	3	$\langle w_3, w_{10}, w_{14}, w_{22}\rangle$	858	5	$\langle w_2, w_3, w_5, w_{11}, w_{13}\rangle$
1155	2	$\langle w_2, w_3, w_5, w_7, w_{11}\rangle$	1190	3	$\langle w_2, w_3, w_5, w_7, w_{17}\rangle$	1430	3	$\langle w_2, w_3, w_5, w_{11}, w_{13}\rangle$

Table 8: Hyperelliptic models of the form $y^2 = f(x)$ for 150 non-elliptic genus 1 curves $X = X_0^D(N)/W$. For each X, the hyperelliptic involution with respect to the given model(s) is the image of the Fricke involution w_{DN} on X.

D	N	W	f
6	5	$\langle w_3\rangle$	$-4x^4 - 75456x^2 - 31850496$
6	5	$\langle w_5\rangle$	$-4x^4 + 145728x^2 - 1327104000$
6	7	$\langle w_2\rangle$	$-3x^4 + 36612x^2 - 90699264$
6	7	$\langle w_7\rangle$	$-3x^4 - 10530x^2 - 8680203$
6	11	$\langle w_6\rangle$	$-19x^4 + 4507902x^2 + 3972672x - 268754525019$
6	11	$\langle w_{22}\rangle$	$-19x^4 + 193230x^2 + 1477440x - 619233579$
6	11	$\langle w_{33}\rangle$	$-19x^4 + 375174x^2 + 2725056x - 2245734963$
6	13	$\langle w_6\rangle$	$-3x^4 - 161676x^2 - 2176782336$

6	17	$\langle w_2 \rangle$	$-4x^4 + 167499/2x^2 - 136323/2x - 27467644077/64$
6	19	$\langle w_3 \rangle$	$-3x^4 + 11502x^2 - 886464x - 56234331$
6	19	$\langle w_{19} \rangle$	$-19x^4 + 3189150x^2 - 55157760x - 149960715579$
6	19	$\langle w_{57} \rangle$	$-3x^4 + 10854x^2 - 10097379$
6	23	$\langle w_{46} \rangle$	$-19x^4 - 167922x^2 - 7879680x - 481536171$
6	23	$\langle w_{69} \rangle$	$-19x^4 + 465462x^2 + 92980224x + 7687562013$
6	23	$\langle w_2, w_{23} \rangle$	$-19x^4 + 943236x^2 + 81012960x + 96821568$
6	23	$\langle w_3, w_{23} \rangle$	$-19x^4 + 3428550x^2 + 606308544x + 24159368205$
6	35	$\langle w_6, w_{10} \rangle$	$-19x^4 + 2349198x^2 + 5975424x - 71124246315$
6	35	$\langle w_6, w_{14} \rangle$	$-19x^4 + 3128958x^2 - 319127040x + 4893464421$
6	35	$\langle w_7, w_{10} \rangle$	$-19x^4 - 405954x^2 - 984960x - 2264732379$
6	35	$\langle w_{21}, w_{30} \rangle$	$-19x^4 + 480510x^2 + 48066048x + 1367504613$
6	43	$\langle w_{129} \rangle$	$-3x^4 - 47466x^2 - 205667667$
6	47	$\langle w_{94} \rangle$	$-19x^4 - 471618x^2 - 124104960x - 8341147611$
6	47	$\langle w_2, w_{47} \rangle$	$-19x^4 + 3973014x^2 + 498143520x - 55289795427$
6	55	$\langle w_6, w_{11} \rangle$	$-19x^4 + 855342x^2 - 44323200x + 541295541$
6	55	$\langle w_{30}, w_{55} \rangle$	$-19x^4 + 1096110x^2 + 214327296x + 14335857333$
6	59	$\langle w_6, w_{118} \rangle$	$-43x^4 - 816570x^2 + 247655232x - 12936032667$
6	67	$\langle w_3, w_{67} \rangle$	$-67x^4 - 3118314x^2 - 35586112x - 2750077779$
6	95	$\langle w_{10}, w_{114} \rangle$	$-19x^4 + 695286x^2 + 1319500125$
6	107	$\langle w_6, w_{214} \rangle$	$-43x^4 - 1875402x^2 + 101424960x - 13594570443$
6	143	$\langle w_6, w_{13}, w_{22} \rangle$	$-43x^4 + 36279702x^2 - 18146820096x + 2553066024597$
10	3	$\langle w_6 \rangle$	$-3x^4 - 19116x^2 - 30233088$
10	7	$\langle w_2 \rangle$	$-3x^4 + 26406x^2 - 635040x - 18098883$
10	7	$\langle w_7 \rangle$	$-3x^4 + 1134x^2 - 51840x + 359397$
10	7	$\langle w_{14} \rangle$	$-3x^4 - 8586x^2 - 64800x - 10225683$
10	13	$\langle w_{10} \rangle$	$-3x^4 + 12798x^2 + 134784x - 7583787$
10	13	$\langle w_{13} \rangle$	$-3x^4 + 42606x^2 + 1036800x - 67291803$
10	21	$\langle w_3, w_5 \rangle$	$-3x^4 + 7614x^2 + 290304x + 3007125$
10	21	$\langle w_3, w_{14} \rangle$	$-3x^4 + 1134x^2 - 51840x + 359397$
10	21	$\langle w_5, w_7 \rangle$	$-3x^4 + 7614x^2 + 31000725$
10	21	$\langle w_{30}, w_{35} \rangle$	$-3x^4 + 11502x^2 - 2066715$
10	31	$\langle w_{62} \rangle$	$-3x^4 + 7614x^2 - 103680x - 7630443$
10	31	$\langle w_2, w_{31} \rangle$	$-3x^4 + 36774x^2 - 12960x - 111644163$
10	39	$\langle w_3, w_{65} \rangle$	$-3x^4 - 16362x^2 + 85293$
10	39	$\langle w_{30}, w_{65} \rangle$	$-3x^4 + 7614x^2 + 767637$
10	57	$\langle w_2, w_3, w_{19} \rangle$	$-3x^4 - 5994x^2 + 12960x - 3343923$
14	3	$\langle w_2 \rangle$	$-8x^4 - 95184x^2 + 4898880x - 181717992$
14	5	$\langle w_2 \rangle$	$-4x^4 + 5184x^2 - 5308416$
14	11	$\langle w_2, w_{11} \rangle$	$-11x^4 + 248292x^2 - 1766375424$ or $-11x^4 + 144342x^2 - 16964640x - 2075574699$
14	15	$\langle w_6, w_{14} \rangle$	$-11x^4 + 219582x^2 + 9313920x - 117862371$
14	15	$\langle w_{10}, w_{14} \rangle$	$-11x^4 + 306702x^2 + 23721984x + 429582285$
15	2	$\langle w_{10} \rangle$	$-7x^4 - 13482x^2 + 54432x - 6763743$
15	7	$\langle w_3 \rangle$	$-3x^4 - 1458x^2 + 90720x - 993627$
15	7	$\langle w_7 \rangle$	$-7x^4 + 62118x^2 - 816480x - 194145903$
15	11	$\langle w_{55} \rangle$	$-7x^4 + 62118x^2 - 816480x - 194145903$

15	11	$\langle w_5, w_{11}\rangle$	$-7x^4+284382x^2-72576x-2877983703$
15	14	$\langle w_7, w_{10}\rangle$	$-7x^4+1638x^2+453600x-17786223$
15	22	$\langle w_{10}, w_{22}\rangle$	$-7x^4+1638x^2+453600x-17786223$
21	2	$\langle w_2\rangle$	$-4x^4+576x^2-1327104$
21	5	$\langle w_{15}\rangle$	$-4x^4+12888x^2-54432x-12803508$
21	5	$\langle w_{21}\rangle$	$-4x^4+7488x^2-1327104$
21	5	$\langle w_3, w_5\rangle$	$-4x^4+86976x^2+3773952x-102518784$
21	10	$\langle w_2, w_{15}\rangle$	$-4x^4+576x^2-1327104$
22	3	$\langle w_3\rangle$	$-3x^4+3726x^2+57024x-2696571$
22	3	$\langle w_{11}\rangle$	$-11x^4-54450x^2-288178803$
22	3	$\langle w_{33}\rangle$	$-3x^4-2106x^2-649539$
22	7	$\langle w_{14}\rangle$	$-3x^4+16686x^2-228096x-37571931$
22	7	$\langle w_{77}\rangle$	$-3x^4+5670x^2+57024x-1139427$
22	7	$\langle w_2, w_7\rangle$	$-3x^4+80838x^2-28512x-541229283$
22	7	$\langle w_7, w_{11}\rangle$	$-3x^4+17334x^2-10368x-24479091$
22	15	$\langle w_{15}, w_{66}\rangle$	$-11x^4+50094x^2+53366445$
26	5	$\langle w_{26}\rangle$	$-11x^4+146718x^2-570240x-435058371$
26	15	$\langle w_5, w_6, w_{26}\rangle$	$-11x^4+888822x^2+112679424x+3994116885$
33	5	$\langle w_{55}\rangle$	$-4x^4-10944x^2-1327104$
34	1	$\langle w_2\rangle$	$-3x^4-2196x^2+568752x-15685344$
34	3	$\langle w_{17}\rangle$	$-3x^4-7938x^2-3011499$
34	7	$\langle w_{14}, w_{34}\rangle$	$-3x^4+1782x^2+176256x+4494285$
35	3	$\langle w_{35}\rangle$	$-8x^4+4032x^2-622080x-19792512$
38	3	$\langle w_{38}\rangle$	$-11x^4-21186x^2+1596672x-51942627$
39	1	$\langle w_{13}\rangle$	$-7x^4-1386x^2-435456x-8015679$
39	7	$\langle w_7, w_{13}\rangle$	$-7x^4-1386x^2-435456x-8015679$
46	3	$\langle w_2, w_3\rangle$	$-3x^4-2268x^2-2239488$ or $-3x^4-5994x^2+59616x-1384371$
55	1	$\langle w_5\rangle$	$-3x^4-2754x^2-28512x-375435$
55	3	$\langle w_3, w_5\rangle$	$-3x^4-2754x^2-28512x-375435$
62	1	$\langle w_2\rangle$	$-4x^4-7776x^2+428544x-8921664$
69	1	$\langle w_3\rangle$	$-3x^4+4212x^2-1679616$
77	1	$\langle w_{11}\rangle$	$-11x^4-82764x^2-303595776$
85	1	$\langle w_{17}\rangle$	$-3x^4+3726x^2-12960x-807003$
94	1	$\langle w_2\rangle$	$-4x^4+5184x^2-2654208$
178	1	$\langle w_{89}\rangle$	$-3x^4-5994x^2-82944x-5233491$
210	1	$\langle w_{30}\rangle$	$-43x^4+79146918x^2+955735200x-37340341770267$
210	1	$\langle w_{42}\rangle$	$-43x^4+35836974x^2+15291763200x+1375667271117$
210	1	$\langle w_{70}\rangle$	$-43x^4+7347582x^2-44582400x-328155195267$
210	1	$\langle w_{105}\rangle$	$-43x^4+33617142x^2-407780352x-6857905126347$
210	1	$\langle w_2, w_{15}\rangle$	$-43x^4+12273318x^2+4388580000x-2819921955867$
210	1	$\langle w_2, w_{21}\rangle$	$-43x^4+31543596x^2+17867790000x+2228750746272$
210	1	$\langle w_2, w_{35}\rangle$	$-43x^4+37401228x^2-18076770000x+2307023620128$
210	1	$\langle w_3, w_{14}\rangle$	$-43x^4+82407393/2x^2-11610191565x+10101157035777/16$
210	1	$\langle w_3, w_{35}\rangle$	$-43x^4+205352262x^2-236028364992x+73335354773733$
210	1	$\langle w_5, w_6\rangle$	$-43x^4+319576086x^2+5981472x-593760997118475$
210	1	$\langle w_5, w_{14}\rangle$	$-43x^4+59775633/2x^2-498069x-83091244509855/16$

210	1	$\langle w_5, w_{21}\rangle$	$-43x^4+136658214x^2-4383936x-108572735488635$
210	1	$\langle w_6, w_7\rangle$	$-43x^4+14325966x^2+1224529920x-707282085843$
210	1	$\langle w_7, w_{10}\rangle$	$-43x^4-1383138x^2-406591488x-25152585507$
330	1	$\langle w_3\rangle$	$-3x^4-20898x^2-1425600x-23563467$
330	1	$\langle w_{22}\rangle$	$-3x^4+109998x^2-1162261467$
330	1	$\langle w_{33}\rangle$	$-3x^4+230526x^2-15625959723$
330	1	$\langle w_{165}\rangle$	$-3x^4+49734x^2-129140163$
330	1	$\langle w_2, w_3\rangle$	$-3x^4+171558x^2+17820000x+273781053$
330	1	$\langle w_2, w_{11}\rangle$	$-3x^4+464292x^2-17958454272$
330	1	$\langle w_3, w_5\rangle$	$-3x^4-9234x^2+114048x-8131995$
330	1	$\langle w_{10}, w_{22}\rangle$	$-3x^4-61074x^2-39858075$
330	1	$\langle w_{10}, w_{66}\rangle$	$-3x^4-68364x^2+272097792$
330	1	$\langle w_{15}, w_{33}\rangle$	$-3x^4-267138x^2-3653656875$
390	1	$\langle w_6, w_{13}\rangle$	$-67x^4+29210526x^2+10984826880x+1145424134133$
390	1	$\langle w_6, w_{130}\rangle$	$-67x^4+8304918x^2-1158103040x+3897509997$
390	1	$\langle w_{10}, w_{13}\rangle$	$-67x^4+10464462x^2+1055877120x-744755073723$
390	1	$\langle w_{10}, w_{78}\rangle$	$-67x^4+106418646x^2+48162816000x+5653007192493$
390	1	$\langle w_{13}, w_{15}\rangle$	$-67x^4+331863462x^2+329581854720x+53757938405277$
390	1	$\langle w_{15}, w_{78}\rangle$	$-67x^4+7888446x^2+1528243200x+83601627093$
462	1	$\langle w_{154}\rangle$	$-4x^4-163008x^2-1358954496$
462	1	$\langle w_2, w_{11}\rangle$	$-4x^4-96192x^2-642318336$
462	1	$\langle w_{21}, w_{66}\rangle$	$-4x^4+279360x^2+1146617856$
510	1	$\langle w_3, w_{10}\rangle$	$-3x^4+69822x^2+793152x-322978347$
510	1	$\langle w_3, w_{34}\rangle$	$-3x^4+14094x^2-440640x+3275397$
510	1	$\langle w_3, w_{85}\rangle$	$-3x^4-60426x^2+295245$
510	1	$\langle w_{10}, w_{34}\rangle$	$-3x^4-189378x^2-14348907$
510	1	$\langle w_{10}, w_{102}\rangle$	$-3x^4+119718x^2+23914845$
510	1	$\langle w_{30}, w_{85}\rangle$	$-3x^4+5022x^2+2657205$
570	1	$\langle w_5, w_{57}\rangle$	$-43x^4-8420346x^2+220862097765$ or $-43x^4-395514x^2-2428849152x+83757843045$
570	1	$\langle w_6, w_{190}\rangle$	$-43x^4-4959018x^2+15295159125$ or $-43x^4+3065814x^2+588487680x-14147949483$
570	1	$\langle w_{30}, w_{38}\rangle$	$-43x^4+148472550x^2+115268686848x+10471935300741$
570	1	$\langle w_{30}, w_{57}\rangle$	$-43x^4-955890x^2+246094848x-19258179219$
690	1	$\langle w_3, w_{46}\rangle$	$-3x^4-7938x^2+20503125$
690	1	$\langle w_3, w_{115}\rangle$	$-3x^4+8262x^2-119232x-11053827$
690	1	$\langle w_{10}, w_{138}\rangle$	$-3x^4-4698x^2+23914845$
690	1	$\langle w_{30}, w_{115}\rangle$	$-3x^4+335502x^2-8968066875$
690	1	$\langle w_3, w_5, w_{23}\rangle$	$-3x^4+43254x^2+20736x-154136115$
770	1	$\langle w_5, w_{14}, w_{22}\rangle$	$-67x^4+68897574x^2-21580993728x-12944847617955$
798	1	$\langle w_2, w_{57}\rangle$	$-4x^4+14400x^2-21233664$
798	1	$\langle w_{42}, w_{114}\rangle$	$-4x^4-47808x^2-84934656$
858	1	$\langle w_6, w_{286}\rangle$	$-67x^4+11549862x^2-362957760x-324467310627$
858	1	$\langle w_{33}, w_{78}\rangle$	$-67x^4-22879026x^2+7748193024x-1511524007739$
870	1	$\langle w_3, w_{145}\rangle$	$-3x^4+66582x^2+3006720x-71764947$
870	1	$\langle w_{10}, w_{58}\rangle$	$-3x^4-33858x^2-14348907$
910	1	$\langle w_{65}, w_{70}\rangle$	$-8x^4+877824x^2-10871635968$

1110	1	$\langle w_{15}, w_{222}\rangle$	$-43x^4 + 20610846x^2 + 6687360000x + 350920466397$
1302	1	$\langle w_2, w_{21}, w_{93}\rangle$	$-4x^4 + 272448x^2 - 1358954496$
1410	1	$\langle w_3, w_{10}, w_{94}\rangle$	$-3x^4 - 17658x^2 + 184528125$
1590	1	$\langle w_3, w_{10}, w_{106}\rangle$	$-3x^4 + 39366x^2 + 1373760x + 6862077$
2010	1	$\langle w_6, w_{10}, w_{67}\rangle$	$-67x^4 - 2256426x^2 - 227499840x + 203533334253$

Table 9: Equations for 405 genus 2 curves $X = X_0^D(N)/W$ which are bielliptic via an Atkin-Lehner involution. For each X, we give a single divisor m_1 of DN so that $X/\langle w_{m_1}\rangle$ is a bielliptic quotient of X if X has a single bielliptic Atkin–Lehner quotient, and we similarly give divisors m_1 and m_2 if X has two bielliptic Atkin–Lehner quotients. Our hyperelliptic model for X is $y^2 = f(x)$ for the given polynomial f.

D	N	W	m_1	m_2	f
6	11	$\langle w_2\rangle$	w_3	w_{11}	$-11979/2x^6 - 10917/2x^4 + 1035/2x^2 - 27/2$
6	11	$\langle w_3\rangle$	w_2	w_{11}	$3267x^6 - 7092x^4 + 1143x^2 - 54$
6	11	$\langle w_{11}\rangle$	w_2	w_3	$3267x^6 - 19044x^4 + 36999x^2 - 23958$
6	17	$\langle w_{17}\rangle$	w_3	w_6	$-235824x^6 - 42291x^4 - 2538x^2 - 51$
6	19	$\langle w_2\rangle$	w_3	w_{19}	$-171x^6 + 522x^4 + 2637x^2 - 8748$
6	19	$\langle w_6\rangle$	w_2	w_{38}	$9747/4x^6 - 18945/4x^4 + 11385/4x^2 - 2187/4$
6	19	$\langle w_{38}\rangle$	w_2	w_6	$38988x^6 - 20043x^4 + 3258x^2 - 171$
6	29	$\langle w_2\rangle$	w_3	w_{29}	$-219501/8x^6 - 21825/8x^4 + 1737/8x^2 - 27/8$
6	29	$\langle w_6\rangle$	w_2	w_{58}	$24389x^6 - 11874x^4 + 1581x^2 - 64$
6	29	$\langle w_{58}\rangle$	w_2	w_6	$-27x^6 + 126x^4 + 45x^2 - 576$
6	31	$\langle w_3\rangle$	w_2	w_{31}	$-2883x^6 - 1386x^4 + 405x^2 - 24$
6	31	$\langle w_6\rangle$	w_2	w_{62}	$-8649/16x^6 - 30825/16x^4 + 16893/16x^2 - 2187/16$
6	31	$\langle w_{93}\rangle$	w_3	w_6	$72x^6 - 747x^4 + 2286x^2 - 2187$
6	35	$\langle w_5, w_6\rangle$	w_2	w_7	$-6125/2x^6 - 5451/2x^4 + 669/2x^2 - 5/2$
6	35	$\langle w_{15}, w_{21}\rangle$	w_2	w_3	$-245x^6 + 1164x^4 - 1473x^2 + 250$
6	37	$\langle w_2\rangle$	w_3	w_{37}	$-333/16x^6 - 12465/16x^4 - 8055/16x^2 - 2187/16$
6	37	$\langle w_3\rangle$	w_2	w_{37}	$37x^6 - 786x^4 + 381x^2 - 64$
6	37	$\langle w_{37}\rangle$	w_2	w_3	$-2187x^6 - 4194x^4 - 2691x^2 - 576$
6	41	$\langle w_{82}\rangle$	w_2	w_6	$-243x^6 - 738x^4 + 693x^2 - 144$
6	43	$\langle w_{258}\rangle$	w_3	w_6	$81x^6 + 306x^4 + 153x^2 + 36$
6	53	$\langle w_{106}\rangle$	w_2	w_6	$-27x^6 - 306x^4 - 1251x^2 - 2304$
6	53	$\langle w_{318}\rangle$	w_2	w_6	$288x^6 + 585x^4 + 342x^2 + 81$
6	55	$\langle w_6, w_{10}\rangle$	w_2	w_{22}	$-121/3x^6 - 6250x^4 + 581x^2 - 40/3$
6	55	$\langle w_6, w_{110}\rangle$	w_2	w_5	$-24x^6 + 585x^4 - 2322x^2 - 15$
6	55	$\langle w_{15}, w_{66}\rangle$	w_2	w_3	$14520x^6 - 5319x^4 + 558x^2 - 15$
6	55	$\langle w_{22}, w_{30}\rangle$	w_2	w_5	$9/8x^6 + 9/8x^4 - 18621/8x^2 - 405/8$
6	59	$\langle w_{354}\rangle$	w_2	w_6	$18x^6 - 333x^4 + 1692x^2 - 81$
6	61	$\langle w_{61}\rangle$	w_2	w_3	$-2187x^6 - 2898x^4 + 2925x^2 - 576$
6	61	$\langle w_{366}\rangle$	w_2	w_3	$144x^6 + 153x^4 - 234x^2 + 81$
6	65	$\langle w_2, w_{15}\rangle$	w_3	w_{39}	$1625x^6 - 1002x^4 + 249x^2 - 40$
6	65	$\langle w_3, w_{10}\rangle$	w_2	w_{26}	$117x^6 - 1314x^4 + 5517x^2 - 11664$

6	65	$\langle w_6, w_{130}\rangle$	w_5	w_{10}	$-648x^6 - 819x^4 - 306x^2 + 45$
6	65	$\langle w_{10}, w_{78}\rangle$	w_3	w_6	$-91125/8x^6 + 2583/8x^4 - 3663/8x^2 + 45/8$
6	67	$\langle w_{402}\rangle$	w_3	w_6	$729x^6 + 1170x^4 + 369x^2 + 36$
6	77	$\langle w_2, w_{33}\rangle$	w_3	w_7	$-27951x^6 - 8298x^4 - 783x^2 - 24$
6	77	$\langle w_6, w_{11}\rangle$	w_2	w_7	$83853/16x^6 - 48915/16x^4 + 6255/16x^2 - 297/16$
6	77	$\langle w_{33}, w_{42}\rangle$	w_3	w_6	$-3x^6 + 54x^4 + 2061x^2 + 10752$
6	79	$\langle w_{474}\rangle$	w_3	w_6	$-27x^6 + 198x^4 - 171x^2 + 576$
6	85	$\langle w_{30}, w_{34}\rangle$	w_3	w_5	$153x^6 - 774x^4 - 4815x^2 - 5508$
6	91	$\langle w_3, w_{91}\rangle$	w_7	w_{14}	$-18x^6 - 261x^4 - 720x^2 - 729$
6	95	$\langle w_2, w_{285}\rangle$	w_3	w_{15}	$9x^6 - 18x^4 - 1143x^2 + 1152$
6	95	$\langle w_5, w_{114}\rangle$	w_3	w_6	$9x^6 - 126x^4 + 801x^2 - 684$
6	101	$\langle w_3, w_{202}\rangle$	w_2		$648x^6 + 657x^4 - 18x^2 + 9$
6	103	$\langle w_2, w_{309}\rangle$	w_3		$1944x^6 + 441x^4 - 90x^2 + 9$
6	109	$\langle w_{654}\rangle$	w_2	w_3	$576x^6 - 1143x^4 + 630x^2 + 81$
6	115	$\langle w_{10}, w_{69}\rangle$	w_2	w_3	$90x^6 - 333x^4 + 468x^2 - 81$
6	127	$\langle w_2, w_{381}\rangle$	w_3		$648x^6 + 1305x^4 + 342x^2 + 9$
6	149	$\langle w_3, w_{298}\rangle$	w_2		$1296x^6 + 225x^4 - 234x^2 + 9$
6	151	$\langle w_2, w_{453}\rangle$	w_3		$432x^6 + 441x^4 - 306x^2 + 9$
6	163	$\langle w_2, w_{489}\rangle$	w_3		$-243x^6 + 90x^4 + 693x^2 + 36$
6	181	$\langle w_6, w_{181}\rangle$	w_2		$648x^6 + 225x^4 - 738x^2 + 9$
6	185	$\langle w_2, w_3, w_{185}\rangle$	w_5		$3240x^6 + 1737x^4 + 198x^2 + 9$
6	185	$\langle w_3, w_{10}, w_{37}\rangle$	w_2		$-1215x^6 + 522x^4 + 225x^2 + 36$
6	217	$\langle w_2, w_3, w_{217}\rangle$	w_7		$3024x^6 + 2169x^4 - 18x^2 + 9$
6	247	$\langle w_2, w_{13}, w_{57}\rangle$	w_3		$-243x^6 + 954x^4 - 171x^2 + 36$
6	265	$\langle w_6, w_{10}, w_{106}\rangle$	w_2		$405x^6 + 1278x^4 + 45x^2 + 144$
6	295	$\langle w_2, w_5, w_{177}\rangle$	w_3		$9720x^6 - 423x^4 - 90x^2 + 9$
10	11	$\langle w_2\rangle$	w_5	w_{11}	$-99/8x^6 - 38439/8x^4 + 1719/8x^2 - 45/8$
10	11	$\langle w_{10}\rangle$	w_2	w_{22}	$-99x^6 - 4842x^4 - 1179x^2 - 72$
10	11	$\langle w_{22}\rangle$	w_2	w_{10}	$45x^6 + 198x^4 + 549x^2 - 72$
10	13	$\langle w_2\rangle$	w_5	w_{13}	$-117x^6 + 432x^4 + 207x^2 - 2250$
10	13	$\langle w_5\rangle$	w_2	w_{26}	$-2925/2x^6 - 6507/2x^4 - 4707/2x^2 - 1125/2$
10	13	$\langle w_{26}\rangle$	w_2	w_5	$-5850x^6 - 5193x^4 - 1368x^2 - 117$
10	17	$\langle w_{17}\rangle$	w_2	w_5	$-5625x^6 - 4392x^4 + 387x^2 - 18$
10	19	$\langle w_2\rangle$	w_5	w_{95}	$-171/5x^6 - 16506/5x^4 - 4347/5x^2 - 288/5$
10	19	$\langle w_{10}\rangle$	w_2	w_{19}	$-171/8x^6 - 26307/8x^4 - 585/8x^2 - 9/8$
10	19	$\langle w_{190}\rangle$	w_2	w_{10}	$-288x^6 + 513x^4 - 90x^2 + 9$
10	21	$\langle w_{14}, w_{30}\rangle$	w_2	w_5	$-9x^6 - 216x^4 + 819x^2 + 2430$
10	33	$\langle w_2, w_{165}\rangle$	w_5	w_{11}	$450x^6 + 99x^4 + 36x^2 - 9$
10	37	$\langle w_{185}\rangle$	w_5	w_{10}	$-18x^6 - 117x^4 - 1152x^2 - 5625$
10	43	$\langle w_2, w_{215}\rangle$	w_5		$-1800x^6 + 2601x^4 - 234x^2 + 9$
10	43	$\langle w_5, w_{86}\rangle$	w_2		$1152x^6 + 2385x^4 + 54x^2 + 9$
10	43	$\langle w_{10}, w_{43}\rangle$	w_2		$14400x^6 - 2799x^4 + 54x^2 + 9$
10	47	$\langle w_2, w_{235}\rangle$	w_5		$1440x^6 - 999x^4 + 126x^2 + 9$
10	47	$\langle w_5, w_{94}\rangle$	w_2		$576x^6 + 1593x^4 + 1206x^2 + 225$
10	47	$\langle w_{10}, w_{47}\rangle$	w_2		$-360x^6 + 81x^4 + 414x^2 + 9$
10	51	$\langle w_3, w_{170}\rangle$	w_2	w_5	$-432x^6 + 369x^4 + 198x^2 + 9$
10	51	$\langle w_{10}, w_{17}\rangle$	w_2	w_6	$-459x^6 + 198x^4 - 99x^2 - 72$

10	51	$\langle w_{17}, w_{30}\rangle$	w_2	w_5	$4608x^6 + 1089x^4 + 198x^2 + 9$
10	57	$\langle w_2, w_{285}\rangle$	w_3	w_5	$-144x^6 + 81x^4 + 414x^2 + 225$
10	57	$\langle w_{30}, w_{38}\rangle$	w_3	w_5	$432x^6 + 45x^4 - 234x^2 - 243$
10	59	$\langle w_5, w_{118}\rangle$	w_2		$360x^6 + 1089x^4 - 162x^2 + 9$
10	69	$\langle w_{15}, w_{138}\rangle$	w_2	w_6	$-1035x^6 + 1062x^4 - 531x^2 + 72$
10	71	$\langle w_2, w_{355}\rangle$	w_5		$9x^6 + 558x^4 + 1449x^2 + 288$
10	71	$\langle w_5, w_{142}\rangle$	w_2		$36x^6 + 477x^4 - 414x^2 + 45$
10	73	$\langle w_2, w_{365}\rangle$	w_5		$2250x^6 - 2061x^4 + 396x^2 - 9$
10	77	$\langle w_{10}, w_{77}\rangle$	w_2	w_{14}	$45x^6 - 306x^4 + 981x^2 - 2304$
10	111	$\langle w_3, w_{10}, w_{74}\rangle$	w_2		$27x^6 + 828x^4 + 2223x^2 + 2250$
10	111	$\langle w_5, w_6, w_{37}\rangle$	w_2		$-135x^6 + 666x^4 + 153x^2 + 36$
10	119	$\langle w_2, w_{17}, w_{35}\rangle$	w_5		$-63x^6 + 1854x^4 + 225x^2 + 288$
10	119	$\langle w_5, w_{14}, w_{17}\rangle$	w_2		$1008x^6 + 1665x^4 - 234x^2 + 9$
10	133	$\langle w_2, w_{19}, w_{35}\rangle$	w_5		$1125x^6 + 954x^4 + 189x^2 + 36$
14	3	$\langle w_3\rangle$	w_2	w_7	$567x^6 - 1656x^4 - 477x^2 - 18$
14	3	$\langle w_6\rangle$	w_2	w_7	$27783/2x^6 + 5193/2x^4 + 153/2x^2 - 9/2$
14	3	$\langle w_7\rangle$	w_3	w_6	$686x^6 + 795x^4 + 240x^2 + 7$
14	5	$\langle w_5\rangle$	w_2	w_7	$-315x^6 + 1998x^4 - 3123x^2 - 144$
14	5	$\langle w_7\rangle$	w_5	w_{10}	$-1008/5x^6 - 16479/5x^4 - 2034/5x^2 - 63/5$
14	5	$\langle w_{10}\rangle$	w_2	w_7	$-315/16x^6 - 16227/16x^4 + 3303/16x^2 - 9/16$
14	11	$\langle w_{77}\rangle$	w_7	w_{14}	$1152x^6 - 1827x^4 + 774x^2 - 99$
14	13	$\langle w_{14}\rangle$	w_2	w_{26}	$117x^6 + 846x^4 - 99x^2 - 7056$
14	15	$\langle w_2, w_{105}\rangle$	w_3	w_5	$108x^6 + 261x^4 + 162x^2 + 45$
14	15	$\langle w_3, w_{35}\rangle$	w_2	w_5	$-27x^6 - 306x^4 - 963x^2 - 288$
14	15	$\langle w_5, w_{14}\rangle$	w_2	w_3	$-1620/7x^6 + 7731/7x^4 + 1854/7x^2 - 45/7$
14	15	$\langle w_5, w_{21}\rangle$	w_2	w_3	$-45/7x^6 + 594/7x^4 - 5877/7x^2 + 1152/7$
14	15	$\langle w_{15}, w_{21}\rangle$	w_2	w_3	$-405x^6 - 396x^4 - 801x^2 + 18$
14	15	$\langle w_{21}, w_{30}\rangle$	w_2	w_3	$51840/7x^6 - 6417/7x^4 + 522/7x^2 - 9/7$
14	17	$\langle w_2, w_{17}\rangle$	w_7		$-1224/7x^6 + 11511/7x^4 + 234/7x^2 - 153/7$
14	17	$\langle w_{14}, w_{17}\rangle$	w_2		$-504x^6 + 1521x^4 + 270x^2 + 9$
14	31	$\langle w_7, w_{62}\rangle$	w_2		$-558x^6 + 1251x^4 - 612x^2 + 63$
14	31	$\langle w_{14}, w_{31}\rangle$	w_2		$882x^6 - 909x^4 + 108x^2 + 63$
14	33	$\langle w_2, w_{21}, w_{33}\rangle$	w_3		$-3168x^6 + 6849x^4 - 4770x^2 + 1089$
14	33	$\langle w_6, w_{14}, w_{22}\rangle$	w_2		$-6048x^6 - 1791x^4 - 90x^2 + 9$
14	51	$\langle w_2, w_{17}, w_{21}\rangle$	w_3		$-576x^6 + 873x^4 - 306x^2 + 9$
14	55	$\langle w_5, w_7, w_{22}\rangle$	w_2		$-3960x^6 + 657x^4 - 18x^2 + 9$
14	61	$\langle w_2, w_{427}\rangle$	w_7		$3087x^6 - 684x^4 - 117x^2 + 18$
14	61	$\langle w_{14}, w_{61}\rangle$	w_2		$441x^6 + 1962x^4 + 1737x^2 + 36$
14	65	$\langle w_{10}, w_{14}, w_{26}\rangle$	w_2		$360x^6 - 2223x^4 + 3870x^2 + 441$
14	69	$\langle w_6, w_{14}, w_{46}\rangle$	w_2		$864x^6 + 153x^4 + 198x^2 + 81$
15	2	$\langle w_2\rangle$	w_3	w_5	$-360x^6 - 963x^4 + 342x^2 - 27$
15	14	$\langle w_3, w_7\rangle$	w_2	w_5	$-45x^6 - 1368x^4 - 4257x^2 - 3402$
15	14	$\langle w_3, w_{70}\rangle$	w_2	w_5	$-162x^6 - 45x^4 + 144x^2 + 63$
15	14	$\langle w_6, w_7\rangle$	w_2	w_5	$-360x^6 - 963x^4 + 342x^2 - 27$
15	14	$\langle w_7, w_{15}\rangle$	w_2	w_3	$9x^6 + 126x^4 - 2799x^2 + 9720$
15	14	$\langle w_7, w_{30}\rangle$	w_2	w_5	$126x^6 + 531x^4 + 504x^2 + 135$
15	22	$\langle w_3, w_{55}\rangle$	w_2	w_5	$27x^6 + 360x^4 + 711x^2 + 198$

15	22	$\langle w_6, w_{55}\rangle$	w_2	w_5	$-54x^6+279x^4-36x^2-45$
15	22	$\langle w_{15}, w_{33}\rangle$	w_2	w_3	$1215x^6-2628x^4+387x^2+18$
15	22	$\langle w_{30}, w_{55}\rangle$	w_2	w_5	$-5346x^6-3357x^4-360x^2-9$
15	23	$\langle w_3, w_{115}\rangle$	w_5		$1215x^6+171x^4-99x^2+9$
15	23	$\langle w_{15}, w_{69}\rangle$	w_3		$-243x^6-1287x^4-1737x^2-45$
15	34	$\langle w_2, w_{255}\rangle$	w_3	w_5	$-54x^6+279x^4-36x^2-45$
15	34	$\langle w_{15}, w_{17}\rangle$	w_3	w_6	$450x^6-1989x^4+1044x^2-81$
21	2	$\langle w_3\rangle$	w_2	w_{14}	$189/4x^6-1611/4x^4-801/4x^2-81/4$
21	2	$\langle w_{14}\rangle$	w_3	w_6	$21x^6+234x^4+621x^2+84$
21	10	$\langle w_2, w_{105}\rangle$	w_3	w_5	$27x^6-180x^4+207x^2+90$
21	10	$\langle w_7, w_{15}\rangle$	w_3	w_6	$162x^6+1035x^4-684x^2+63$
21	10	$\langle w_{14}, w_{15}\rangle$	w_3	w_6	$-9x^6+36x^4+531x^2-1134$
21	10	$\langle w_{15}, w_{21}\rangle$	w_3	w_6	$180x^6+1413x^4+1098x^2+189$
21	10	$\langle w_{15}, w_{42}\rangle$	w_3	w_6	$-27x^6-342x^4-1251x^2-1260$
21	10	$\langle w_{21}, w_{30}\rangle$	w_3	w_5	$45x^6+306x^4+477x^2-108$
21	17	$\langle w_7, w_{51}\rangle$	w_3		$-243x^6+117x^4+207x^2+63$
21	17	$\langle w_{21}, w_{51}\rangle$	w_3		$-9x^6+495x^4-2979x^2+15309$
21	22	$\langle w_6, w_{154}\rangle$	w_2	w_{11}	$-9x^6+36x^4+531x^2-1134$
21	22	$\langle w_7, w_{33}\rangle$	w_2	w_3	$63x^6+396x^4-1773x^2+1458$
21	38	$\langle w_2, w_7, w_{57}\rangle$	w_3		$756x^6-423x^4-198x^2+9$
22	3	$\langle w_2\rangle$	w_3	w_{11}	$-99x^6-630x^4-1179x^2-972$
22	3	$\langle w_6\rangle$	w_2	w_{22}	$891/4x^6-441/4x^4-207/4x^2-243/4$
22	3	$\langle w_{22}\rangle$	w_2	w_6	$3564x^6-3123x^4+954x^2-99$
22	5	$\langle w_2\rangle$	w_5	w_{11}	$-99x^6-126x^4-171x^2-180$
22	5	$\langle w_5\rangle$	w_2	w_{11}	$-99x^6-1314x^4-5931x^2-9216$
22	5	$\langle w_{11}\rangle$	w_2	w_5	$45x^6-306x^4+981x^2-2304$
22	13	$\langle w_{26}\rangle$	w_2	w_{22}	$-99x^6-306x^4+549x^2-576$
22	13	$\langle w_{286}\rangle$	w_2	w_{13}	$288x^6-495x^4+342x^2+9$
22	15	$\langle w_3, w_{110}\rangle$	w_2	w_{10}	$9x^6+18x^4-279x^2-180$
22	15	$\langle w_5, w_{66}\rangle$	w_2	w_6	$9x^6-126x^4+441x^2+396$
22	21	$\langle w_{42}, w_{66}\rangle$	w_2	w_6	$-486x^6-261x^4-252x^2-9$
22	23	$\langle w_2, w_{253}\rangle$	w_{11}		$288x^6+441x^4-162x^2+9$
22	23	$\langle w_{11}, w_{46}\rangle$	w_2		$144x^6-855x^4+918x^2+1089$
22	23	$\langle w_{22}, w_{23}\rangle$	w_2		$-72x^6+657x^4-450x^2+9$
22	31	$\langle w_2, w_{341}\rangle$	w_{11}		$3168x^6+1593x^4+414x^2+9$
22	39	$\langle w_2, w_{13}, w_{33}\rangle$	w_3		$216x^6+1233x^4+2070x^2+1089$
22	39	$\langle w_3, w_{22}, w_{26}\rangle$	w_2		$-27x^6+342x^4+765x^2+792$
26	3	$\langle w_2\rangle$	w_3	w_{39}	$-6591x^6-3258x^4-495x^2-24$
26	3	$\langle w_6\rangle$	w_2	w_{13}	$19773/8x^6-6291/8x^4-225/8x^2-9/8$
26	3	$\langle w_{78}\rangle$	w_2	w_6	$72x^6+81x^4-18x^2+9$
26	5	$\langle w_{65}\rangle$	w_2	w_5	$-9x^6-216x^4-1341x^2-18$
26	7	$\langle w_{26}\rangle$	w_2	w_{14}	$-63x^6-990x^4-1215x^2-468$
26	7	$\langle w_{182}\rangle$	w_2	w_{13}	$-36x^6-27x^4+90x^2+117$
26	11	$\langle w_2, w_{143}\rangle$	w_{11}		$792x^6-279x^4+54x^2+9$
26	11	$\langle w_{11}, w_{26}\rangle$	w_2		$288x^6-495x^4+342x^2+9$
26	15	$\langle w_6, w_{65}\rangle$	w_2	w_5	$-36x^6-27x^4+90x^2+117$
26	19	$\langle w_2, w_{247}\rangle$	w_{13}		$-17784x^6+4905x^4-378x^2+9$

26	19	$\langle w_{13}, w_{38}\rangle$	w_2		$-5472x^6+801x^4+198x^2+9$
26	19	$\langle w_{19}, w_{26}\rangle$	w_2		$-1872x^6+2097x^4-90x^2+9$
26	21	$\langle w_{21}, w_{26}\rangle$	w_2	w_6	$18x^6-45x^4+540x^2-1521$
33	10	$\langle w_{15}, w_{33}\rangle$	w_3	w_6	$-972x^6-1179x^4-630x^2-99$
33	10	$\langle w_{30}, w_{55}\rangle$	w_3	w_5	$180x^6+549x^4+18x^2-27$
34	5	$\langle w_{170}\rangle$	w_2	w_{10}	$-18x^6+63x^4+144x^2-45$
35	2	$\langle w_7\rangle$	w_2	w_{10}	$5625/2x^6+261/2x^4-117/2x^2-9/2$
35	6	$\langle w_{15}, w_{21}\rangle$	w_3	w_6	$-378x^6-873x^4-432x^2-45$
35	6	$\langle w_{30}, w_{35}\rangle$	w_3	w_5	$-90x^6-297x^4-72x^2+27$
38	3	$\langle w_{114}\rangle$	w_2	w_6	$18x^6-117x^4+252x^2-9$
38	5	$\langle w_2, w_5\rangle$	w_{19}		$1440x^6+5697x^4+7470x^2+3249$
38	5	$\langle w_2, w_{19}\rangle$	w_5		$-144x^6+81x^4+414x^2+225$
38	5	$\langle w_2, w_{95}\rangle$	w_5		$360x^6+297x^4-90x^2+9$
38	5	$\langle w_5, w_{38}\rangle$	w_2		$-288x^6+513x^4-90x^2+9$
38	7	$\langle w_{266}\rangle$	w_2	w_{14}	$9x^6+18x^4+9x^2+684$
38	11	$\langle w_2, w_{209}\rangle$	w_{11}		$1584x^6+585x^4+126x^2+9$
38	15	$\langle w_2, w_5, w_{57}\rangle$	w_3		$360x^6+297x^4-90x^2+9$
38	15	$\langle w_5, w_6, w_{19}\rangle$	w_2		$-90x^6+567x^4-504x^2+171$
39	14	$\langle w_6, w_7, w_{13}\rangle$	w_2		$-2268x^6-639x^4+18x^2+9$
46	3	$\langle w_{138}\rangle$	w_3	w_6	$-27x^6-90x^4-27x^2+144$
46	7	$\langle w_7, w_{23}\rangle$	w_2		$-1449/2x^6-2655/2x^4-999/2x^2+207/2$
46	7	$\langle w_{14}, w_{46}\rangle$	w_2		$-9x^6+792x^4+1827x^2+414$
46	11	$\langle w_2, w_{253}\rangle$	w_{11}		$9x^6-162x^4+441x^2+288$
46	11	$\langle w_{11}, w_{46}\rangle$	w_2		$9x^6-54x^4-423x^2+36$
46	15	$\langle w_3, w_{10}, w_{23}\rangle$	w_2		$-135x^6-198x^4-135x^2+36$
51	2	$\langle w_{51}\rangle$	w_2	w_3	$9x^6+126x^4+225x^2-216$
55	2	$\langle w_{55}\rangle$	w_5	w_{10}	$450x^6+99x^4+36x^2-9$
55	6	$\langle w_3, w_5, w_{22}\rangle$	w_2		$-540x^6+369x^4+162x^2+9$
58	3	$\langle w_{29}\rangle$	w_2	w_3	$-243x^6-666x^4-603x^2-72$
58	3	$\langle w_{174}\rangle$	w_2	w_3	$144x^6-63x^4+54x^2+9$
58	7	$\langle w_2, w_{203}\rangle$	w_7		$63x^6+612x^4-117x^2+18$
58	7	$\langle w_{14}, w_{29}\rangle$	w_2		$-36x^6+693x^4-774x^2+261$
62	3	$\langle w_{62}\rangle$	w_2	w_6	$-81x^6-72x^4-261x^2-18$
69	5	$\langle w_{15}, w_{69}\rangle$	w_3		$-9x^6+63x^4+45x^2+621$
74	3	$\langle w_{74}\rangle$	w_2	w_6	$-27x^6-18x^4-99x^2-288$
82	7	$\langle w_{14}, w_{41}\rangle$	w_2		$72x^6-279x^4-18x^2+369$
85	2	$\langle w_{10}, w_{17}\rangle$	w_2		$-36x^6+225x^4-198x^2+9$
86	5	$\langle w_5, w_{86}\rangle$	w_2		$72x^6-207x^4+270x^2+9$
87	2	$\langle w_{87}\rangle$	w_3	w_6	$-648x^6-423x^4-90x^2+9$
87	4	$\langle w_4, w_{87}\rangle$	w_3		$9x^6-90x^4-423x^2-648$
91	1	$\langle w_{91}\rangle$	w_7		$-9x^6+171x^4-27x^2+9$
91	2	$\langle w_2, w_{91}\rangle$	w_7		$819x^6+387x^4+81x^2+9$
93	2	$\langle w_{31}\rangle$	w_2	w_3	$-243x^6-630x^4-99x^2-36$
94	5	$\langle w_5, w_{94}\rangle$	w_2		$9x^6+234x^4+441x^2+36$
94	7	$\langle w_{14}, w_{94}\rangle$	w_2		$-72x^6-63x^4-450x^2-423$
95	2	$\langle w_{95}\rangle$	w_2	w_5	$-9x^6-36x^4-261x^2+450$
95	4	$\langle w_4, w_{95}\rangle$	w_5		$-9x^6-36x^4-261x^2+450$

106	3	$\langle w_6, w_{53}\rangle$	w_2		$-216x^6 - 495x^4 - 306x^2 + 9$
106	5	$\langle w_5, w_{106}\rangle$	w_2		$90x^6 - 117x^4 + 180x^2 - 9$
111	2	$\langle w_{111}\rangle$	w_3	w_6	$18x^6 - 45x^4 + 180x^2 - 729$
111	4	$\langle w_4, w_{111}\rangle$	w_3		$-729x^6 + 180x^4 - 45x^2 + 18$
118	3	$\langle w_6, w_{59}\rangle$	w_2		$-27x^6 + 126x^4 + 45x^2 + 576$
119	3	$\langle w_{21}, w_{51}\rangle$	w_3		$-63x^6 - 171x^4 - 45x^2 - 153$
122	3	$\langle w_3, w_{122}\rangle$	w_2		$-216x^6 + 369x^4 - 18x^2 + 9$
123	1	$\langle w_{123}\rangle$	w_3		$-81x^6 + 171x^4 + 45x^2 + 9$
141	1	$\langle w_{141}\rangle$	w_3		$243x^6 - 45x^4 - 63x^2 + 9$
142	1	$\langle w_2\rangle$	w_{71}		$-144x^6 - 783x^4 - 1314x^2 - 639$
142	1	$\langle w_{142}\rangle$	w_2		$144x^6 + 81x^4 - 90x^2 + 9$
142	3	$\langle w_2, w_{213}\rangle$	w_3		$72x^6 + 297x^4 + 198x^2 + 9$
155	1	$\langle w_{155}\rangle$	w_5		$225x^6 - 171x^4 + 99x^2 - 9$
158	1	$\langle w_{158}\rangle$	w_2		$-72x^6 + 81x^4 + 126x^2 + 9$
210	1	$\langle w_2, w_3\rangle$	w_5	w_7	$-672x^6 - 21303x^4 + 6417x^2 - 480$
210	1	$\langle w_2, w_5\rangle$	w_3	w_7	$-98784/5x^6 - 80307/5x^4 + 17613/5x^2 - 864/5$
210	1	$\langle w_2, w_7\rangle$	w_3	w_5	$-15435/32x^6 - 117351/32x^4 - 65169/32x^2 - 189/32$
210	1	$\langle w_3, w_5\rangle$	w_2	w_7	$4725x^6 - 27666x^4 + 21069x^2 - 4320$
210	1	$\langle w_3, w_7\rangle$	w_2	w_5	$675/7x^6 - 19566/7x^4 - 65061/7x^2 - 864/7$
210	1	$\langle w_5, w_7\rangle$	w_2	w_3	$4725x^6 - 66834x^4 + 314829x^2 - 493920$
210	1	$\langle w_6, w_{10}\rangle$	w_2	w_{14}	$-231525/32x^6 - 701037/32x^4 - 19431/32x^2 - 135/32$
210	1	$\langle w_{10}, w_{14}\rangle$	w_2	w_6	$-21600/343x^6 - 687537/343x^4 + 117171/343x^2 - 1440/343$
210	1	$\langle w_{15}, w_{21}\rangle$	w_3	w_6	$-32928/25x^6 - 229563/25x^4 + 6534/25x^2 + 21/25$
210	11	$\langle w_2, w_7, w_{165}\rangle$	w_3	w_5	$162x^6 + 1035x^4 - 684x^2 + 63$
210	11	$\langle w_2, w_{11}, w_{105}\rangle$	w_3	w_5	$5346x^6 - 2421x^4 - 1116x^2 + 495$
210	11	$\langle w_5, w_6, w_{77}\rangle$	w_2	w_7	$1620x^6 + 2061x^4 + 18x^2 - 99$
210	11	$\langle w_5, w_{21}, w_{22}\rangle$	w_2	w_3	$-396x^6 + 1557x^4 + 2034x^2 + 405$
210	11	$\langle w_6, w_{22}, w_{70}\rangle$	w_2	w_7	$-972x^6 + 4869x^4 + 18x^2 - 315$
210	11	$\langle w_7, w_{15}, w_{22}\rangle$	w_3	w_6	$-405x^6 - 396x^4 + 1719x^2 + 1386$
210	11	$\langle w_{21}, w_{22}, w_{30}\rangle$	w_2	w_3	$-243x^6 + 1854x^4 - 2043x^2 - 5760$
210	13	$\langle w_3, w_{10}, w_{91}\rangle$	w_2	w_7	$-810x^6 - 369x^4 + 1656x^2 + 819$
210	13	$\langle w_3, w_{26}, w_{35}\rangle$	w_2	w_5	$-234x^6 - 369x^4 + 1224x^2 + 675$
210	13	$\langle w_{14}, w_{30}, w_{65}\rangle$	w_2	w_5	$-5625x^6 - 20232x^4 - 21933x^2 - 7938$
210	17	$\langle w_{21}, w_{30}, w_{85}\rangle$	w_3	w_5	$-900x^6 - 7767x^4 - 10998x^2 - 5103$
210	23	$\langle w_6, w_7, w_{10}, w_{46}\rangle$	w_2		$-7560x^6 + 16641x^4 - 2034x^2 + 9$
254	1	$\langle w_{254}\rangle$	w_2		$72x^6 + 225x^4 - 162x^2 + 9$
326	1	$\langle w_{326}\rangle$	w_2		$9x^6 + 90x^4 - 567x^2 + 36$
330	1	$\langle w_2, w_5\rangle$	w_3	w_{11}	$-3168/25x^6 + 39321/25x^4 - 97479/25x^2 - 139968/25$
330	1	$\langle w_2, w_{15}\rangle$	w_3	w_{33}	$297x^6 - 20322x^4 + 462393x^2 - 3499200$
330	1	$\langle w_2, w_{55}\rangle$	w_5	w_{15}	$297x^6 - 9378x^4 + 51993x^2 - 79200$
330	1	$\langle w_5, w_6\rangle$	w_2	w_{11}	$-8800/27x^6 - 4951/9x^4 + 6341/9x^2 - 1600/27$
330	1	$\langle w_5, w_{11}\rangle$	w_2	w_6	$-601425/32x^6 - 381177/32x^4 - 56907/32x^2 - 2475/32$
330	1	$\langle w_6, w_{10}\rangle$	w_2	w_{11}	$-7425/64x^6 - 91521/64x^4 - 193707/64x^2 - 54675/64$
330	1	$\langle w_6, w_{11}\rangle$	w_5	w_{10}	$-19008/25x^6 - 197163/25x^4 - 677331/25x^2 - 769824/25$
330	1	$\langle w_{10}, w_{11}\rangle$	w_2	w_6	$-43200/11x^6 - 24021/11x^4 + 34407/11x^2 - 7200/11$
330	1	$\langle w_{15}, w_{66}\rangle$	w_2	w_6	$-52800x^6 - 27207x^4 - 1674x^2 + 33$
330	1	$\langle w_{30}, w_{55}\rangle$	w_2	w_5	$-1749600/11x^6 - 162477/11x^4 + 5922/11x^2 + 27/11$

330	7	$\langle w_2, w_3, w_{385}\rangle$	w_5	w_7	$9x^6 + 630x^4 + 3897x^2 - 4536$
330	7	$\langle w_2, w_{33}, w_{35}\rangle$	w_5	w_{15}	$-63x^6 + 342x^4 + 24993x^2 + 145800$
330	7	$\langle w_3, w_{22}, w_{70}\rangle$	w_5	w_7	$-2187x^6 + 2574x^4 - 315x^2 - 72$
330	7	$\langle w_5, w_{11}, w_{42}\rangle$	w_2	w_6	$3564x^6 - 2547x^4 + 234x^2 + 45$
330	7	$\langle w_5, w_{14}, w_{33}\rangle$	w_2	w_6	$1260x^6 + 909x^4 - 1062x^2 + 189$
330	7	$\langle w_6, w_{22}, w_{70}\rangle$	w_5	w_7	$-27x^6 + 558x^4 + 3717x^2 - 19800$
330	7	$\langle w_{15}, w_{33}, w_{42}\rangle$	w_2	w_6	$-14580x^6 - 18747x^4 + 2826x^2 + 693$
390	1	$\langle w_2, w_{13}\rangle$	w_3	w_5	$-98865/128x^6 + 547659/128x^4 - 754371/128x^2 - 3159/128$
390	1	$\langle w_2, w_{39}\rangle$	w_3	w_{15}	$98865/4x^6 - 66303/4x^4 + 14571/4x^2 - 1053/4$
390	1	$\langle w_2, w_{65}\rangle$	w_5	w_{15}	$-351/4x^6 - 23607/4x^4 + 3339/4x^2 - 117/4$
390	1	$\langle w_3, w_{13}\rangle$	w_2	w_5	$-135/13x^6 - 33066/13x^4 - 759231/13x^2 - 31104/13$
390	1	$\langle w_3, w_{26}\rangle$	w_2	w_{10}	$-135x^6 - 1422x^4 + 6489x^2 - 4212$
390	1	$\langle w_3, w_{65}\rangle$	w_5	w_{10}	$-864x^6 - 58203x^4 + 1926x^2 + 117$
390	1	$\langle w_5, w_{13}\rangle$	w_2	w_3	$-351/5x^6 - 19242/5x^4 + 480537/5x^2 - 2530944/5$
390	1	$\langle w_5, w_{26}\rangle$	w_2	w_6	$-3159x^6 - 3006x^4 + 2745x^2 - 468$
390	1	$\langle w_5, w_{39}\rangle$	w_3	w_6	$-5694624x^6 + 475749x^4 - 13050x^2 + 117$
390	1	$\langle w_{26}, w_{30}\rangle$	w_3	w_5	$-177957/32x^6 + 236079/32x^4 - 58095/32x^2 - 27/32$
390	1	$\langle w_{30}, w_{39}\rangle$	w_2	w_5	$-243/13x^6 - 6894/13x^4 - 23715/13x^2 - 108/13$
390	1	$\langle w_{30}, w_{65}\rangle$	w_2	w_3	$-27/5x^6 - 1854/5x^4 + 12789/5x^2 + 79092/5$
390	7	$\langle w_3, w_{13}, w_{70}\rangle$	w_2	w_5	$9x^6 + 306x^4 + 2601x^2 + 24300$
390	7	$\langle w_{13}, w_{21}, w_{30}\rangle$	w_2	w_5	$-900x^6 - 7083x^4 - 1062x^2 - 27$
446	1	$\langle w_{446}\rangle$	w_2		$-144x^6 - 63x^4 + 342x^2 + 9$
462	1	$\langle w_2, w_3\rangle$	w_7	w_{11}	$-891/8x^6 - 58167/8x^4 + 49887/8x^2 - 9261/8$
462	1	$\langle w_2, w_7\rangle$	w_3	w_{33}	$-11319x^6 - 32058x^4 - 28647x^2 - 8232$
462	1	$\langle w_2, w_{33}\rangle$	w_3	w_7	$-9261/11x^6 + 2610/11x^4 + 59139/11x^2 - 648/11$
462	1	$\langle w_3, w_{14}\rangle$	w_2	w_{22}	$-99x^6 - 9090x^4 - 93195x^2 - 254016$
462	1	$\langle w_3, w_{22}\rangle$	w_2	w_{14}	$189x^6 + 5598x^4 + 31221x^2 - 46656$
462	1	$\langle w_6, w_7\rangle$	w_2	w_{11}	$305613/64x^6 - 387819/64x^4 - 81441/64x^2 - 3969/64$
462	1	$\langle w_6, w_{22}\rangle$	w_2	w_{14}	$453789/64x^6 + 37917/64x^4 - 39969/64x^2 - 729/64$
462	1	$\langle w_6, w_{77}\rangle$	w_7	w_{14}	$441/8x^6 + 4869/8x^4 - 5877/8x^2 + 567/8$
462	1	$\langle w_7, w_{66}\rangle$	w_2	w_6	$4536x^6 + 3609x^4 - 1170x^2 + 81$
462	1	$\langle w_{14}, w_{22}\rangle$	w_3	w_6	$3226944x^6 + 189045x^4 + 3510x^2 + 21$
462	1	$\langle w_{14}, w_{33}\rangle$	w_2	w_6	$3528x^6 + 6633x^4 + 1422x^2 + 81$
462	1	$\langle w_{42}, w_{66}\rangle$	w_3	w_6	$1778112/11x^6 + 498951/11x^4 + 6786/11x^2 - 9/11$
462	5	$\langle w_2, w_7, w_{165}\rangle$	w_3	w_5	$81x^6 - 18x^4 + 1233x^2 + 1008$
462	5	$\langle w_2, w_{15}, w_{77}\rangle$	w_3	w_{21}	$3240x^6 + 11673x^4 + 11574x^2 + 3465$
462	5	$\langle w_2, w_{77}, w_{105}\rangle$	w_3	w_5	$-144x^6 - 1287x^4 - 2034x^2 - 567$
462	5	$\langle w_{11}, w_{14}, w_{15}\rangle$	w_3	w_6	$405x^6 + 3438x^4 + 4149x^2 - 504$
510	1	$\langle w_2, w_3\rangle$	w_5	w_{17}	$-459/8x^6 - 114111/8x^4 + 13599/8x^2 - 405/8$
510	1	$\langle w_2, w_5\rangle$	w_3	w_{17}	$-765/16x^6 - 225009/16x^4 + 667161/16x^2 - 492075/16$
510	1	$\langle w_2, w_{51}\rangle$	w_3	w_{15}	$-405/16x^6 - 12897/16x^4 + 15489/16x^2 - 2187/16$
510	1	$\langle w_2, w_{85}\rangle$	w_5	w_{15}	$-6075/8x^6 - 167607/8x^4 + 7839/8x^2 - 45/8$
510	1	$\langle w_3, w_5\rangle$	w_2	w_{17}	$765x^6 - 15354x^4 + 13941x^2 - 3240$
510	1	$\langle w_3, w_{17}\rangle$	w_2	w_5	$45/17x^6 - 13914/17x^4 - 114219/17x^2 - 216/17$
510	1	$\langle w_5, w_{17}\rangle$	w_2	w_3	$-54675x^6 + 31446x^4 - 3339x^2 - 360$
510	1	$\langle w_6, w_{10}\rangle$	w_2	w_{17}	$-459/25x^6 - 358146/25x^4 - 671283/25x^2 - 314928/25$
510	1	$\langle w_6, w_{17}\rangle$	w_2	w_{10}	$45x^6 + 1278x^4 + 6741x^2 - 34992$

510	1	$\langle w_{10}, w_{17}\rangle$	w_2	w_6	$-675/17x^6 - 356418/17x^4 + 229509/17x^2 - 720/17$
510	1	$\langle w_{15}, w_{51}\rangle$	w_2	w_3	$-2187/25x^6 - 38538/25x^4 - 168579/25x^2 - 1944/25$
510	1	$\langle w_{15}, w_{102}\rangle$	w_2	w_6	$45x^6 - 1602x^4 + 14517x^2 - 6480$
546	1	$\langle w_3, w_{91}\rangle$	w_7	w_{14}	$-6144x^6 - 9747x^4 + 342x^2 - 3$
546	1	$\langle w_{13}, w_{21}\rangle$	w_3	w_6	$73728/13x^6 - 145143/13x^4 + 17838/13x^2 - 567/13$
546	1	$\langle w_{14}, w_{78}\rangle$	w_3	w_6	$-36x^6 + 225x^4 - 198x^2 - 567$
546	1	$\langle w_{21}, w_{39}\rangle$	w_2	w_3	$-41067x^6 + 42894x^4 - 11763x^2 + 288$
546	1	$\langle w_{21}, w_{78}\rangle$	w_2	w_6	$1168128x^6 - 135063x^4 + 5094x^2 - 63$
546	1	$\langle w_{42}, w_{78}\rangle$	w_2	w_3	$-2628288/7x^6 - 69345/7x^4 + 1818/7x^2 - 9/7$
570	1	$\langle w_3, w_{95}\rangle$	w_5	w_{10}	$90x^6 - 5769x^4 + 15588x^2 + 38475$
570	1	$\langle w_5, w_{114}\rangle$	w_3	w_6	$-243/2x^6 + 2259/2x^4 + 2547/2x^2 + 45/2$
570	1	$\langle w_6, w_{95}\rangle$	w_5	w_{10}	$675x^6 - 2844x^4 + 3231x^2 - 486$
570	1	$\langle w_{10}, w_{114}\rangle$	w_3	w_6	$-555579/2x^6 - 102717/2x^4 - 4149/2x^2 + 45/2$
570	1	$\langle w_{15}, w_{19}\rangle$	w_3	w_6	$-13718x^6 + 16023x^4 - 5532x^2 + 475$
570	1	$\langle w_{15}, w_{38}\rangle$	w_3	w_6	$675x^6 - 2556x^4 + 2367x^2 + 90$
690	1	$\langle w_2, w_{69}\rangle$	w_3	w_{15}	$45x^6 - 342x^4 - 21771x^2 - 78732$
690	1	$\langle w_6, w_{23}\rangle$	w_2	w_{10}	$16875/4x^6 - 54153/4x^4 + 56961/4x^2 - 19683/4$
690	1	$\langle w_{10}, w_{69}\rangle$	w_3	w_6	$-1863x^6 + 4194x^4 + 4257x^2 + 900$
690	1	$\langle w_{15}, w_{46}\rangle$	w_2	w_6	$-23x^6 + 834x^4 - 3543x^2 + 2300$
690	1	$\langle w_{15}, w_{138}\rangle$	w_2	w_6	$67500x^6 - 13347x^4 - 1062x^2 + 45$
690	1	$\langle w_{23}, w_{30}\rangle$	w_2	w_3	$225/4x^6 - 3843/4x^4 + 11187/4x^2 + 46575/4$
714	1	$\langle w_6, w_{119}\rangle$	w_7	w_{14}	$153x^6 - 4302x^4 + 37089x^2 - 89964$
714	1	$\langle w_6, w_{238}\rangle$	w_7	w_{14}	$56448x^6 - 16155x^4 + 1206x^2 - 27$
714	1	$\langle w_{17}, w_{42}\rangle$	w_2	w_6	$-2499/4x^6 - 2367/4x^4 + 4887/4x^2 - 21/4$
714	1	$\langle w_{21}, w_{34}\rangle$	w_2	w_6	$153x^6 + 3042x^4 + 14409x^2 - 756$
714	1	$\langle w_{21}, w_{102}\rangle$	w_2	w_6	$-10404x^6 + 1125x^4 + 234x^2 - 27$
714	1	$\langle w_{42}, w_{51}\rangle$	w_2	w_6	$-867x^6 + 3726x^4 - 2043x^2 - 4704$
770	1	$\langle w_5, w_{154}\rangle$	w_7	w_{11}	$-3087/2x^6 + 351/2x^4 + 3663/2x^2 + 225/2$
770	1	$\langle w_7, w_{110}\rangle$	w_2	w_5	$450x^6 + 2763x^4 - 4644x^2 + 1575$
770	1	$\langle w_{14}, w_{55}\rangle$	w_2	w_5	$-6174x^6 + 12699x^4 - 7956x^2 + 1575$
798	1	$\langle w_2, w_{399}\rangle$	w_3	w_{19}	$486x^6 + 1359x^4 - 1440x^2 + 171$
798	1	$\langle w_6, w_{133}\rangle$	w_2	w_{14}	$3888x^6 + 2817x^4 + 342x^2 + 9$
798	1	$\langle w_{14}, w_{57}\rangle$	w_3	w_6	$-81x^6 + 576x^4 + 99x^2 - 18$
798	1	$\langle w_{19}, w_{42}\rangle$	w_2	w_3	$2736x^6 + 3177x^4 + 1062x^2 + 81$
858	1	$\langle w_2, w_{429}\rangle$	w_{11}	w_{13}	$-9x^6 + 612x^4 - 7245x^2 + 27378$
870	1	$\langle w_2, w_{435}\rangle$	w_5	w_{15}	$3600x^6 + 1881x^4 - 306x^2 + 9$
870	1	$\langle w_3, w_{290}\rangle$	w_5	w_{10}	$45x^6 - 666x^4 + 1629x^2 + 4176$
870	1	$\langle w_{10}, w_{87}\rangle$	w_2	w_3	$900x^6 - 819x^4 - 2286x^2 + 2349$
870	1	$\langle w_{15}, w_{58}\rangle$	w_2	w_3	$36x^6 - 171x^4 - 126x^2 + 405$
910	1	$\langle w_5, w_{182}\rangle$	w_2	w_7	$2880x^6 + 1089x^4 - 378x^2 + 9$
910	3	$\langle w_2, w_{15}, w_{21}, w_{39}\rangle$	w_3		$7560x^6 + 11169x^4 + 1926x^2 + 81$
930	1	$\langle w_6, w_{310}\rangle$	w_5	w_{10}	$-162x^6 - 333x^4 - 1008x^2 - 225$
966	1	$\langle w_7, w_{46}\rangle$	w_2	w_6	$-243x^6 + 558x^4 + 2853x^2 - 7056$
1110	1	$\langle w_3, w_{370}\rangle$	w_2	w_{10}	$-324x^6 + 765x^4 + 522x^2 + 333$
1110	1	$\langle w_5, w_{222}\rangle$	w_2	w_6	$-36x^6 + 693x^4 + 234x^2 + 405$
1155	1	$\langle w_3, w_5, w_{77}\rangle$	w_7		$-4851x^6 + 6813x^4 - 801x^2 + 135$
1155	1	$\langle w_3, w_7, w_{55}\rangle$	w_5		$51975x^6 + 10971x^4 + 549x^2 + 9$

1155	1	$\langle w_7, w_{11}, w_{15}\rangle$	w_3		$11025x^6+1521x^4-981x^2+99$
1190	1	$\langle w_5, w_{238}\rangle$	w_2	w_{14}	$-36x^6+1773x^4+3114x^2+2205$
1218	1	$\langle w_6, w_{406}\rangle$	w_7	w_{14}	$-9x^6-576x^4-10917x^2-71442$
1230	1	$\langle w_6, w_{205}\rangle$	w_2	w_5	$-864x^6+9x^4+774x^2+225$
1230	1	$\langle w_{15}, w_{82}\rangle$	w_2	w_3	$144x^6+369x^4-234x^2-135$
1326	1	$\langle w_3, w_{442}\rangle$	w_2	w_{17}	$-7776x^6-1503x^4+198x^2+9$
1326	1	$\langle w_{34}, w_{39}\rangle$	w_2	w_6	$72x^6+1665x^4-7650x^2-3159$
1330	1	$\langle w_2, w_5, w_{133}\rangle$	w_7		$-2016x^6+4113x^4-1746x^2+225$
1330	1	$\langle w_2, w_7, w_{95}\rangle$	w_5		$2520x^6+2601x^4+54x^2+9$
1330	1	$\langle w_2, w_{19}, w_{35}\rangle$	w_5		$720x^6-639x^4-2754x^2+3249$
1410	1	$\langle w_5, w_{282}\rangle$	w_3	w_6	$2025x^6+306x^4-63x^2+36$
1430	1	$\langle w_2, w_{13}, w_{55}\rangle$	w_5		$7920x^6-7479x^4+1854x^2+9$
1430	1	$\langle w_5, w_{13}, w_{22}\rangle$	w_2		$-1440x^6+3321x^4+198x^2+1521$
1430	1	$\langle w_{10}, w_{11}, w_{13}\rangle$	w_2		$19800x^6+4401x^4+126x^2+9$
1482	1	$\langle w_2, w_3, w_{247}\rangle$	w_{13}		$22464x^6-2583x^4+846x^2+9$
1482	1	$\langle w_2, w_{13}, w_{57}\rangle$	w_3		$-62208x^6+2601x^4-306x^2+9$
1482	1	$\langle w_2, w_{19}, w_{39}\rangle$	w_3		$151632x^6+14265x^4+558x^2+9$
1518	1	$\langle w_2, w_3, w_{253}\rangle$	w_{11}		$50094x^6-2961x^4-504x^2+27$
1518	1	$\langle w_2, w_{11}, w_{69}\rangle$	w_3		$-6831x^6+3870x^4+3393x^2+144$
1518	1	$\langle w_3, w_{22}, w_{23}\rangle$	w_2		$-8712x^6+16641x^4-7362x^2+729$
1518	1	$\langle w_6, w_{22}, w_{46}\rangle$	w_2		$-54648x^6-16623x^4-1170x^2+9$
1554	1	$\langle w_2, w_3, w_{259}\rangle$	w_7		$1701x^6+3978x^4-531x^2+36$
1554	1	$\langle w_2, w_{21}, w_{37}\rangle$	w_3		$-2268x^6+2277x^4+1962x^2+333$
1554	1	$\langle w_6, w_7, w_{37}\rangle$	w_2		$1458x^6+4707x^4+828x^2+63$
1590	1	$\langle w_5, w_{318}\rangle$	w_2	w_3	$36x^6+477x^4-4014x^2+3645$
1610	1	$\langle w_2, w_5, w_{161}\rangle$	w_7		$-14112x^6+7785x^4-594x^2+9$
1610	1	$\langle w_2, w_{23}, w_{35}\rangle$	w_5		$-2205x^6+3816x^4+3159x^2+414$
1770	1	$\langle w_2, w_{885}\rangle$	w_3	w_5	$-1296x^6+1521x^4+126x^2+225$
1770	1	$\langle w_{15}, w_{118}\rangle$	w_3	w_6	$-6075x^6-234x^4-171x^2+144$
1806	1	$\langle w_2, w_3, w_{301}\rangle$	w_7		$95256x^6-12519x^4+198x^2+9$
1806	1	$\langle w_2, w_{21}, w_{43}\rangle$	w_3		$-11907x^6+23202x^4-12267x^2+1548$
1806	1	$\langle w_6, w_7, w_{43}\rangle$	w_2		$-244944x^6+28305x^4-954x^2+9$
1806	1	$\langle w_6, w_{14}, w_{86}\rangle$	w_2		$18144x^6+1089x^4-1818x^2+9$
1830	1	$\langle w_2, w_3, w_{305}\rangle$	w_5		$16200x^6+4761x^4-234x^2+9$
1830	1	$\langle w_3, w_{10}, w_{61}\rangle$	w_2		$-4860x^6+9621x^4-4014x^2+549$
2046	1	$\langle w_3, w_{22}, w_{31}\rangle$	w_2		$14256x^6+17073x^4+1062x^2+9$
2090	1	$\langle w_5, w_{11}, w_{38}\rangle$	w_2		$1710x^6+2799x^4-1008x^2+99$
2190	1	$\langle w_3, w_{10}, w_{73}\rangle$	w_2		$-432x^6+441x^4+630x^2+657$
2226	1	$\langle w_2, w_{21}, w_{53}\rangle$	w_3		$81x^6+1926x^4+3393x^2+3816$
2226	1	$\langle w_3, w_7, w_{106}\rangle$	w_2		$-486x^6+1683x^4+36x^2+63$
2262	1	$\langle w_3, w_{13}, w_{58}\rangle$	w_2		$324x^6+981x^4-126x^2+117$
2370	1	$\langle w_3, w_{10}, w_{79}\rangle$	w_2		$810x^6+1899x^4-2124x^2+711$
2478	1	$\langle w_3, w_7, w_{118}\rangle$	w_2		$-216x^6-207x^4+1278x^2+441$
2490	1	$\langle w_2, w_5, w_{249}\rangle$	w_3		$-3240x^6+4329x^4-522x^2+9$
2670	1	$\langle w_6, w_{10}, w_{178}\rangle$	w_2		$-2160x^6+1521x^4+774x^2+9$

Table 10: All 469 genus 2 curves $X = X_0^D(N)/W$ which are not bielliptic via an Atkin–Lehner involution for which we remain unsure of whether X is geometrically bielliptic.

D	N	W	D	N	W	D	N	W
6	29	$\langle w_3, w_{29}\rangle$	6	37	$\langle w_6, w_{74}\rangle$	6	41	$\langle w_2, w_3\rangle$
6	41	$\langle w_3, w_{41}\rangle$	6	43	$\langle w_2, w_3\rangle$	6	43	$\langle w_2, w_{43}\rangle$
6	49	$\langle w_2, w_3\rangle$	6	49	$\langle w_2, w_{49}\rangle$	6	49	$\langle w_6, w_{98}\rangle$
6	55	$\langle w_2, w_5, w_{11}\rangle$	6	55	$\langle w_3, w_5, w_{11}\rangle$	6	59	$\langle w_2, w_{59}\rangle$
6	61	$\langle w_2, w_3\rangle$	6	65	$\langle w_3, w_5, w_{13}\rangle$	6	73	$\langle w_2, w_3\rangle$
6	73	$\langle w_3, w_{146}\rangle$	6	77	$\langle w_2, w_3, w_7\rangle$	6	77	$\langle w_2, w_7, w_{11}\rangle$
6	77	$\langle w_2, w_{11}, w_{21}\rangle$	6	77	$\langle w_3, w_7, w_{11}\rangle$	6	85	$\langle w_2, w_3, w_5\rangle$
6	85	$\langle w_3, w_5, w_{17}\rangle$	6	91	$\langle w_2, w_3, w_{13}\rangle$	6	91	$\langle w_2, w_7, w_{13}\rangle$
6	91	$\langle w_6, w_7, w_{26}\rangle$	6	95	$\langle w_2, w_3, w_{19}\rangle$	6	95	$\langle w_2, w_5, w_{19}\rangle$
6	95	$\langle w_2, w_{15}, w_{57}\rangle$	6	95	$\langle w_3, w_5, w_{19}\rangle$	6	95	$\langle w_5, w_6, w_{38}\rangle$
6	95	$\langle w_6, w_{10}, w_{38}\rangle$	6	115	$\langle w_2, w_3, w_{23}\rangle$	6	115	$\langle w_2, w_5, w_{23}\rangle$
6	115	$\langle w_3, w_5, w_{23}\rangle$	6	115	$\langle w_3, w_5, w_{46}\rangle$	6	115	$\langle w_5, w_6, w_{23}\rangle$
6	119	$\langle w_2, w_7, w_{17}\rangle$	6	119	$\langle w_2, w_7, w_{51}\rangle$	6	119	$\langle w_2, w_{21}, w_{51}\rangle$
6	119	$\langle w_6, w_7, w_{17}\rangle$	6	119	$\langle w_6, w_{14}, w_{17}\rangle$	6	143	$\langle w_2, w_{11}, w_{13}\rangle$
6	143	$\langle w_2, w_{11}, w_{39}\rangle$	6	143	$\langle w_2, w_{33}, w_{39}\rangle$	6	143	$\langle w_3, w_{11}, w_{26}\rangle$
6	143	$\langle w_3, w_{22}, w_{26}\rangle$	6	145	$\langle w_2, w_3, w_{29}\rangle$	6	145	$\langle w_2, w_{15}, w_{29}\rangle$
6	145	$\langle w_3, w_5, w_{58}\rangle$	6	145	$\langle w_6, w_{10}, w_{58}\rangle$	6	155	$\langle w_3, w_{10}, w_{31}\rangle$
6	155	$\langle w_3, w_{10}, w_{62}\rangle$	6	155	$\langle w_5, w_6, w_{62}\rangle$	6	157	$\langle w_2, w_3, w_{157}\rangle$
6	161	$\langle w_2, w_{21}, w_{23}\rangle$	6	161	$\langle w_6, w_7, w_{23}\rangle$	6	161	$\langle w_6, w_{14}, w_{23}\rangle$
6	161	$\langle w_6, w_{14}, w_{46}\rangle$	6	173	$\langle w_2, w_3, w_{173}\rangle$	6	203	$\langle w_2, w_7, w_{87}\rangle$
6	203	$\langle w_3, w_{14}, w_{29}\rangle$	6	203	$\langle w_6, w_7, w_{58}\rangle$	6	235	$\langle w_2, w_5, w_{141}\rangle$
6	239	$\langle w_2, w_3, w_{239}\rangle$	6	251	$\langle w_2, w_3, w_{251}\rangle$	6	263	$\langle w_2, w_3, w_{263}\rangle$
6	319	$\langle w_2, w_3, w_{11}, w_{29}\rangle$	6	329	$\langle w_2, w_3, w_7, w_{47}\rangle$	6	335	$\langle w_2, w_3, w_5, w_{67}\rangle$
6	391	$\langle w_2, w_3, w_{17}, w_{23}\rangle$	6	473	$\langle w_2, w_3, w_{11}, w_{43}\rangle$	6	497	$\langle w_2, w_3, w_7, w_{71}\rangle$
6	665	$\langle w_2, w_3, w_5, w_7, w_{19}\rangle$	10	17	$\langle w_{10}, w_{34}\rangle$	10	33	$\langle w_2, w_3, w_5\rangle$
10	33	$\langle w_2, w_3, w_{11}\rangle$	10	33	$\langle w_3, w_{10}, w_{22}\rangle$	10	33	$\langle w_6, w_{10}, w_{22}\rangle$
10	39	$\langle w_2, w_3, w_5\rangle$	10	39	$\langle w_2, w_3, w_{13}\rangle$	10	39	$\langle w_5, w_6, w_{26}\rangle$
10	39	$\langle w_6, w_{10}, w_{13}\rangle$	10	49	$\langle w_2, w_{245}\rangle$	10	51	$\langle w_2, w_3, w_{17}\rangle$
10	51	$\langle w_5, w_6, w_{34}\rangle$	10	53	$\langle w_5, w_{106}\rangle$	10	57	$\langle w_2, w_5, w_{19}\rangle$
10	57	$\langle w_3, w_{10}, w_{19}\rangle$	10	69	$\langle w_2, w_5, w_{23}\rangle$	10	77	$\langle w_2, w_{11}, w_{35}\rangle$
10	77	$\langle w_{10}, w_{11}, w_{14}\rangle$	10	81	$\langle w_2, w_5, w_{81}\rangle$	10	83	$\langle w_2, w_5, w_{83}\rangle$
10	93	$\langle w_2, w_3, w_{155}\rangle$	10	93	$\langle w_3, w_5, w_{62}\rangle$	10	93	$\langle w_5, w_6, w_{62}\rangle$
10	101	$\langle w_2, w_5, w_{101}\rangle$	10	121	$\langle w_2, w_5, w_{121}\rangle$	10	129	$\langle w_2, w_3, w_5, w_{43}\rangle$
10	147	$\langle w_2, w_3, w_5, w_{49}\rangle$	10	153	$\langle w_2, w_5, w_9, w_{17}\rangle$	10	177	$\langle w_2, w_3, w_5, w_{59}\rangle$
10	221	$\langle w_2, w_5, w_{13}, w_{17}\rangle$	10	273	$\langle w_2, w_3, w_5, w_7, w_{13}\rangle$	14	9	$\langle w_7, w_9\rangle$
14	13	$\langle w_2, w_{13}\rangle$	14	13	$\langle w_7, w_{13}\rangle$	14	15	$\langle w_2, w_3, w_5\rangle$
14	15	$\langle w_6, w_7, w_{10}\rangle$	14	23	$\langle w_7, w_{23}\rangle$	14	23	$\langle w_7, w_{46}\rangle$
14	25	$\langle w_2, w_{25}\rangle$	14	25	$\langle w_2, w_{175}\rangle$	14	29	$\langle w_7, w_{29}\rangle$
14	29	$\langle w_7, w_{58}\rangle$	14	39	$\langle w_2, w_7, w_{13}\rangle$	14	39	$\langle w_2, w_{13}, w_{21}\rangle$
14	39	$\langle w_3, w_{14}, w_{26}\rangle$	14	39	$\langle w_6, w_7, w_{13}\rangle$	14	39	$\langle w_6, w_7, w_{26}\rangle$
14	39	$\langle w_6, w_{14}, w_{26}\rangle$	14	41	$\langle w_2, w_7, w_{41}\rangle$	14	47	$\langle w_2, w_7, w_{47}\rangle$
14	57	$\langle w_2, w_3, w_{133}\rangle$	14	71	$\langle w_2, w_7, w_{71}\rangle$	14	99	$\langle w_2, w_7, w_9, w_{11}\rangle$
14	109	$\langle w_2, w_7, w_{109}\rangle$	14	111	$\langle w_2, w_3, w_7, w_{37}\rangle$	14	115	$\langle w_2, w_5, w_7, w_{23}\rangle$

14	129	$\langle w_2, w_3, w_7, w_{43}\rangle$	14	165	$\langle w_2, w_3, w_5, w_7, w_{11}\rangle$	15	4	$\langle w_4, w_5\rangle$
15	8	$\langle w_5, w_8\rangle$	15	8	$\langle w_5, w_{24}\rangle$	15	13	$\langle w_3, w_5\rangle$
15	13	$\langle w_5, w_{13}\rangle$	15	14	$\langle w_2, w_3, w_5\rangle$	15	14	$\langle w_5, w_6, w_{14}\rangle$
15	22	$\langle w_2, w_5, w_{33}\rangle$	15	22	$\langle w_5, w_6, w_{22}\rangle$	15	26	$\langle w_3, w_5, w_{26}\rangle$
15	26	$\langle w_5, w_6, w_{13}\rangle$	15	26	$\langle w_5, w_6, w_{26}\rangle$	15	32	$\langle w_3, w_5, w_{32}\rangle$
15	34	$\langle w_5, w_6, w_{34}\rangle$	15	43	$\langle w_3, w_5, w_{43}\rangle$	15	49	$\langle w_3, w_5, w_{49}\rangle$
15	56	$\langle w_3, w_5, w_7, w_8\rangle$	15	59	$\langle w_3, w_5, w_{59}\rangle$	15	62	$\langle w_2, w_3, w_5, w_{31}\rangle$
21	4	$\langle w_3, w_4\rangle$	21	4	$\langle w_{12}, w_{21}\rangle$	21	10	$\langle w_2, w_3, w_7\rangle$
21	10	$\langle w_3, w_{10}, w_{14}\rangle$	21	13	$\langle w_3, w_{13}\rangle$	21	22	$\langle w_3, w_{14}, w_{22}\rangle$
21	26	$\langle w_2, w_3, w_{91}\rangle$	21	26	$\langle w_3, w_7, w_{13}\rangle$	21	26	$\langle w_3, w_{13}, w_{14}\rangle$
21	52	$\langle w_3, w_4, w_7, w_{13}\rangle$	22	13	$\langle w_2, w_{11}\rangle$	22	15	$\langle w_2, w_3, w_{11}\rangle$
22	15	$\langle w_2, w_{11}, w_{15}\rangle$	22	15	$\langle w_3, w_5, w_{11}\rangle$	22	21	$\langle w_2, w_3, w_7\rangle$
22	21	$\langle w_2, w_3, w_{77}\rangle$	22	21	$\langle w_2, w_7, w_{11}\rangle$	22	21	$\langle w_3, w_7, w_{11}\rangle$
22	21	$\langle w_3, w_7, w_{22}\rangle$	22	25	$\langle w_2, w_{11}, w_{25}\rangle$	22	27	$\langle w_2, w_{11}, w_{27}\rangle$
22	35	$\langle w_2, w_5, w_{77}\rangle$	22	35	$\langle w_5, w_{14}, w_{22}\rangle$	22	35	$\langle w_7, w_{10}, w_{11}\rangle$
22	45	$\langle w_2, w_5, w_9, w_{11}\rangle$	22	47	$\langle w_2, w_{11}, w_{47}\rangle$	22	69	$\langle w_2, w_3, w_{11}, w_{23}\rangle$
26	3	$\langle w_3, w_{13}\rangle$	26	5	$\langle w_2, w_5\rangle$	26	5	$\langle w_{10}, w_{13}\rangle$
26	7	$\langle w_7, w_{13}\rangle$	26	9	$\langle w_2, w_{13}\rangle$	26	9	$\langle w_{13}, w_{18}\rangle$
26	15	$\langle w_2, w_3, w_{13}\rangle$	26	15	$\langle w_2, w_5, w_{39}\rangle$	26	15	$\langle w_3, w_5, w_{13}\rangle$
26	17	$\langle w_2, w_{221}\rangle$	26	21	$\langle w_3, w_{13}, w_{14}\rangle$	26	21	$\langle w_6, w_{13}, w_{14}\rangle$
26	31	$\langle w_2, w_{13}, w_{31}\rangle$	26	33	$\langle w_2, w_3, w_{11}, w_{13}\rangle$	26	35	$\langle w_2, w_5, w_7, w_{13}\rangle$
26	45	$\langle w_2, w_5, w_9, w_{13}\rangle$	26	55	$\langle w_2, w_5, w_{11}, w_{13}\rangle$	26	57	$\langle w_2, w_3, w_{13}, w_{19}\rangle$
33	8	$\langle w_3, w_8, w_{11}\rangle$	33	10	$\langle w_2, w_5, w_{11}\rangle$	33	10	$\langle w_2, w_{11}, w_{15}\rangle$
33	10	$\langle w_6, w_{10}, w_{11}\rangle$	33	14	$\langle w_2, w_{11}, w_{21}\rangle$	33	14	$\langle w_3, w_7, w_{11}\rangle$
33	17	$\langle w_3, w_{11}, w_{17}\rangle$	33	28	$\langle w_3, w_4, w_7, w_{11}\rangle$	33	35	$\langle w_3, w_5, w_7, w_{11}\rangle$
34	5	$\langle w_2, w_5\rangle$	34	5	$\langle w_5, w_{17}\rangle$	34	7	$\langle w_2, w_7\rangle$
34	7	$\langle w_2, w_{119}\rangle$	34	35	$\langle w_2, w_5, w_7, w_{17}\rangle$	34	39	$\langle w_2, w_3, w_{13}, w_{17}\rangle$
35	4	$\langle w_4, w_7\rangle$	35	4	$\langle w_{20}, w_{28}\rangle$	35	6	$\langle w_2, w_5, w_7\rangle$
35	6	$\langle w_6, w_7, w_{10}\rangle$	35	8	$\langle w_5, w_7, w_8\rangle$	35	18	$\langle w_2, w_5, w_7, w_9\rangle$
35	19	$\langle w_5, w_7, w_{19}\rangle$	35	22	$\langle w_2, w_5, w_7, w_{11}\rangle$	38	3	$\langle w_2, w_3\rangle$
38	3	$\langle w_3, w_{19}\rangle$	38	13	$\langle w_2, w_{247}\rangle$	38	33	$\langle w_2, w_3, w_{11}, w_{19}\rangle$
38	35	$\langle w_2, w_5, w_7, w_{19}\rangle$	39	2	$\langle w_2, w_{13}\rangle$	39	5	$\langle w_3, w_{13}\rangle$
39	7	$\langle w_{13}, w_{21}\rangle$	39	10	$\langle w_2, w_{13}, w_{15}\rangle$	39	10	$\langle w_2, w_{15}, w_{39}\rangle$
39	10	$\langle w_3, w_{10}, w_{13}\rangle$	39	10	$\langle w_6, w_{10}, w_{13}\rangle$	39	22	$\langle w_2, w_3, w_{11}, w_{13}\rangle$
46	19	$\langle w_2, w_{19}, w_{23}\rangle$	46	35	$\langle w_2, w_5, w_7, w_{23}\rangle$	51	2	$\langle w_2, w_3\rangle$
51	10	$\langle w_2, w_3, w_{85}\rangle$	51	10	$\langle w_3, w_5, w_{17}\rangle$	51	10	$\langle w_3, w_{10}, w_{34}\rangle$
51	20	$\langle w_3, w_4, w_5, w_{17}\rangle$	51	26	$\langle w_2, w_3, w_{13}, w_{17}\rangle$	55	8	$\langle w_5, w_8, w_{11}\rangle$
55	14	$\langle w_2, w_5, w_7, w_{11}\rangle$	57	1	$\langle w_3\rangle$	57	2	$\langle w_2, w_3\rangle$
57	2	$\langle w_3, w_{38}\rangle$	57	2	$\langle w_6, w_{38}\rangle$	57	7	$\langle w_3, w_{133}\rangle$
57	7	$\langle w_7, w_{57}\rangle$	58	3	$\langle w_2, w_3\rangle$	58	5	$\langle w_5, w_{58}\rangle$
58	9	$\langle w_2, w_9, w_{29}\rangle$	58	17	$\langle w_2, w_{17}, w_{29}\rangle$	58	21	$\langle w_2, w_3, w_7, w_{29}\rangle$
62	7	$\langle w_2, w_{217}\rangle$	62	9	$\langle w_2, w_9, w_{31}\rangle$	62	15	$\langle w_2, w_3, w_5, w_{31}\rangle$
62	21	$\langle w_2, w_3, w_7, w_{31}\rangle$	65	3	$\langle w_5, w_{13}\rangle$	65	3	$\langle w_{15}, w_{39}\rangle$
65	4	$\langle w_4, w_5, w_{13}\rangle$	65	11	$\langle w_5, w_{11}, w_{13}\rangle$	65	14	$\langle w_2, w_5, w_7, w_{13}\rangle$
69	2	$\langle w_2, w_3\rangle$	69	10	$\langle w_2, w_3, w_5, w_{23}\rangle$	74	9	$\langle w_2, w_9, w_{37}\rangle$
77	2	$\langle w_2, w_{11}\rangle$	77	2	$\langle w_2, w_{77}\rangle$	77	2	$\langle w_{14}, w_{22}\rangle$
82	3	$\langle w_2, w_{41}\rangle$	82	3	$\langle w_3, w_{82}\rangle$	82	15	$\langle w_2, w_3, w_5, w_{41}\rangle$

86	3	$\langle w_2, w_{43}\rangle$	91	3	$\langle w_3, w_7, w_{13}\rangle$	91	6	$\langle w_2, w_3, w_7, w_{13}\rangle$
106	1	$\langle w_2\rangle$	111	7	$\langle w_3, w_7, w_{37}\rangle$	118	1	$\langle w_2\rangle$
119	6	$\langle w_2, w_3, w_7, w_{17}\rangle$	123	5	$\langle w_3, w_5, w_{41}\rangle$	134	5	$\langle w_2, w_5, w_{67}\rangle$
142	5	$\langle w_2, w_5, w_{71}\rangle$	143	2	$\langle w_2, w_{143}\rangle$	143	2	$\langle w_{11}, w_{13}\rangle$
143	2	$\langle w_{22}, w_{26}\rangle$	143	4	$\langle w_4, w_{11}, w_{13}\rangle$	146	7	$\langle w_2, w_7, w_{73}\rangle$
155	3	$\langle w_3, w_5, w_{31}\rangle$	201	2	$\langle w_2, w_3, w_{67}\rangle$	203	2	$\langle w_2, w_7, w_{29}\rangle$
210	11	$\langle w_2, w_3, w_5, w_{11}\rangle$	210	11	$\langle w_2, w_3, w_{35}, w_{55}\rangle$	210	11	$\langle w_3, w_5, w_7, w_{11}\rangle$
210	11	$\langle w_3, w_5, w_{11}, w_{14}\rangle$	210	11	$\langle w_3, w_7, w_{10}, w_{11}\rangle$	210	11	$\langle w_6, w_{10}, w_{11}, w_{14}\rangle$
210	11	$\langle w_6, w_{10}, w_{14}, w_{22}\rangle$	210	13	$\langle w_2, w_5, w_7, w_{39}\rangle$	210	13	$\langle w_2, w_5, w_{21}, w_{39}\rangle$
210	13	$\langle w_2, w_{13}, w_{15}, w_{21}\rangle$	210	13	$\langle w_3, w_{10}, w_{13}, w_{14}\rangle$	210	13	$\langle w_5, w_6, w_7, w_{13}\rangle$
210	13	$\langle w_6, w_7, w_{10}, w_{26}\rangle$	210	13	$\langle w_6, w_{10}, w_{13}, w_{14}\rangle$	210	17	$\langle w_2, w_3, w_7, w_{85}\rangle$
210	17	$\langle w_2, w_5, w_7, w_{51}\rangle$	210	17	$\langle w_2, w_7, w_{15}, w_{17}\rangle$	210	17	$\langle w_3, w_5, w_{14}, w_{17}\rangle$
210	17	$\langle w_5, w_6, w_7, w_{34}\rangle$	210	17	$\langle w_6, w_{10}, w_{14}, w_{17}\rangle$	210	19	$\langle w_2, w_3, w_7, w_{95}\rangle$
210	19	$\langle w_2, w_5, w_7, w_{57}\rangle$	210	19	$\langle w_2, w_5, w_{19}, w_{21}\rangle$	210	19	$\langle w_2, w_7, w_{15}, w_{57}\rangle$
210	19	$\langle w_3, w_{10}, w_{14}, w_{38}\rangle$	210	19	$\langle w_6, w_{10}, w_{14}, w_{38}\rangle$	210	43	$\langle w_2, w_3, w_5, w_7, w_{43}\rangle$
214	3	$\langle w_2, w_3, w_{107}\rangle$	218	3	$\langle w_2, w_3, w_{109}\rangle$	226	3	$\langle w_2, w_3, w_{113}\rangle$
235	2	$\langle w_2, w_5, w_{47}\rangle$	254	3	$\langle w_2, w_3, w_{127}\rangle$	302	1	$\langle w_2, w_{151}\rangle$
303	1	$\langle w_3, w_{101}\rangle$	326	3	$\langle w_2, w_3, w_{163}\rangle$	327	2	$\langle w_2, w_3, w_{109}\rangle$
330	7	$\langle w_2, w_3, w_5, w_7\rangle$	330	7	$\langle w_2, w_3, w_7, w_{11}\rangle$	330	7	$\langle w_2, w_3, w_{35}, w_{55}\rangle$
330	7	$\langle w_2, w_5, w_{21}, w_{33}\rangle$	330	7	$\langle w_2, w_7, w_{15}, w_{33}\rangle$	330	7	$\langle w_3, w_5, w_7, w_{22}\rangle$
330	7	$\langle w_3, w_{10}, w_{14}, w_{22}\rangle$	330	7	$\langle w_6, w_7, w_{10}, w_{11}\rangle$	330	7	$\langle w_6, w_{10}, w_{11}, w_{14}\rangle$
330	13	$\langle w_2, w_3, w_5, w_{143}\rangle$	330	13	$\langle w_2, w_{11}, w_{13}, w_{15}\rangle$	330	13	$\langle w_3, w_5, w_{22}, w_{26}\rangle$
330	13	$\langle w_5, w_6, w_{11}, w_{13}\rangle$	339	1	$\langle w_3, w_{113}\rangle$	390	7	$\langle w_2, w_3, w_5, w_{91}\rangle$
390	7	$\langle w_2, w_3, w_{35}, w_{65}\rangle$	390	7	$\langle w_2, w_5, w_{13}, w_{21}\rangle$	390	7	$\langle w_3, w_{10}, w_{14}, w_{26}\rangle$
390	7	$\langle w_5, w_6, w_7, w_{26}\rangle$	390	7	$\langle w_6, w_7, w_{10}, w_{13}\rangle$	390	7	$\langle w_6, w_7, w_{10}, w_{26}\rangle$
390	7	$\langle w_6, w_{10}, w_{13}, w_{14}\rangle$	390	7	$\langle w_6, w_{10}, w_{14}, w_{26}\rangle$	390	17	$\langle w_2, w_3, w_5, w_{13}, w_{17}\rangle$
462	5	$\langle w_3, w_5, w_7, w_{22}\rangle$	462	5	$\langle w_3, w_5, w_{14}, w_{22}\rangle$	462	5	$\langle w_3, w_7, w_{10}, w_{11}\rangle$
462	5	$\langle w_5, w_6, w_7, w_{11}\rangle$	462	5	$\langle w_5, w_6, w_{11}, w_{14}\rangle$	462	5	$\langle w_5, w_6, w_{14}, w_{22}\rangle$
462	17	$\langle w_2, w_3, w_7, w_{11}, w_{17}\rangle$	510	7	$\langle w_2, w_3, w_{17}, w_{35}\rangle$	510	7	$\langle w_2, w_3, w_{35}, w_{85}\rangle$
510	7	$\langle w_2, w_5, w_{17}, w_{21}\rangle$	510	7	$\langle w_3, w_5, w_7, w_{34}\rangle$	510	7	$\langle w_3, w_5, w_{14}, w_{34}\rangle$
510	7	$\langle w_5, w_6, w_{14}, w_{17}\rangle$	510	7	$\langle w_6, w_{10}, w_{14}, w_{17}\rangle$	514	1	$\langle w_2, w_{257}\rangle$
546	1	$\langle w_2, w_3, w_7\rangle$	546	5	$\langle w_2, w_3, w_7, w_{65}\rangle$	546	5	$\langle w_2, w_3, w_{13}, w_{35}\rangle$
546	5	$\langle w_2, w_3, w_{35}, w_{65}\rangle$	546	5	$\langle w_2, w_{13}, w_{15}, w_{21}\rangle$	546	5	$\langle w_2, w_{15}, w_{21}, w_{39}\rangle$
546	5	$\langle w_5, w_6, w_7, w_{13}\rangle$	570	1	$\langle w_2, w_3, w_5\rangle$	570	1	$\langle w_2, w_3, w_{19}\rangle$
570	1	$\langle w_2, w_5, w_{19}\rangle$	570	7	$\langle w_2, w_3, w_5, w_{133}\rangle$	570	7	$\langle w_2, w_5, w_7, w_{57}\rangle$
570	7	$\langle w_2, w_{15}, w_{19}, w_{21}\rangle$	570	7	$\langle w_3, w_5, w_7, w_{38}\rangle$	570	7	$\langle w_3, w_7, w_{10}, w_{19}\rangle$
570	7	$\langle w_3, w_{10}, w_{14}, w_{38}\rangle$	570	7	$\langle w_5, w_6, w_7, w_{19}\rangle$	570	7	$\langle w_6, w_7, w_{10}, w_{19}\rangle$
570	7	$\langle w_6, w_{10}, w_{14}, w_{38}\rangle$	586	1	$\langle w_2, w_{293}\rangle$	614	1	$\langle w_2, w_{307}\rangle$
690	1	$\langle w_2, w_3, w_5\rangle$	690	1	$\langle w_5, w_6, w_{46}\rangle$	690	11	$\langle w_2, w_3, w_5, w_{11}, w_{23}\rangle$
714	1	$\langle w_2, w_7, w_{17}\rangle$	714	11	$\langle w_2, w_3, w_7, w_{11}, w_{17}\rangle$	770	1	$\langle w_7, w_{10}, w_{22}\rangle$
770	3	$\langle w_2, w_7, w_{11}, w_{15}\rangle$	770	3	$\langle w_2, w_{11}, w_{15}, w_{21}\rangle$	770	3	$\langle w_3, w_5, w_7, w_{22}\rangle$
770	3	$\langle w_5, w_6, w_7, w_{11}\rangle$	770	3	$\langle w_5, w_6, w_7, w_{22}\rangle$	798	1	$\langle w_3, w_7, w_{19}\rangle$
858	1	$\langle w_2, w_{11}, w_{13}\rangle$	858	1	$\langle w_3, w_{11}, w_{13}\rangle$	858	7	$\langle w_2, w_3, w_7, w_{11}, w_{13}\rangle$
870	1	$\langle w_2, w_3, w_5\rangle$	870	7	$\langle w_2, w_3, w_5, w_7, w_{29}\rangle$	910	1	$\langle w_2, w_7, w_{13}\rangle$
910	1	$\langle w_2, w_{13}, w_{35}\rangle$	910	1	$\langle w_7, w_{10}, w_{13}\rangle$	930	1	$\langle w_2, w_{15}, w_{93}\rangle$
930	1	$\langle w_3, w_5, w_{31}\rangle$	930	1	$\langle w_3, w_{10}, w_{62}\rangle$	930	1	$\langle w_5, w_6, w_{31}\rangle$
930	7	$\langle w_2, w_3, w_5, w_7, w_{31}\rangle$	966	1	$\langle w_2, w_3, w_{23}\rangle$	966	1	$\langle w_6, w_7, w_{23}\rangle$

1110	1	$\langle w_2, w_3, w_{37}\rangle$	1110	1	$\langle w_2, w_5, w_{37}\rangle$	1110	1	$\langle w_3, w_{10}, w_{74}\rangle$
1110	7	$\langle w_2, w_3, w_5, w_7, w_{37}\rangle$	1122	1	$\langle w_2, w_3, w_{17}\rangle$	1122	1	$\langle w_2, w_{11}, w_{17}\rangle$
1122	1	$\langle w_6, w_{11}, w_{34}\rangle$	1122	5	$\langle w_2, w_3, w_5, w_{11}, w_{17}\rangle$	1190	1	$\langle w_2, w_{17}, w_{35}\rangle$
1218	1	$\langle w_2, w_{21}, w_{87}\rangle$	1230	1	$\langle w_2, w_3, w_{41}\rangle$	1230	1	$\langle w_2, w_5, w_{41}\rangle$
1254	1	$\langle w_6, w_{19}, w_{22}\rangle$	1254	5	$\langle w_2, w_3, w_5, w_{11}, w_{19}\rangle$	1290	1	$\langle w_2, w_{15}, w_{129}\rangle$
1290	1	$\langle w_3, w_{10}, w_{86}\rangle$	1290	1	$\langle w_5, w_6, w_{86}\rangle$	1302	1	$\langle w_2, w_3, w_{31}\rangle$
1302	1	$\langle w_2, w_{21}, w_{31}\rangle$	1302	1	$\langle w_3, w_7, w_{31}\rangle$	1302	1	$\langle w_3, w_7, w_{62}\rangle$
1302	1	$\langle w_6, w_{14}, w_{31}\rangle$	1330	3	$\langle w_2, w_3, w_5, w_7, w_{19}\rangle$	1365	2	$\langle w_2, w_3, w_5, w_7, w_{13}\rangle$
1410	1	$\langle w_6, w_{10}, w_{94}\rangle$	1590	1	$\langle w_2, w_{15}, w_{53}\rangle$	1590	1	$\langle w_2, w_{15}, w_{159}\rangle$
1590	1	$\langle w_3, w_{10}, w_{53}\rangle$	1590	1	$\langle w_6, w_{10}, w_{106}\rangle$	1722	1	$\langle w_2, w_{21}, w_{123}\rangle$
1722	1	$\langle w_6, w_7, w_{82}\rangle$	1770	1	$\langle w_3, w_{10}, w_{59}\rangle$	1794	1	$\langle w_2, w_{23}, w_{39}\rangle$
1794	1	$\langle w_6, w_{13}, w_{23}\rangle$	1794	1	$\langle w_6, w_{13}, w_{46}\rangle$	1870	1	$\langle w_2, w_5, w_{11}, w_{17}\rangle$
1914	1	$\langle w_2, w_3, w_{319}\rangle$	1938	1	$\langle w_3, w_{17}, w_{38}\rangle$	1938	1	$\langle w_6, w_{19}, w_{34}\rangle$
1974	1	$\langle w_2, w_3, w_{329}\rangle$	1974	1	$\langle w_2, w_{21}, w_{47}\rangle$	1974	1	$\langle w_3, w_7, w_{94}\rangle$
2010	1	$\langle w_2, w_3, w_{335}\rangle$	2010	1	$\langle w_5, w_6, w_{67}\rangle$	2130	1	$\langle w_2, w_5, w_{213}\rangle$
2130	1	$\langle w_2, w_{15}, w_{71}\rangle$	2130	1	$\langle w_3, w_{10}, w_{71}\rangle$	2130	1	$\langle w_3, w_{10}, w_{142}\rangle$
3210	1	$\langle w_2, w_3, w_5, w_{107}\rangle$	3458	1	$\langle w_2, w_7, w_{13}, w_{19}\rangle$	3486	1	$\langle w_2, w_3, w_7, w_{83}\rangle$
3542	1	$\langle w_2, w_7, w_{11}, w_{23}\rangle$	3930	1	$\langle w_2, w_3, w_5, w_{131}\rangle$	4002	1	$\langle w_2, w_3, w_{23}, w_{29}\rangle$
4278	1	$\langle w_2, w_3, w_{23}, w_{31}\rangle$	4326	1	$\langle w_2, w_3, w_7, w_{103}\rangle$	4710	1	$\langle w_2, w_3, w_5, w_{157}\rangle$
4890	1	$\langle w_2, w_3, w_5, w_{163}\rangle$						

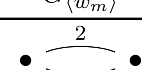
10. Appendix: Dual graphs and Kodaira symbols in depth one

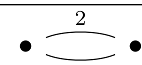
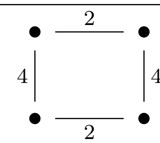
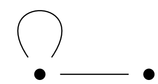
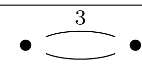
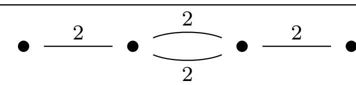
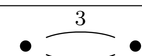
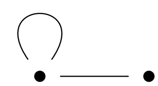
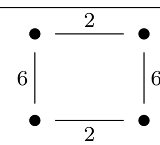
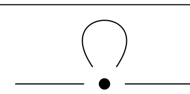
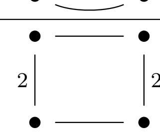
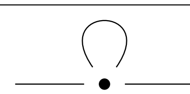
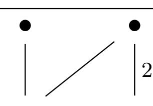
Let $X = X_0^D(N)/\langle w_m\rangle$ be an Atkin–Lehner quotient of genus 1 with $w_m \in W_0(D,N)$ a non-trivial involution. In Table 11, we draw the graph $G_{\langle w_m\rangle}$, as defined in Section 2.3, associated to X over $\mathbb{Z}_p$ for all primes $p \mid D$.

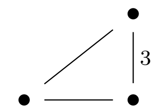
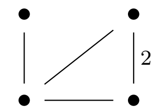
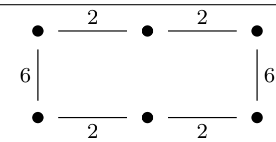
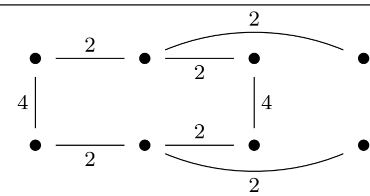
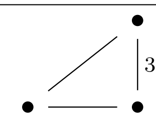
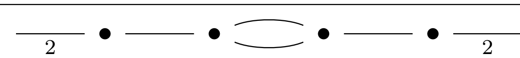
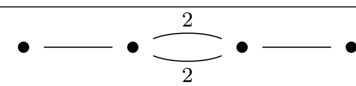
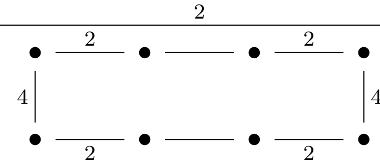
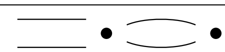
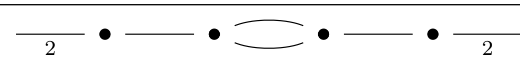
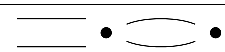
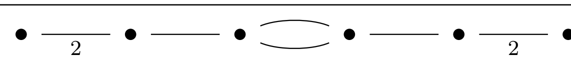
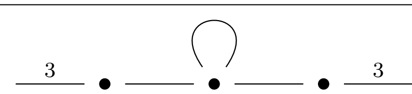
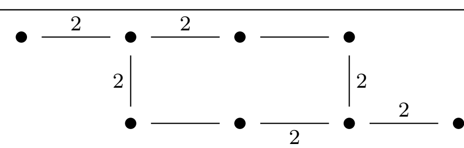
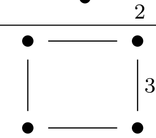
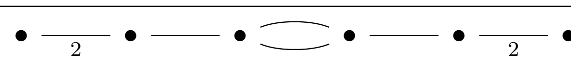
From these graphs, the Kodaira symbols for $J = \mathrm{Jac}(X)$ at all such primes are determined by minimizing and resolving. The isomorphism class of J is then determined just from this information and its isogeny class (see Proposition 5.2), as described in the proof of Theorem 6.2, for all but 5 such curves X. For these 5 curves, we list the two Cremona references for the isomorphism classes matching the Kodaira symbols of J in blue, and we strikethrough the one which we prove is incorrect by other means as discussed in the proof of the referenced theorem.

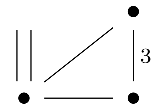
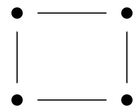
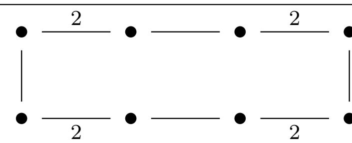
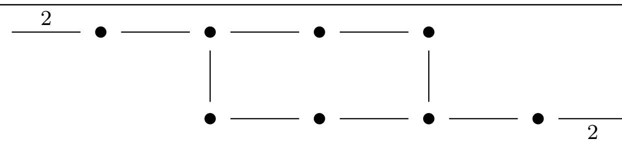
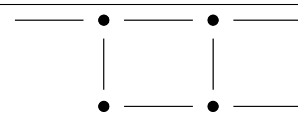
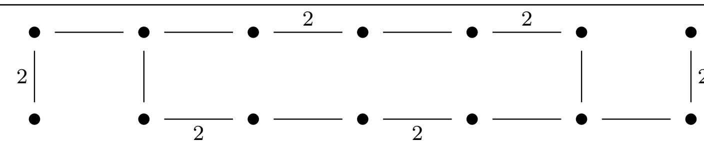
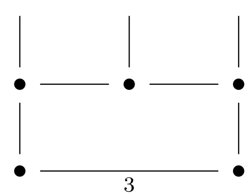
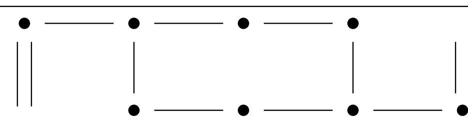
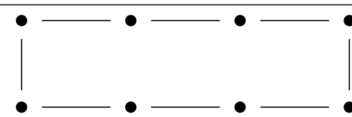
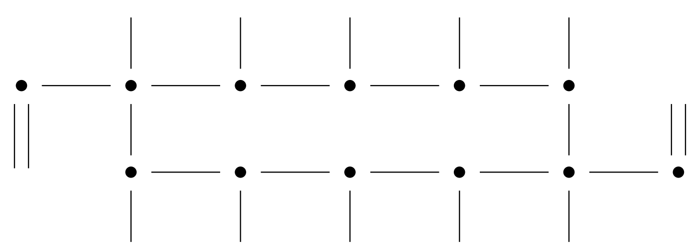
It is hoped that these explicitly worked examples will be useful to the reader seeking to gain comfort with Čerednik–Drinfeld uniformizations, given that there are not plentiful such examples in the literature (especially in the level of generality of this work, allowing squarefree $N > 1$ and general Atkin–Lehner subgroups W).

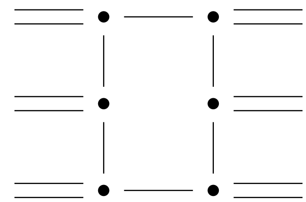
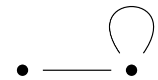
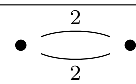
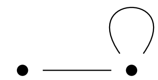
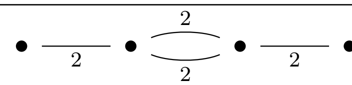
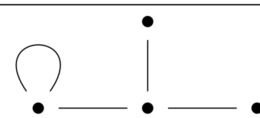
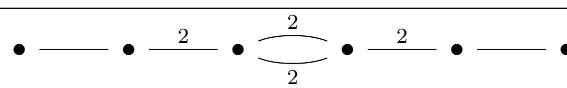
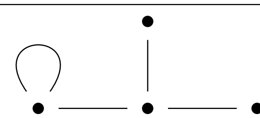
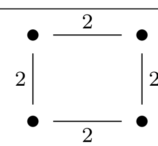
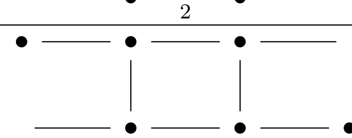
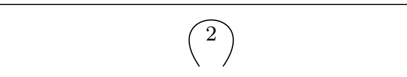
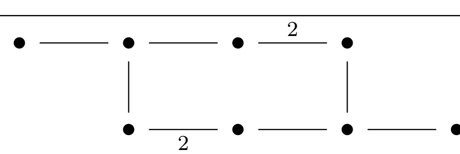
Table 11: Kodaira symbols at $p \mid D$ and the isomorphism class of $J = \mathrm{Jac}(X)$, via dual graphs of regular models over $\mathbb{Z}_p$, for all 129 Atkin–Lehner quotients $X = X_0^D(N)/\langle w_m\rangle$ with genus 1

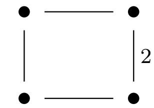
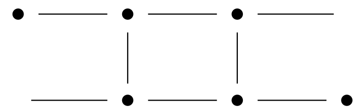
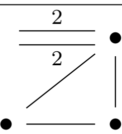
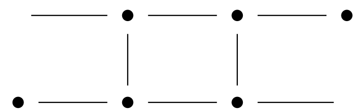
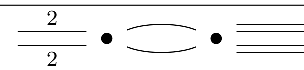
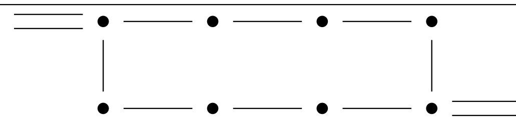
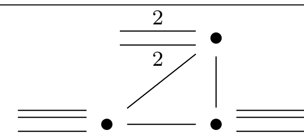
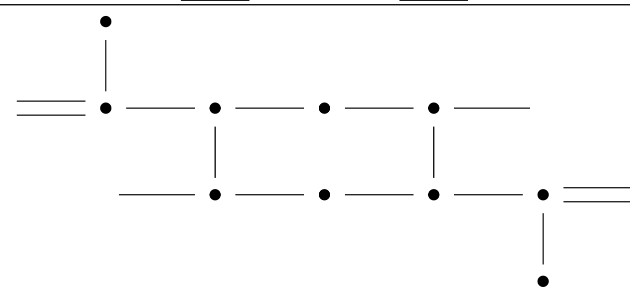
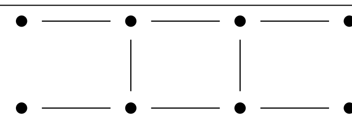
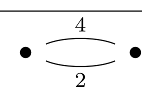
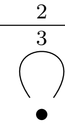
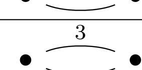
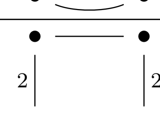
(D,N,m)	Class of J	p	$G_{\langle w_m\rangle}$	Symbol of J at p
$(6,5,3)$	30a8	2		I_3

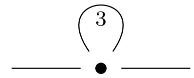
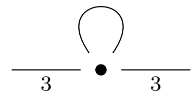
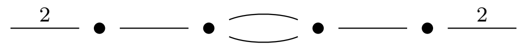
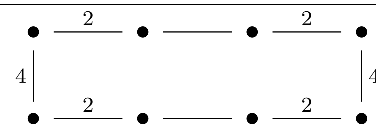
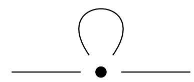
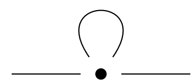
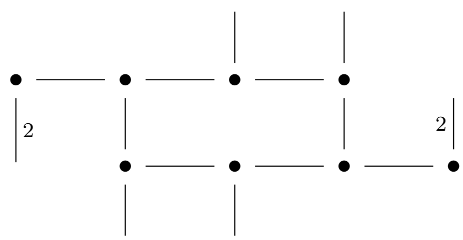
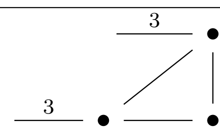
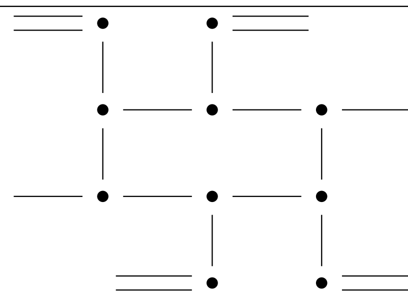
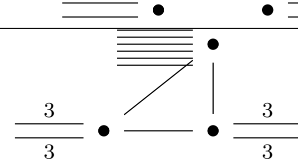
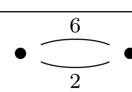
		3		I_1
$(6,5,5)$	30a7	2		I_3
		3		I_4
$(6,5,15)$	30a3	2		I_{12}
		3		I_1
$(6,7,2)$	42a5	2		I_1
		3		I_4
$(6,7,7)$	42a2	2		I_4
		3		I_4
$(6,7,14)$	42a6	2		I_1
		3		I_{16}
$(6,11,6)$	66c3	2		I_2
		3		I_1
$(6,11,22)$	66b2	2		I_2
		3		I_2
$(6,11,33)$	66a3	2		I_6
		3		I_1
$(6,13,6)$	78a4	2		I_4

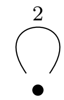
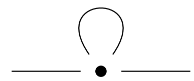
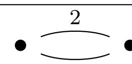
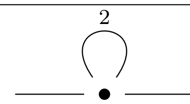
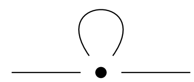
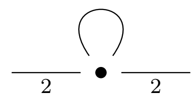
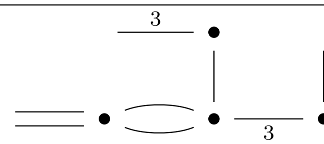
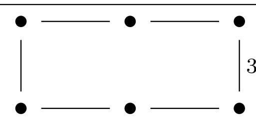
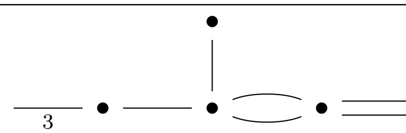
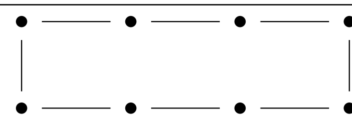
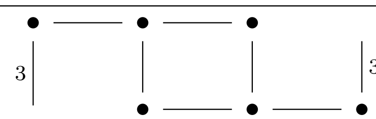
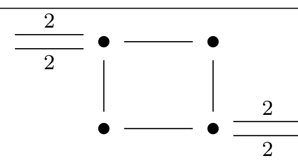
		3		I_5
$(6,13,26)$	78a3	2		I_4
		3		I_{20}
$(6,13,39)$	78a1	2		I_{16}
		3		I_5
$(6,17,2)$	102b3	2		I_2
		3		I_4
$(6,17,51)$	102c3	2		I_{18}
		3		I_2
$(6,17,102)$	102a1	2		I_2
		3		I_2
$(6,19,3)$	114a3	2		I_2
		3		I_1
$(6,19,19)$	114c2	2		I_{10}
		3		I_6
$(6,19,57)$	114b1	2		I_2

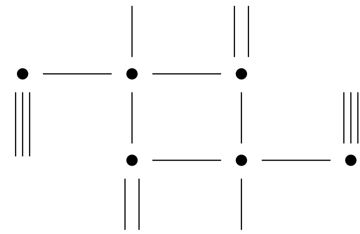
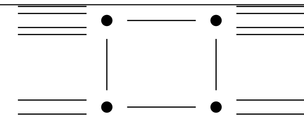
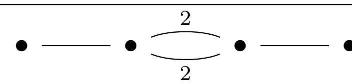
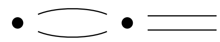
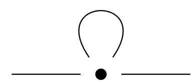
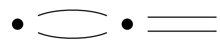
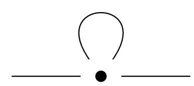
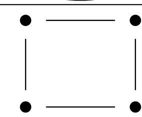
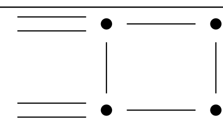
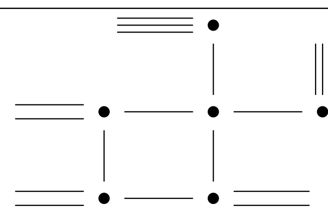
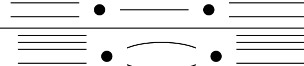
		3		I_5
$(6,23,46)$	138c2	2		I_2
		3		I_4
$(6,23,69)$	138b3	2		I_{12}
		3		I_2
$(6,23,138)$	138a1	2		I_2
		3		I_2
$(6,41,246)$	246d1	2		I_6
		3		I_4
$(6,43,129)$	258b1	2		I_{14}
		3		I_7
$(6,47,94)$	282a2	2		I_6
		3		I_8
$(6,71,426)$	426b1	2		I_{10}

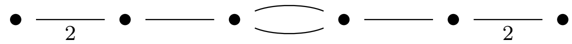
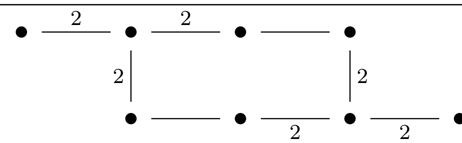
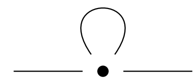
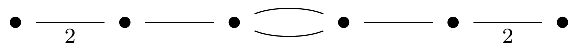
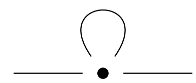
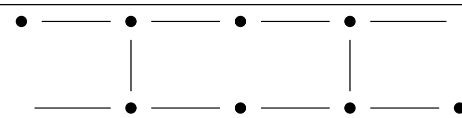
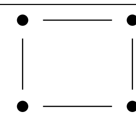
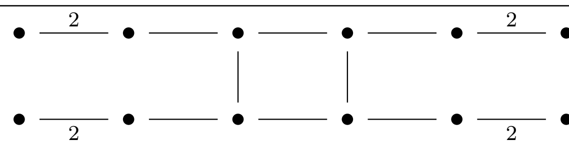
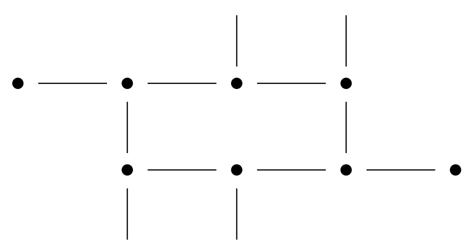
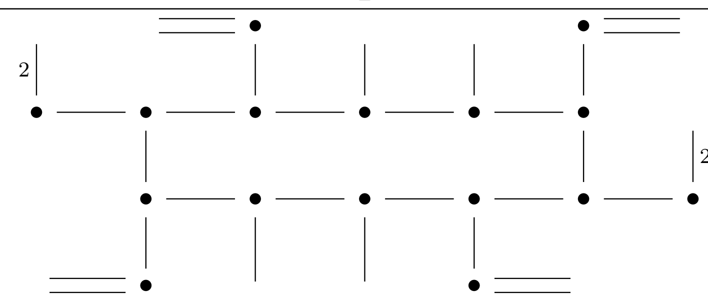
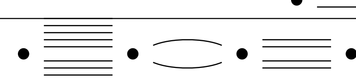
		3		I_6
$(10,3,6)$	30a5	2		I_1
		5		I_4
$(10,3,10)$	30a4	2		I_1
		5		I_1
$(10,3,15)$	30a1	2		I_4
		5		I_1
$(10,7,2)$	70a3	2		I_1
		5		I_2
$(10,7,7)$	70a1	2		I_4
		5		I_2
$(10,7,14)$	70a4	2		I_1
		5		I_8
$(10,13,10)$	130b2	2		I_4
		5		I_2
$(10,13,13)$	130c1	2		I_8

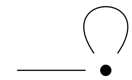
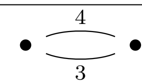
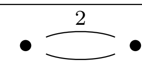
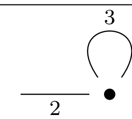
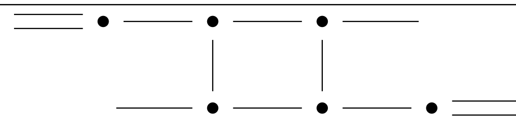
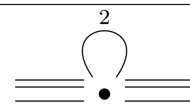
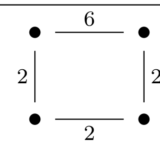
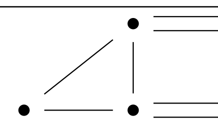
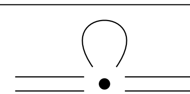
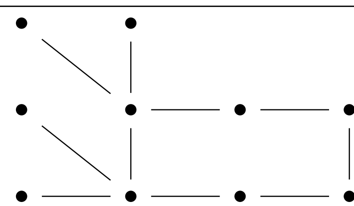
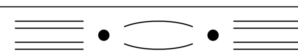
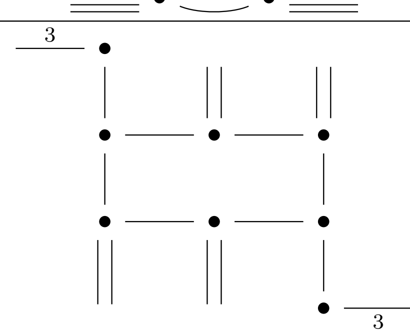
		5		I_5
$(10, 13, 130)$	130a1	2		I_4
		5		I_3
$(10, 17, 170)$	170a1	2		I_4
		5		I_2
$(10, 29, 290)$	290a1	2		I_8
		5		I_3
$(10, 31, 62)$	310a1	2		I_6
		5		I_4
$(14, 1, 7)$	14a1	2		I_6
		7		I_3
$(14, 3, 2)$	42a3	2		I_2
		7		I_4
$(14, 3, 21)$	14a1	2		I_6

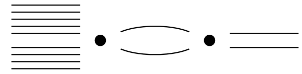
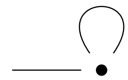
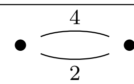
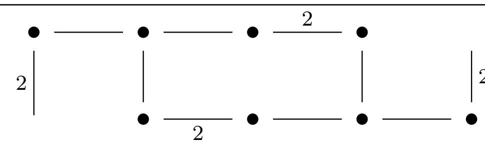
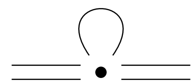
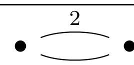
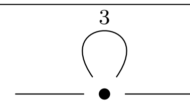
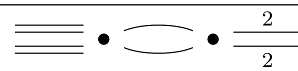
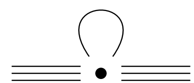
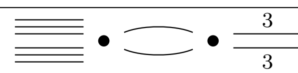
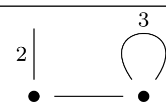
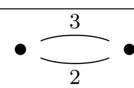
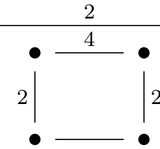
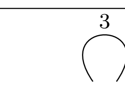
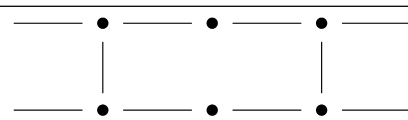
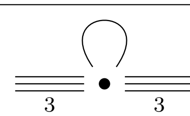
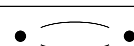
		7		I_1
$(14,3,42)$	14a4	2		I_2
		7		I_1
$(14,5,2)$	70a2	2		I_2
		7		I_2
$(14,5,35)$	14a3	2		I_{18}
		7		I_1
$(14,5,70)$	14a4	2		I_2
		7		I_1
$(14,13,182)$	14a1	2		I_6
		7		I_3
$(14,19,266)$	14a1	2		I_6
		7		I_3
$(15,1,5)$	15a1	3		I_8

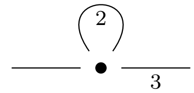
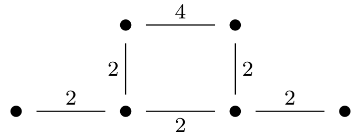
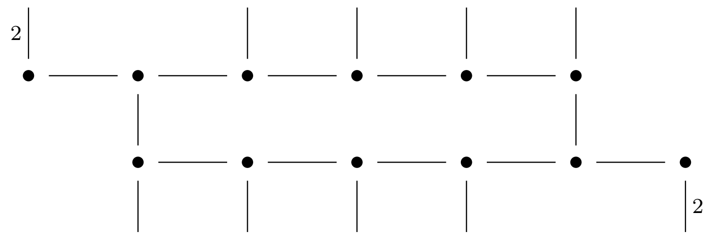
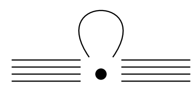
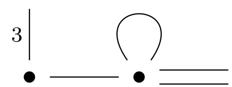
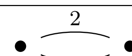
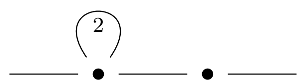
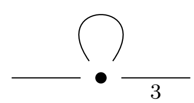
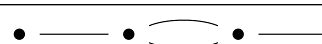
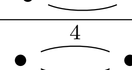
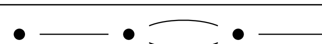
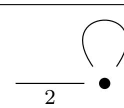
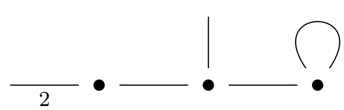
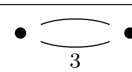
		5		I_2
$(15,2,3)$	30a3	3		I_1
		5		I_3
$(15,2,10)$	15a3	3		I_2
		5		I_2
$(15,2,30)$	~~15a7~~ ~~or~~ 15a8	3		I_1
		5		I_1
$(15,7,3)$	105a2	3		I_2
		5		I_2
$(15,7,7)$	15a2	3		I_8
		5		I_2
$(15,7,105)$	15a3	3		I_2
		5		I_2
$(15,11,55)$	15a2	3		I_8
		5		I_2
$(15,13,195)$	15a1	3		I_4
		5		I_4

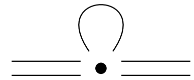
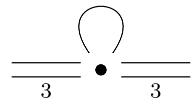
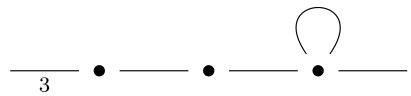
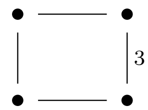
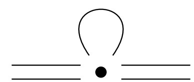
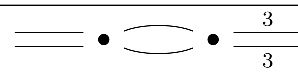
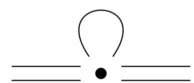
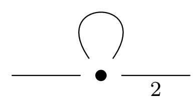
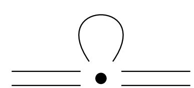
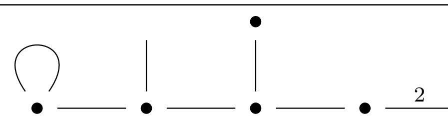
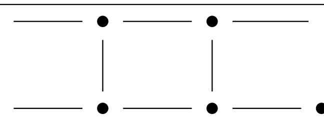
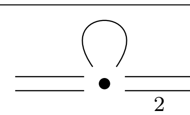
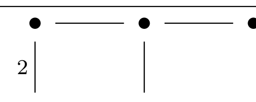
$(15, 17, 255)$	15a1	3		I_4
		5		I_4
$(21, 2, 2)$	21a1	3		I_4
		7		I_2
$(21, 2, 21)$	42a1 or ~~42a4~~	3		I_2
		7		I_1
$(21, 2, 42)$	21a4	3		I_2
		7		I_1
$(21, 5, 15)$	21a2	3		I_2
		7		I_4
$(21, 5, 21)$	105a2	3		I_2
		7		I_2
$(21, 5, 105)$	21a1	3		I_4
		7		I_2
$(21, 11, 231)$	21a1	3		I_4
		7		I_2

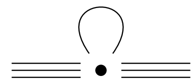
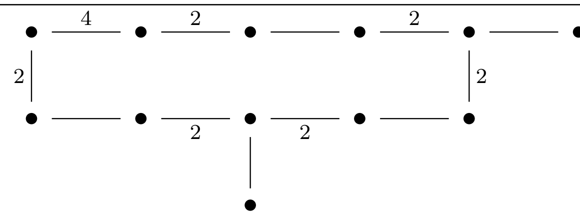
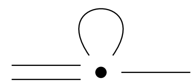
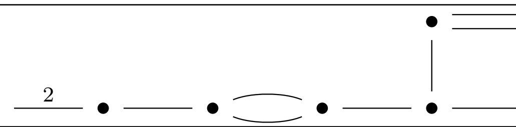
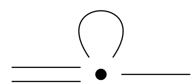
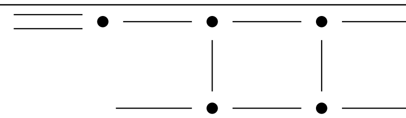
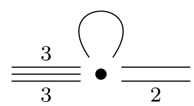
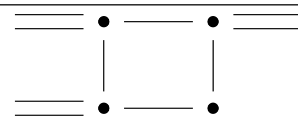
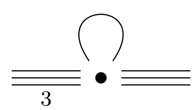
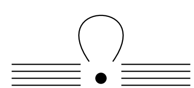
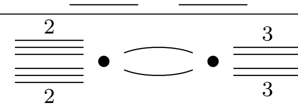
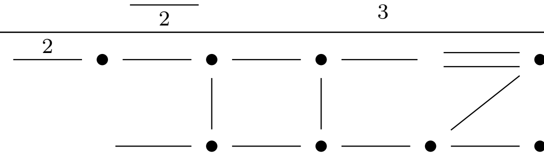
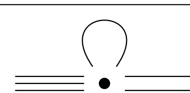
$(22, 3, 3)$	66b2	2		I_2
		11		I_2
$(22, 3, 11)$	66c1	2		I_{10}
		11		I_1
$(22, 3, 33)$	66a1	2		I_2
		11		I_1
$(22, 7, 14)$	154b2	2		I_6
		11		I_4
$(22, 7, 77)$	154c1	2		I_4
		11		I_2
$(22, 7, 154)$	154a1	2		I_6
		11		I_2
$(22, 17, 374)$	374a1	2		I_{10}
		11		I_2

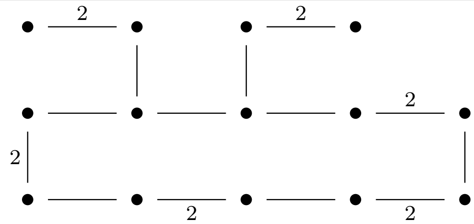
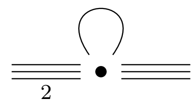
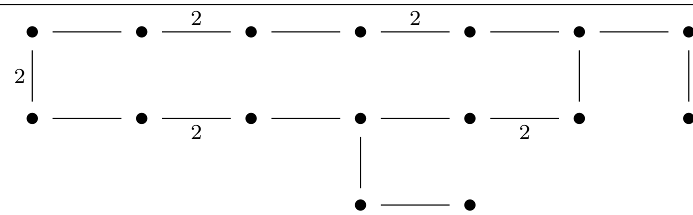
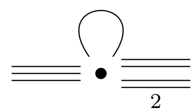
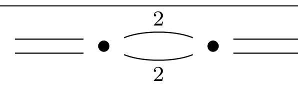
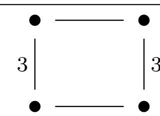
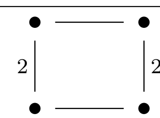
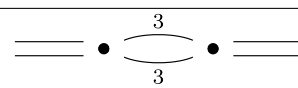
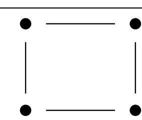
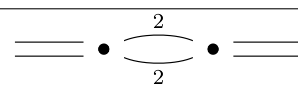
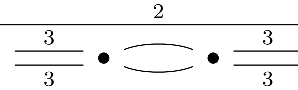
$(26, 1, 2)$	26b2	2		I_1
		13		I_7
$(26, 1, 13)$	26a1	2		I_3
		13		I_3
$(26, 5, 26)$	130b2	2		I_4
		13		I_2
$(33, 1, 11)$	33a4	3		I_{12}
		11		I_1
$(33, 2, 66)$	33a2	3		I_3
		11		I_1
$(33, 5, 55)$	33a1	3		I_6
		11		I_2
$(33, 7, 231)$	33a1	3		I_6

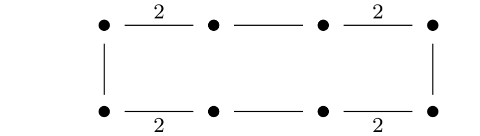
		11		I_2
$(34, 1, 2)$	34a4	2		I_1
		17		I_6
$(34, 3, 17)$	102b1	2		I_8
		17		I_1
$(35, 1, 7)$	35a1	5		I_3
		7		I_3
$(35, 2, 35)$	70a1	5		I_2
		7		I_1
$(35, 3, 35)$	105a2	5		I_2
		7		I_2
$(38, 1, 2)$	38b2	2		I_1
		19		I_5
$(38, 1, 19)$	38a1	2		I_9
		19		I_3
$(38, 3, 38)$	114a1	2		I_6
		19		I_1
$(39, 1, 13)$	39a1	3		I_2

		13		I_2
$(46, 1, 23)$	46a1	2		I_{10}
		23		I_1
$(46, 5, 230)$	46a1	2		I_{10}
		23		I_1
$(51, 1, 3)$	51a2	3		I_1
		17		I_3
$(55, 1, 5)$	55a1	5		I_2
		11		I_2
$(57, 1, 57)$	57a1	3		I_2
		19		I_1
$(58, 1, 2)$	58b1	2		I_2
		29		I_5
$(58, 1, 58)$	58a1	2		I_2
		29		I_1
$(62, 1, 2)$	62a3	2		I_1
		31		I_4

$(65, 1, 65)$	65a1	5		I_1
		13		I_1
$(69, 1, 3)$	69a2	3		I_1
		23		I_2
$(77, 1, 11)$	77c2	7		I_6
		11		I_1
$(77, 1, 77)$	77a1	7		I_2
		11		I_1
$(82, 1, 82)$	82a1	2		I_2
		41		I_1
$(85, 1, 17)$	85a1	5		I_2
		17		I_1
$(94, 1, 2)$	94a2	2		I_1
		47		I_2
$(106, 1, 106)$	106b1	2		I_4
		53		I_1
$(115, 1, 23)$	115a1	5		I_5

		23		I_1
$(118, 1, 59)$	118d1	2		I_{19}
		59		I_1
$(118, 1, 118)$	118a1	2		I_2
		59		I_1
$(122, 1, 122)$	122a1	2		I_4
		61		I_1
$(129, 1, 129)$	129a1	3		I_4
		43		I_1
$(143, 1, 143)$	143a1	11		I_1
		13		I_2
$(166, 1, 166)$	166a1	2		I_4
		83		I_1

$(178, 1, 89)$	178b1	2		I_{14}
		89		I_1
$(202, 1, 101)$	202a1	2		I_{17}
		101		I_1
$(210, 1, 30)$	210e5	2		I_2
		3		I_2
		5		I_4
		7		I_8
$(210, 1, 42)$	210a6	2		I_2
		3		I_2
		5		I_6
		7		I_6
$(210, 1, 70)$	210c3	2		I_2
		3		I_4
		5		I_4
		7		I_2

$(210, 1, 105)$	210b6	2		I_{12}
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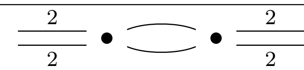
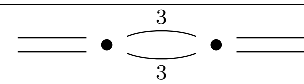
		5		I_2
		7		I_6
$(210, 1, 210)$	210d2	2		I_2
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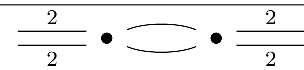
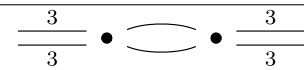
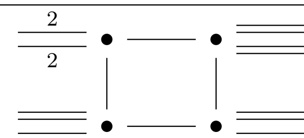
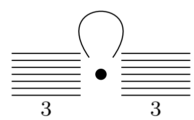
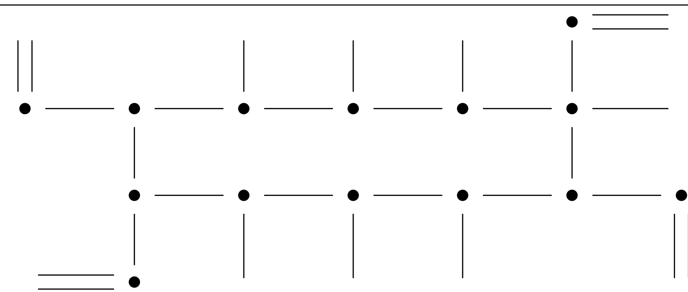
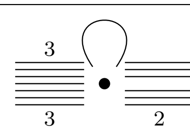
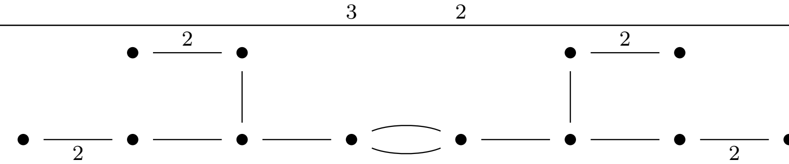
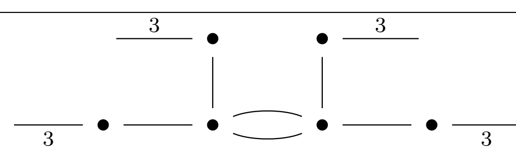
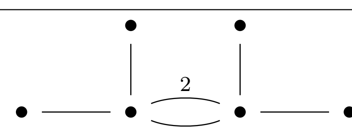
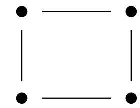
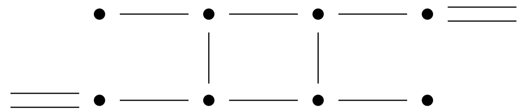
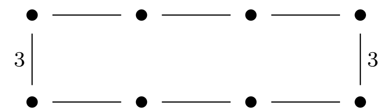
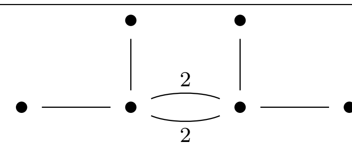
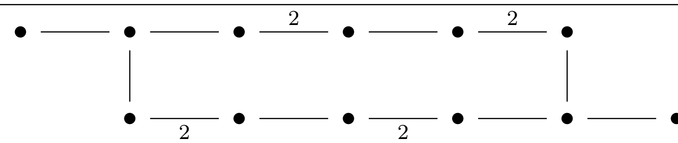
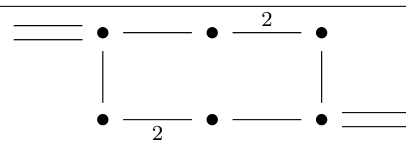
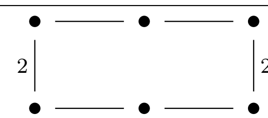
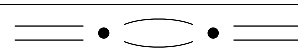
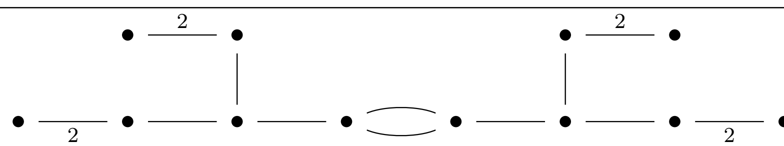
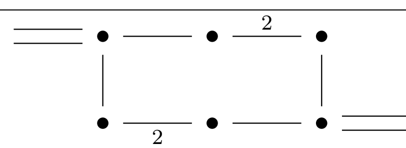
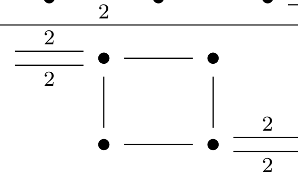
		5		I_2
		7		I_2
$(215, 1, 215)$	215a1	5		I_4
		43		I_1
$(314, 1, 314)$	314a1	2		I_{10}
		157		I_1
$(330, 1, 3)$	330b3	2		I_2
		3		I_2
		5		I_4

		11		I_4
$(330, 1, 22)$	330c3	2		I_4
		3		I_{12}
		5		I_4
		11		I_2
$(330, 1, 33)$	330d2	2		I_{14}
		3		I_{10}
		5		I_8
		11		I_2
$(330, 1, 165)$	330a2	2		I_2
		3		I_{10}
		5		I_4
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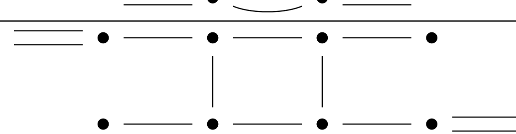
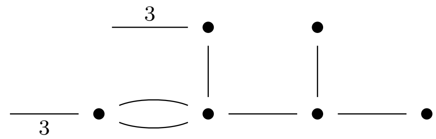
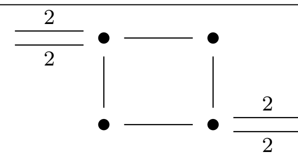
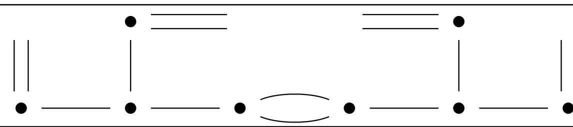
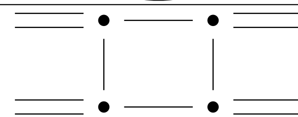
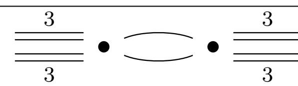
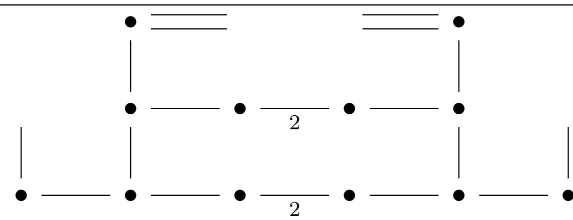
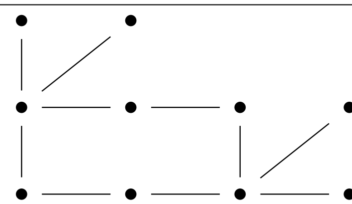
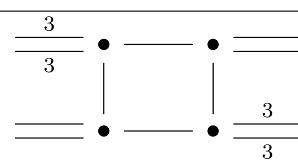
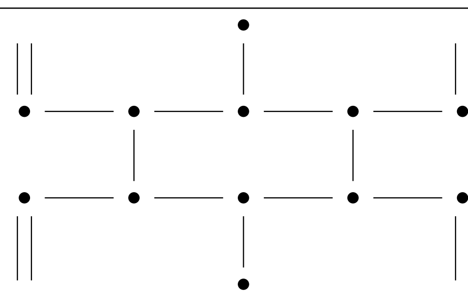
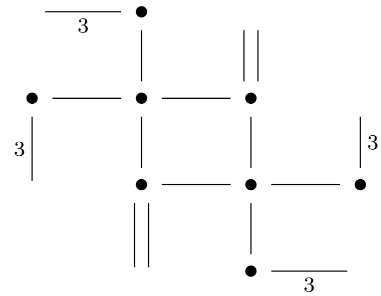
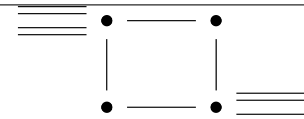
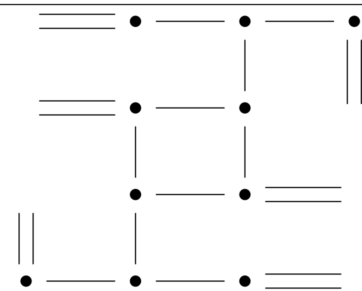
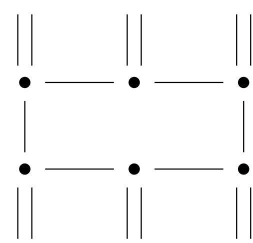
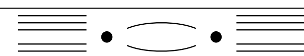
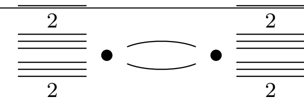
$(330, 1, 330)$	330e2	2		I_4

		3		I_2
		5		I_4
		11		I_2
$(390, 1, 390)$	390a2	2		I_2
		3		I_4
		5		I_2
		13		I_2
$(462, 1, 154)$	462b2	2		I_{10}
		3		I_6
		7		I_4
		11		I_2
$(510, 1, 510)$	510d2	2		I_6

		3		I_4
		5		I_4
		17		I_2
$(546, 1, 510)$	546c2	2		I_4
		3		I_6
		7		I_2
		13		I_2

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Oana Padurariu, Max-Planck-Institut für Mathematik Bonn, Germany
URL: https://sites.google.com/view/oanapadurariu/home
Email address: oana.padurariu11@gmail.com

Frederick Saia, University of Illinois Chicago, USA
URL: https://fsaia.github.io/site/
Email address: fsaia@uic.edu