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From Dark Radiation to Dark Energy: Unified Cosmological Evolution in K-essence Models

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Abstract

We study a class of Unified Dark Matter (UDM) models based on generalized K-essence, where a single scalar field with non-canonical kinetic terms accounts for dark radiation, dark matter, and dark energy. Starting from the purely kinetic Lagrangian proposed by Scherrer in [1], we extend the analysis to quadratic and exponential scalar potentials and explore their phenomenology. All models are implemented in a modified version of `Hi_CLASS` and confronted with data from *Planck* 2018, DESI DR1, and Big Bang Nucleosynthesis. The scenarios reproduce the full sequence of cosmic epochs: an early radiation-like phase, a matter-dominated era, and late-time accelerated expansion. The new models predict slightly higher values of the Hubble constant compared to Λ CDM, thereby partially alleviating the respective tensions from $\sim 4.4\sigma$ to $\sim 3.4\sigma$. The quadratic potential requires an ultralight mass that makes it effectively indistinguishable from the Scherrer solution. Overall, generalized K-essence provides a minimal and observationally viable realization of UDM, offering a unified description of the dark sector with distinctive signatures in both early- and late-time cosmology.

1 Introduction

The Λ CDM framework has long served as the reference model in cosmology, providing an excellent fit to many key observations, including the cosmic microwave background (CMB), baryon acoustic oscillations (BAO), and large-scale structure. Its minimal parameter set and consistency with early-Universe probes have established it as the prevailing benchmark for precision cosmology. However, recent analyses, particularly those incorporating the latest DESI measurements [2], indicate that the cosmological constant may not fully account for the data. Another notable discrepancy in the cosmology measurements is the H_0 tension. Which refers to the $\sim 5\sigma$ difference between the Hubble constant inferred from Planck measurements of the CMB [3] and local distance-ladder determinations such as those reported by the SH0ES collaboration [4]. This would also be relaxed by a change in the dark sector behavior [5]. This has renewed interest in exploring alternative scenarios capable of addressing emerging tensions between early- and late-time observables.

Scalar field theories provide a natural framework for such extensions. Canonical quintessence scenarios [6, 7, 8] describe dark energy through a scalar potential, while generalized models with non-canonical kinetic terms, known as K-essence [9, 10, 11, 12], broaden the phenomenology and allow cosmic acceleration to arise from the kinetic structure itself. A minimal and analytically tractable realization is the purely kinetic model proposed by Scherrer [1], in which the scalar Lagrangian is expanded quadratically around a background value X_0 . This construction allows the field to behave as radiation at early times, as matter at intermediate epochs, and as vacuum energy at late times, thereby realizing the unified dark matter (UDM) picture. Related approaches have shown that tachyon and Chaplygin-type scenarios [13], models unifying inflation with dark matter and dark energy [14], and more general K-essence functions with a minimum [15] all fall within this broad class, underscoring its generality and robustness [16].

Despite its interesting properties, a purely kinetic model reduces to a barotropic fluid model which reduces the rich phenomenology of a field theory of the dark sector [17]. In this work, we revisit the Scherrer's model and consider two extensions that incorporate scalar potentials: a quadratic potential and an exponential one, that are introduced as additive terms in the Lagrangian as adopted previous works [18, 19, 20]. These scenarios allow us to test if potentials can provide a richer phenomenology or leave distinctive observational imprints.

Our analysis has three main objectives: (i) to test the observational viability of the Scherrer model and its potential-extended generalizations; (ii) to quantify their impact on key cosmological parameters, including ω_{dm} , H_0 , and S_8 ; and (iii) to identify distinctive features in the background and perturbation dynamics, with particular emphasis on the early-time relativistic phase of the scalar field and its imprint on N_{eff} . These results are then compared with current data and critically assessed in light of their ability to address the H_0 and S_8 tensions.

The remainder of this paper is organized as follows. In Section 2 we introduce the theoretical framework of generalized K-essence as a realization of Unified Dark Matter, starting from the Scherrer solution and extending it with quadratic and exponential potentials. Section 3 describes the observational datasets and the numerical methodology, including the implementation of the models in **Hi.CLASS** and the parameter inference with **MontePython**. Our main results are presented in Section 4, where we discuss the background evolution, perturbation behavior, and cosmological constraints for the different scenarios. Finally, Section 5 summarizes our findings, assesses their implications for current cosmological tensions, and outlines possible directions for future work.

2 K-Essence Models in Cosmology

K-essence theories provide a general class of scalar field models characterized by non-canonical kinetic terms. Originally developed to drive inflation [9], they were later extended to describe dark energy and unified dark sector dynamics [10, 11].

The action for a minimally coupled K-essence field reads:

$$\mathcal{L} = \int d^4x \sqrt{-g} \left(\frac{R}{2\kappa^2} + G_2(X, \phi) + \mathcal{L}_m \right), \quad (1)$$

where $G_2(X, \phi)$ is the scalar field Lagrangian, $\mathcal{L}_m$ corresponds to standard matter, and $\kappa^2 = 8\pi G$. We assume a functional form

$$G_2(X, \phi) = F(X) - V(\phi), \quad (2)$$

with $X \equiv -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$. This decomposition is widely adopted in scalar field cosmologies and effective field theory approaches [18, 19, 20, 21, 22].

This field has an energy-momentum tensor of:

$$T^\mu_\nu = G_{2,X}\partial^\mu\phi\partial_\nu\phi + \delta^\mu_\nu G_2, \quad (3)$$

with energy density and pressure:

$$\rho_\phi = -G_2 + 2XG_{2,X}, \quad P_\phi = G_2. \quad (4)$$

In a flat Friedmann-Lemaître-Robertson-Walker (FLRW) background, the field evolution is governed by the generalized Klein-Gordon equation:

$$(F_X + 2XF_{XX})\phi'' + 2\mathcal{H}(F_X - XF_{XX})\phi' = a^2V_\phi, \quad (5)$$

where primes denote conformal time derivatives and $\mathcal{H} = a'/a$.

2.1 Purely Kinetic K-Essence

In the absence of a scalar potential, the dynamics of the field are governed entirely by the non-canonical kinetic term $F(X)$. For purely kinetic, the scalar field evolution admits a first integral of motion given by:

$$XF_X^2 = ka^{-6}, \quad (6)$$

where k is a constant of integration. This describes how X evolves as the Universe expands.

If $F(X)$ has a minimum at a certain value X_0 , it can be approximated as a quadratic function close to this minimum. This led Scherrer in [1] to introduce the purely kinetic quadratic model

$$F(X) = -F_0 + F_2(X - X_0)^2, \quad (7)$$

where F_0 , F_2 , and X_0 are constants. In the regime where $(X - X_0)/X_0 \ll 1$, the kinetic term X evolves as:

$$X = X_0 \left[1 + \frac{\theta}{a^3} \right], \quad (8)$$

where θ is a constant dependent on the initial conditions of the field, and $\theta/a^3 \ll 1$ for this approximation to be valid. The corresponding energy density takes the form

$$\rho_\phi \simeq F_0 + 4F_2 X_0^2 \frac{\theta}{a^3}, \quad (9)$$

which represents a superposition of a cosmological constant and a pressureless matter component. Identifying these contributions with present-day density parameters, we obtain

$$F_0 = \Omega_{F_0} \rho_c^{(0)}, \quad F_2 = \frac{\Omega_{\text{dm}} \rho_c^{(0)}}{4X_0^2 \theta}, \quad (10)$$

where Ω_{F_0} denotes the present-day density fraction associated with the constant term (effectively playing the role of dark energy), and $\rho_c^{(0)} = 3H_0^2/(8\pi G)$ is the critical density today.

2.2 Early-Time Evolution and BBN Constraints

In the early Universe, when the scale factor satisfies $a \ll 1$, the kinetic term is much larger than its present value, i.e., $X \gg X_0$. Under this approximation, the asymptotic behavior of the solution becomes:

$$X(a) \approx X_0 \left(1 + \frac{\theta^{2/3}}{a^2} \right). \quad (11)$$

In this limit, the scalar field dynamics are dominated by the quadratic kinetic term. The energy density then scales as $\rho_\phi \propto a^{-4}$, mimicking the behavior of a radiation fluid.

During the early Universe, the scalar field behaves as a radiation-like component with equation of state $w_\phi = 1/3$. In this regime, it contributes to the total radiation density and thus to the parameter N_{eff} . The latter is tightly constrained by primordial nucleosynthesis and the CMB, which are sensitive to any additional relativistic degrees of freedom photons and the three neutrino families. Therefore, it is essential to quantify the scalar contribution to N_{eff} in order to ensure that the radiation-like density remains compatible with BBN and recombination constraints.

Defining the total relativistic density of the universe as proportional to the photon density ρ_γ , gives

$$\rho_{\text{rel}} = \left[1 + \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} N_{\text{eff}} \right] \rho_\gamma, \quad (12)$$

with $N_{\text{eff}} \simeq 3.046$ for the 3 standard model neutrino flavors in the absence of other relativistic components [23, 24, 25, 26]. In our case, the scalar field adds an extra component to the N_{eff} as

$$\Delta N_{\text{eff}} = \frac{8}{7} \left(\frac{11}{4} \right)^{4/3} \left(\frac{3P_\phi}{\rho_\gamma} \right), \quad (13)$$

This expression makes explicit how the scalar field modifies N_{eff} during the radiation era. From the solution (11) and using F_2 from eq. (10), we see that

$$\Delta N_{\text{eff}} \simeq \frac{6}{7} \left(\frac{11}{4} \right)^{4/3} \frac{\Omega_{\text{dm}} \theta^{1/3}}{\Omega_\gamma}, \quad (14)$$

which explicitly depends on the initial conditions parameter θ . In section 4.1, we will use this modification on the relativistic degrees of freedom to constrain the model against observations, where we will use a more precise numerical solution and the expression (13) to determine N_{eff} .

2.3 Extensions with Scalar Potentials

Adding a non zero scalar potential to the Lagrangian (2) can add new phenomenology to the scalar field models. Here consider two potential forms:

$$V_{\text{exp}}(\phi) = V_0 e^{-\lambda\phi}, \quad (15)$$

$$V_m(\phi) = \frac{1}{2} m_\phi^2 \phi^2. \quad (16)$$

From the full Klein-Gordon equation, the evolution of X obeys:

$$X' = \frac{\phi' V_\phi - 6\mathcal{H}F_X}{F_X + 2XF_{XX}}, \quad (17)$$

which now its not solvable as the purely kinetic model. In order obtain cosmological predictions from this model and to constrain its parameters, in the next section we will solve equation (17) numerically with the help of `Hi_CLASS` [27, 28, 29]. It will solve the background field equation (17) with the condition that ρ_ϕ today be equal to the dark sector component

$$\frac{\rho_\phi^{(0)}}{\rho_{\text{crit}}^{(0)}} = \Omega_\Lambda + \Omega_{dm}. \quad (18)$$

To satisfy this condition the code performs a shooting method where the first step is approximated by the purely kinetic early universe conditions

$$X' = \frac{\phi' V_\phi / F_2 - 12\mathcal{H}X(X - X_0)}{6X - 2X_0}. \quad (19)$$

where F_0 and F_2 come from equation (10). And the subsequent iterations modify the initial conditions of the field until the condition (18) is satisfied.

3 Data and Methodology

This section describes the observational datasets and numerical tools employed to test the K-essence models introduced in Section 2. We aim to assess the viability of both purely kinetic and potential-extended scenarios by confronting them with current cosmological observations spanning early and late times.

Observations include baryon acoustic oscillation (BAO) measurements from the first data release of the Dark Energy Spectroscopic Instrument (DESI DR1) [2]. These measurements constrain the angular diameter distance $D_A(z)$ and the Hubble distance $D_H(z)$ across seven redshift bins extending to $z \sim 1.7$. The dataset includes the full covariance matrix between D_A and D_H , providing robust sensitivity to the late-time expansion history and the shape of the matter power spectrum.

For early Universe constraints, we use the *Planck* 2018 legacy release (PR3), incorporating temperature and polarization anisotropies (TT, TE, EE), as well as the CMB lensing potential power spectrum [3].

In all K-essence scenarios, the scalar field behaves as a radiation-like fluid during the early Universe, contributing non-negligibly to the total energy density before the matter-radiation equality and thereby affecting the inferred value of N_{eff} and therefore the nucleosynthesis processes.

To constrain the radiation content at early times, we include Big Bang Nucleosynthesis (BBN) likelihoods in our analysis, making use of both the primordial helium abundance Y_p and the primordial deuterium fraction D/H. In practice, we adopt the implementation provided in `MontePython`, which combines measurements of deuterium from high-redshift quasar absorption systems [30], helium from recombination lines [31], and helium from emission line measurements in metal-poor H₂ regions [32]. This combined dataset implies the conservative upper bound $N_{\text{eff}} < 3.15$ at 95% C.L., as summarized in Ref. [33], which we adopt as a prior in our analysis.

Theoretical predictions for the CMB power spectra and the linear matter power spectrum are computed with `Hi_CLASS` [27, 28, 29]. In our analysis we work within the shift-symmetric subclass already supported by `Hi_CLASS`, $G_2(X, \phi) = F(X) - V(\phi)$, and implement specifically the Scherrer kinetic function together with the quadratic and exponential potentials. The modified code evolves both the background and linear perturbations consistently, using exact solutions at early times as initial conditions when appropriate. In our analysis, we fix the effective number of relativistic neutrino species to $N_{\text{eff},\nu} = 3.046$ while the contribution to the relativistic degrees of freedom varies with time and with the initial conditions of the field.

An important aspect of the quadratic potential implementation is the choice of an ultralight scalar field mass. This follows from basic order of magnitude arguments. If we assume that the field starts at the minimum of its potential at early times, $V(t=0) = 0$, which is equivalent to setting $\phi(t=0) = 0$ as an initial condition. In the regime where the kinetic term X remains close to its minimum value X_0 , which we will show it's the case to satisfy the BBN constrain, the field evolves approximately as $\phi(t) \simeq \sqrt{2X_0} t$. The potential energy at the present epoch then takes the

form $V(\phi_0) \simeq m_\phi^2 X_0 t_0^2$. Requiring that this does not exceed the observed dark energy density, $V_{\max} \simeq 0.7\rho_c \simeq 6 \times 10^{-11} \text{ eV}^4$, yields an upper limit on the scalar field mass:

$$m_\phi \lesssim \left(\frac{V_{\max}}{X_0 t_0^2} \right)^{1/2} \simeq \frac{1.18 \times 10^{-38} \text{ eV}^3}{\sqrt{X_0}}, \quad (20)$$

where we have used $t_0 \approx 6.57 \times 10^{32} \text{ eV}^{-1}$ as the age of the Universe. This estimate is consistent with the values naturally obtained in our MCMC chains and with similar results in related quintessence models [34]. In practice, we fix $m_\phi = 10^{-38} \text{ eV}$, which both respects this physical prior and ensures numerical stability in `Hi_CLASS`. This choice maintains the potential subdominant throughout cosmic history, allowing it to influence the dynamics only at very late times.

Bayesian parameter inference is performed using the Markov Chain Monte Carlo (MCMC) engine `MontePython` [35, 36], with our modified `Hi_CLASS`. Beside the nuisance parameters, we sample the standard 6 Λ CDM parameters together with θ for the purely kinetic case and (θ, V_0, λ) or (θ, m_ϕ) for the exponential and quadratic potentials respectively. The total likelihood function is given by the product of the Planck, DESI BAO, and BBN contributions. Posterior distributions are analyzed using `GetDist` [37], which provides high-precision kernel density estimation and visualization tools. This framework allows for a consistent exploration of how K-essence dynamics affect both the expansion history and the growth of structure, while remaining compatible with constraints on radiation content and primordial nucleosynthesis.

Finally, we emphasize that a consistent treatment of K-essence cosmologies requires evolving both the background and linear perturbations. In particular, the clustering properties of the scalar field can leave imprints on the matter power spectrum and CMB anisotropies, making the perturbative sector essential for testing these models against observations. A comprehensive overview of perturbation dynamics in K-essence and related scalar field scenarios can be found in the review by Amendola and Tsujikawa [38] and [39], which highlights their relevance for distinguishing these models from Λ CDM.

4 Results and Discussion

This section presents the main results of our analysis, including both background and linear perturbation evolution, as well as cosmological parameter constraints. We assess the viability of generalized K-essence models as alternatives to the Λ CDM paradigm in the light of current observational data, with three different Lagrangians, the purely kinetic quadratic Lagrangian originally introduced by Scherrer in [1] and our extended versions incorporating exponential and quadratic potentials.

4.1 Background Evolution

We begin with the purely kinetic model, which provides a minimal realization of a unified dark sector. Figure 1 shows the redshift evolution of the scalar field equation of state w_ϕ for various values of the initial condition θ . At large redshifts, the field initially behaves as a radiation fluid ($w_\phi \rightarrow 1/3$) that later transitions to a matter behavior ($w_\phi \simeq 0$) before recombination, and eventually drives cosmic acceleration through the constant term F_0 , which acts effectively as a cosmological constant.

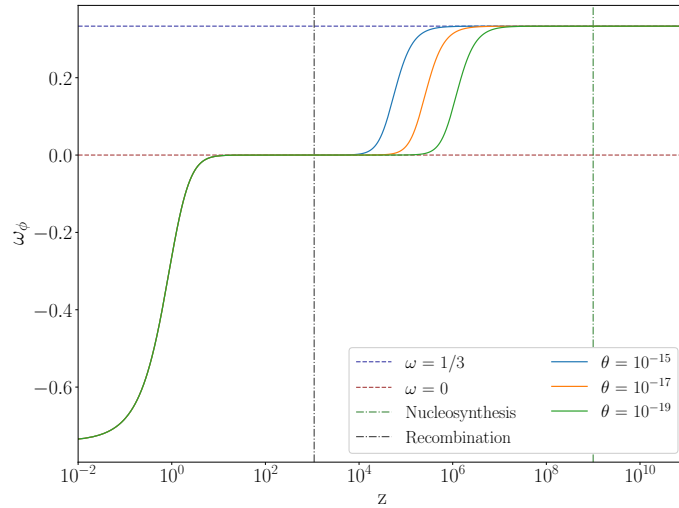


Figure 1: Redshift evolution of the scalar field equation of state w_ϕ for various values of θ . Vertical dashed lines indicate the epochs of BBN and recombination. We see that the field behaves as radiation at high redshifts, transitioning to a matter-like behavior before recombination. Therefore the bounds over extra radiation components on BBN are very important to constrain the model.

In the early Universe, the scalar field contributes as an additional relativistic species, enhancing the total radiation density. As discussed in Sec. 3, this behavior is constrained by Big Bang Nucleosynthesis (BBN), which imposes the bound $N_{\text{eff}} < 3.15$ at 95% C.L. [40, 31, 32, 33]. This bound leads to a $\Delta N_{\text{eff}} < 0.104$ for the contribution of the K-essence field to the early radiation density which from (14) give a bound to the initial condition parameter of

$$\theta < 2.97 \times 10^{-16}, \quad (21)$$

for the purely kinetic model. This condition comes only from the BBN observations and order of magnitude assumptions for Ω_γ and Ω_{dm} , and is close to the condition obtained in the next subsection combining data from Planck+DESI DR1+BBN, which gives $\theta \leq 1.97 \times 10^{-16}$ at 95% C.L. as derived from our MontePython chains.

From equation (8) evaluated at recombination we see that the field needs to be very close to its minimum already at this epoch

$$\frac{X(z_{\text{rec}}) - X_0}{X_0} \leq 2.63 \times 10^{-7} \quad (22)$$

Figure 2 illustrates the evolution of N_{eff} for representative values of θ . The gray band marks the range allowed by BBN, showing that the relativistic phase of the field must fade sufficiently early so that its contribution vanishes before recombination, restoring the standard value of N_{eff} .

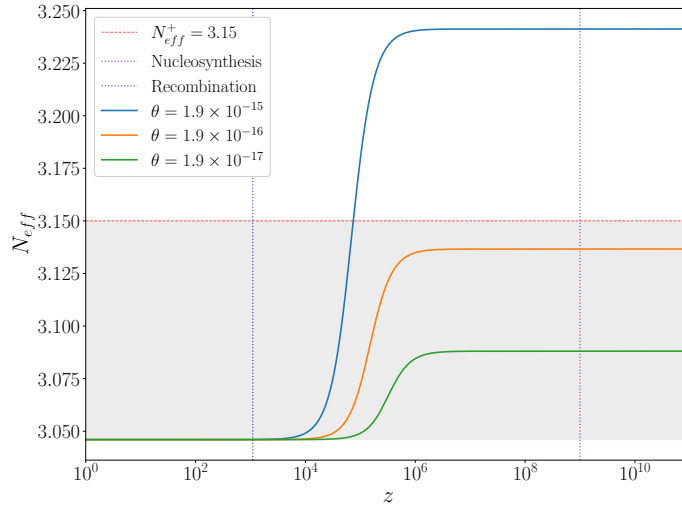


Figure 2: Redshift evolution of the effective number of relativistic species N_{eff} . The gray shaded region denotes the upper bound from BBN constraints [40, 31, 32, 33]. As the scalar field becomes non-relativistic, its contribution to the radiation density vanishes, restoring the standard value of N_{eff} .

4.2 Perturbations

The behavior of linear perturbations in K-essence models reveals distinct signatures in the matter power spectrum $P(k)$, driven by the background dynamics of each scenario. Throughout this section we fix $X_0 = (0.12 \text{ eV})^4$. This choice ensures direct comparability between the models and avoids introducing additional parameter degeneracies as X_0 is very poorly constrained by the data. In the purely kinetic scenario, the qualitative shape of $P(k)$ is insensitive to the specific choice of X_0 , whereas in models with scalar potentials the variation of X_0 can affect the small-scale behavior. The latter case will be further discussed in the parameter constraints analysis.

Figure 3 summarizes the linear matter power spectra at $z = 0$ for the three K-essence scenarios. The three cases present a suppression in the power spectra above a certain k , corresponding to small scale perturbations. Panel (a) shows the purely kinetic case, where varying the initial condition parameter θ modulates the scale of the suppression: larger θ values delay the transition from radiation-like to matter-like behavior as can be seen in figure 1, suppressing the perturbations on small scales. Conversely, smaller θ moves the transition to earlier times, recovering the Λ CDM shape at high k 's. This cutoff resembles that found in fuzzy dark matter models, but here it originates purely from the kinetic sector, without invoking scalar masses. A similar effect occurs on fuzzy dark matter [41, 42] and ultralight scalar field models [43, 44] but in that case it is driven by the Compton wavelength of the field.

Panel (b) corresponds to the exponential potential model $V(\phi) = V_0 e^{-\lambda\phi}$, where the slope λ controls the late-time dynamics. Small λ values enhance the effect of the potential, producing stronger suppression at intermediate and small scales. As λ increases, the potential becomes negligible and the evolution approaches the kinetic limit, recovered in the limit $\lambda \rightarrow \infty$.

Panel (c) illustrates the quadratic potential $V(\phi) = \frac{1}{2}m_\phi^2\phi^2$, where the scalar mass m_ϕ strongly impacts perturbation evolution. For $m_\phi \leq 10^{-37} \text{ eV}$, rapid oscillations appear in $P(k)$, producing a sharp suppression of the perturbations on small scales. Masses below 10^{-38} eV yield spectra nearly indistinguishable from the purely kinetic case, as the potential remains subdominant until very late times. This behavior is consistent with the bound derived in Section 3.

On large scales, all models reproduce the same structure as Λ CDM, ensuring compatibility with current observations. Differences arise mainly at small scales, where the transition dynamics and potential contributions modulate the growth of perturbations, leading to a characteristic suppression in $P(k)$. These deviations, while modest at present sensitivity, represent potential observational signatures for next-generation surveys. In particular, forthcoming galaxy surveys such as *Euclid* [45], the Vera C. Rubin Observatory's Legacy Survey of Space and Time (LSST) [46], and the Dark Energy Spectroscopic Instrument (DESI DR2) [47], together with upcoming CMB experiments like CMB-S4 [48] and the Simons Observatory [49], will achieve the precision needed to probe the characteristic suppressions in $P(k)$ predicted by K-essence scenarios. Finally, figure 4 presents the predicted CMB temperature power spectra for all K-essence scenarios, alongside Λ CDM which presents smaller deviations than for the matter power spectrum, but relevant due to

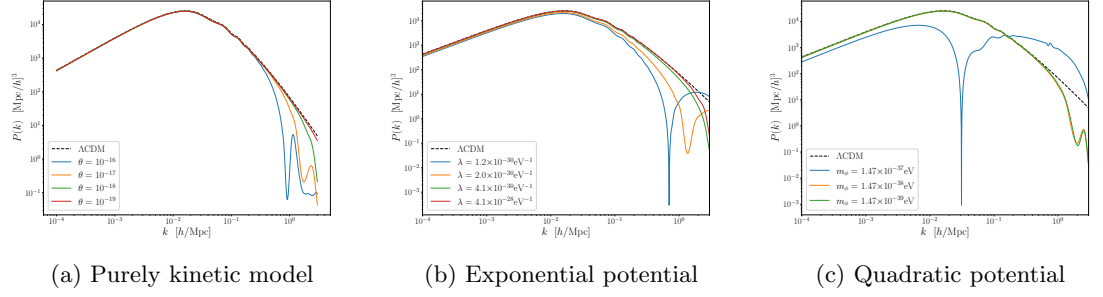


Figure 3: Linear matter power spectrum at $z = 0$ for the three K-essence scenarios. **(a)** Purely kinetic model: larger θ values delay the radiation-to-matter transition and suppressing small scale perturbations (high k values), while smaller θ 's recover the Λ CDM shape. **(b)** Exponential potential: small λ values suppress the perturbations at small scales for a wider range of k 's, while large λ recover the kinetic limit. The results are shown for $\theta = 10^{-7}$ and $V_0 = 10^{-3} \text{ eV}^4$. **(c)** Quadratic potential: large scalar masses induce oscillations and strong suppression, while ultralight masses $m_\phi \leq 10^{-38} \text{ eV}$ approach to the purely kinetic behavior. Here we fixed $\theta = 10^{-7}$.

the high precision at which the Planck team has measured the CMB power spectrum.

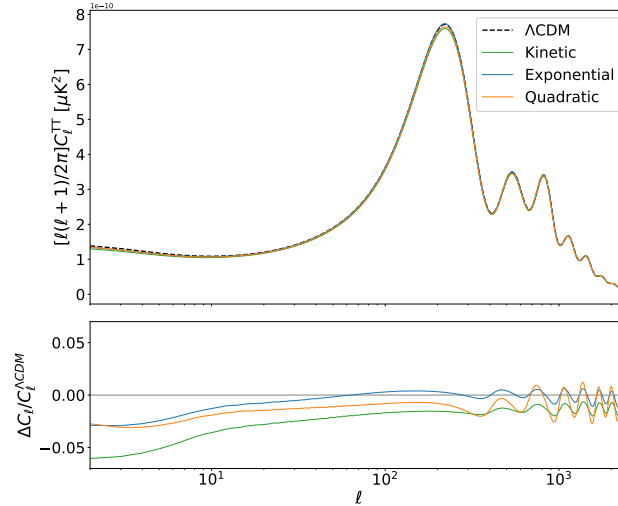


Figure 4: CMB temperature power spectrum C_ℓ^{TT} (upper panel) and relative deviation from Λ CDM (lower panel). All K-essence models agree with *Planck* data across all multipoles, with sub-percent differences at high ℓ . The curves are obtained using the mean values of the cosmological and model-specific parameters reported in Table 1.

4.3 Cosmological Parameter Constraints

We now present the cosmological parameter constraints for the K-essence scenarios, obtained from a joint analysis of Planck 2018, DESI DR1 BAO, and BBN data. Figure 5 displays the two-dimensional marginalized constraints on the parameters ω_{dm} , Ω_{F_0} , H_0 , and S_8 for the purely kinetic, quadratic, and exponential K-essence models, compared to Λ CDM. All three scenarios are consistent with current observations and predict coherent shifts toward higher H_0 , partially alleviating the tension with SH0ES from 4.37σ in Λ CDM to 3.66σ (kinetic), 3.68σ (quadratic), and 3.41σ (exponential), as summarized in table 1. In the quadratic model, the scalar mass m_ϕ is subject to the energetic bound of Eq. (20), obtained by requiring the potential energy to remain subdominant until the present epoch. Using $X_0 = (0.12 \text{ eV})^4$, this yields $m_\phi \lesssim 10^{-37} \text{ eV}$, which we adopt as a prior in the parameter estimation. Within this physically motivated range, the posterior distributions are nearly indistinguishable from those of the purely kinetic case, indicating that the quadratic potential does not lead to novel phenomenology under current observational constraints.

An important feature of the K-essence scenarios is the systematic shift in the cold dark matter

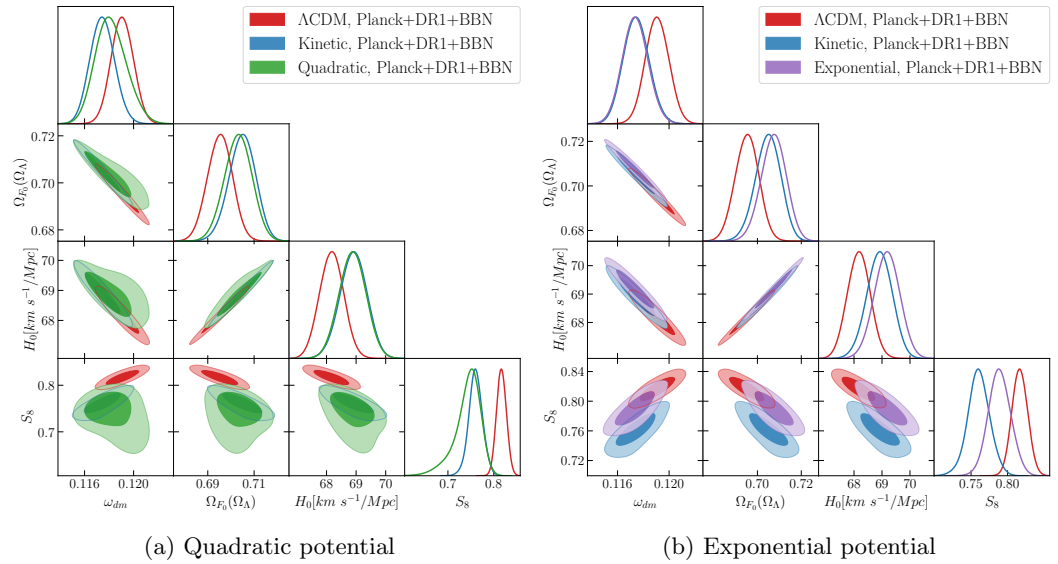


Figure 5: Two-dimensional marginalized posterior distributions for ω_{dm} , Ω_{F_0} , H_0 , and S_8 in the three K-essence models, compared to ΛCDM . We see a shift towards smaller H_0 values reducing the tension with the SH0ES measurement. The contours correspond to 68% and 95% confidence levels.

Model	Kinetic	Quadratic	Exponential	ΛCDM
$\omega_{\text{dm}} (\omega_{\text{cdm}})$	$0.1174^{+0.0020}_{-0.0018}$	$0.1182^{+0.0026}_{-0.0024}$	0.1174 ± 0.0019	0.1191 ± 0.0017
$\Omega_{F_0} (\Omega_\Lambda)$	0.705 ± 0.011	$0.703^{+0.011}_{-0.012}$	0.707 ± 0.011	$0.6952^{+0.0099}_{-0.010}$
H_0 [km s ⁻¹ /Mpc]	$68.91^{+0.83}_{-0.90}$	68.88 ± 0.87	$69.18^{+0.86}_{-0.80}$	$68.17^{+0.77}_{-0.78}$
S_8	$0.761^{+0.030}_{-0.031}$	$0.744^{+0.050}_{-0.064}$	$0.789^{+0.030}_{-0.029}$	0.817 ± 0.020
$10^{17}\theta$	< 19.7	< 14.5	< 17.8	—
$10^{38}m_\phi$ [eV]	—	< 3.48	—	—
$10^3 V_0$ [eV ⁴]	—	—	< 2.49	—
$10^{27}\lambda$ [eV ⁻¹]	—	—	< 4.53	—
χ^2_{min}	2794.46	2794.04	2794.46	2799.44
Hubble Tension	3.66σ	3.68σ	3.41σ	4.37σ
ΔAIC	-2.98	-1.40	-0.48	0.0

Table 1: Summary of the posterior mean values and 1σ uncertainties for key cosmological parameters in the Kinetic, Quadratic, and Exponential K-essence models, along with the ΛCDM baseline. All results are derived from a joint analysis using DESI DR1, Planck 2018, and BBN constraints. The errors are presented at 68% confidence except for the upper bounds that are at 95% confidence level. The lower part of the table lists model-specific parameters, the minimum χ^2 values, the statistical significance of the Hubble tension relative to SH0ES, and the Akaike Information Criterion (AIC) [50, 51] differences with respect to ΛCDM .

density ω_{dm} compared to the ΛCDM baseline. As shown in Table 1, all K-essence models prefer values of ω_{dm} slightly below the ΛCDM determination from *Planck* 2018 [3], with best-fit shifts of order $\Delta\omega_{\text{dm}} \sim -0.0015$ (about $1\text{--}1.5\sigma$). This reduction compensates the additional early-time scalar contribution that mimics a radiation component, ensuring consistency with CMB acoustic peaks and BAO distances. The effect is robust across the kinetic, quadratic, and exponential cases, indicating that it is primarily driven by the modified background dynamics rather than model-specific potential effects.

To assess the impact of the K-essence scenarios on the Hubble tension, Figure 6 shows the one-dimensional posterior probability distributions of H_0 for the three K-essence realizations and ΛCDM , using the combined BBN+Planck+DESI dataset. Relative to the ΛCDM baseline [3], the kinetic and exponential models shift the posterior towards higher values, closer to the SH0ES measurement [4], while the quadratic case essentially overlaps with the kinetic result due to its phenomenological degeneracy. Quantitatively, the Hubble tension is reduced from 4.37σ in ΛCDM to 3.66σ (kinetic), 3.68σ (quadratic), and 3.41σ (exponential). This shift reflects the generic effect of the scalar field's early radiation-like phase on the expansion history, which propagates into higher inferred H_0 values.

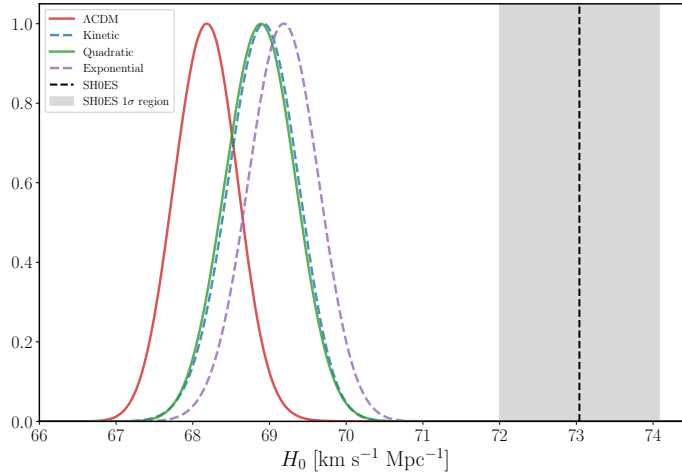


Figure 6: Posterior probability function of H_0 for the three K-essence models and Λ CDM given the BBN+Planck+DESI observations. The shaded region corresponds to the SH0ES measurement [4] at 1σ confidence level. The field models shift the posterior towards higher H_0 reducing the tension to 3.66σ (kinetic), 3.68σ (quadratic), and 3.41σ (exponential), compared to 4.37σ in Λ CDM.

Overall, the parameter constraints confirm that generalized K-essence models can fit current cosmological data at the same statistical level as Λ CDM, while introducing a mild but coherent shift in H_0 bringing its value closer to the region favored by late-Universe probes, while the systematic reduction in ω_{dm} relative to *Planck* 2018 [3] emerges as a distinctive prediction of these scenarios. This combination offers a possible observational handle to test K-essence against the standard Λ CDM paradigm in upcoming surveys.

The Akaike Information Criterion (AIC) given by $AIC = \chi^2_{\text{min}} + 2k$, where k is the number of parameters, is a criterion that evaluates the fit of the models to the data penalizing models with too many free parameters. From table 1, the field models fit better to the data even after penalizing them for the extra parameters, (θ) for the kinetic model, (θ, m_ϕ) for de quadratic potential and (θ, V_0, λ) for the exponential. The purely kinetic model is the better suited according to the criterion.

Figure 7 displays the marginalized posterior distributions for the model-specific parameters in each K-essence scenario: $(\theta, \Omega_{F_0}, F_2)$ for the purely kinetic case, $(\theta, \Omega_{F_0}, m_\phi)$ for the quadratic potential, and $(\theta, \Omega_{F_0}, V_0, \lambda)$ for the exponential potential. A key difference between the purely kinetic model and its extensions is that in the purely kinetic, the coefficient F_2 can be related explicitly to the matter density, since in this case the relations (10) hold, allowing a direct mapping between the scalar Lagrangian and the cosmological density parameters. For the quadratic model, the posterior confirms that viable masses are restricted to $m_\phi \lesssim 10^{-38}$ eV, consistent with the energetic bound in Section 3, which renders the potential dynamically irrelevant. In the exponential case, the posteriors admit a wide range of (V_0, λ) , with only a mild preference for non-zero values, and thus do not yield sharp constraints under current data.

The physical interpretation of our results highlights the central role of the kinetic sector in generalized K-essence cosmologies. In all scenarios considered, the constant term F_0 provides the dominant contribution to the dark energy density today, effectively acting as a cosmological constant. This implies that the late-time acceleration of the Universe is predominantly driven by the non-canonical kinetic term, while scalar potentials play only a secondary role. The scalar potentials explored here mainly modulate the perturbation evolution, without altering the overall mechanism responsible for cosmic acceleration.

The quadratic potential case illustrates this point clearly. The requirement of an ultralight mass, $m_\phi \leq 10^{-38}$ eV, ensures numerical stability as well as a reasonable field density at late times, but simultaneously renders the potential dynamically irrelevant throughout the cosmic history. As shown in Figure 5, the quadratic model essentially mimics the purely kinetic case, producing nearly indistinguishable predictions for both background and perturbations. Small residual differences in the dynamics remain, but these are well below the sensitivity of current observations.

The exponential potential, on the other hand, introduces a larger parameter space through (V_0, λ) , which in principle allows for richer phenomenology. Nevertheless, our constraints indicate no clear preference for a non-vanishing potential. The posterior distributions in Figure 7 remain broad and compatible with the purely kinetic limit, underscoring the robustness of the kinetic

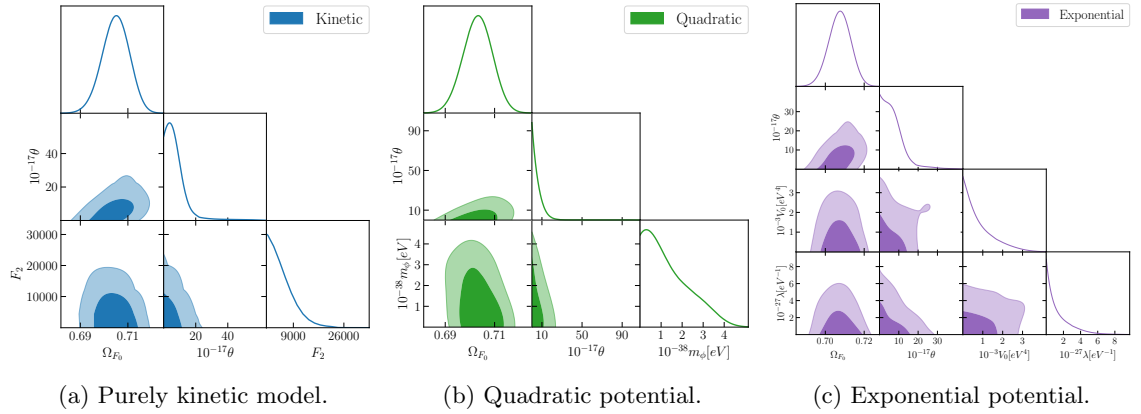


Figure 7: Two-dimensional marginalized posterior distributions for the model-specific parameters in the three K-essence scenarios, compared to Λ CDM. Contours correspond to 68% and 95% confidence levels.

scenario. In this sense, the additional degrees of freedom in the exponential case do not lead to significant improvements in fit quality or distinct observational signatures under present data.

Across all models, the most relevant effects arise from the early-time radiation-like phase of the scalar field. As shown in Figure 2, this phase temporarily increases the effective number of relativistic species, modifies the cold dark matter abundance, and leaves imprints on the growth of structure. Consistently, the two-dimensional posteriors in Figure 5 show that all scenarios predict a systematic reduction in ω_{dm} compared to Λ CDM, compensating the scalar contribution and preserving the CMB acoustic scale and BAO distances. At the same time, the models induce coherent shifts toward higher H_0 , as illustrated in Figures 5 and 6, helping to partially ease the tension between SH0ES and CMB measurements. These signatures are not tied to the details of the potential, but rather stem from the universal features of the non-canonical kinetic sector.

Taken together, these findings reinforce the conclusion that the kinetic core of K-essence is the essential driver of its cosmological phenomenology. Potentials can be added without spoiling consistency with data, but they do not significantly alter the predictions within current observational precision. The distinguishing features of K-essence therefore lie in its early-time behavior and its impact on structure formation, providing a simple yet testable framework for unifying dark matter and dark energy.

5 Conclusions

In this work, we have analyzed a family of generalized K-essence models designed to unify dark matter and dark energy within a single scalar field framework. Building on the purely kinetic quadratic model [1], we studied two extensions with quadratic and exponential scalar potentials respectively. We evaluated whether such additions improve the phenomenological viability or yield distinctive observational signatures. The Scherrer’s Lagrangian (7) is highly general, since any non-canonical Lagrangian $F(X)$ that possesses a minimum can be approximated by its expansion around that point. This universality has been emphasized in previous studies [13, 14, 15]. Our results further explore this framework by confronting these scenarios with current cosmological datasets.

A further assessment of model performance was carried out using the Akaike Information Criterion (AIC), with Λ CDM taken as the reference. All K-essence scenarios yield lower AIC values, indicating a modest statistical preference despite the inclusion of additional parameters. Among them, the purely kinetic Scherrer solution provides the best trade-off between fit quality and complexity, with $\Delta\text{AIC} \simeq -3$ compared to Λ CDM.

These results, summarized in Table 1, confirm that generalized K-essence scenarios remain competitive alternatives to the standard cosmology, providing equally good or marginally better fits to current data while unifying the dark sector under a single scalar degree of freedom.

We have shown that all models reproduce the expected sequence of cosmological epochs: an initial radiation-like phase, a matter-dominated era, and late-time accelerated expansion. In the purely kinetic case, this behavior arises naturally from the non-canonical structure and the constant offset F_0 , which plays the role of a cosmological constant. The exponential potential preserves this structure while producing mild modifications at late times, whereas the quadratic

potential requires an ultralight scalar mass of order 10^{-38} eV, rendering it dynamically indistinguishable from the kinetic scenario.

A key outcome is the scalar field's early-time relativistic behavior, which enhances the radiation density and contributes to the effective number of relativistic species. We explicitly computed this contribution and identified the parameter region $\theta \lesssim 19.7 \times 10^{-17}$ as required to satisfy BBN and *Planck* constraints. This bound applies uniformly across all models, providing a robust early-Universe constraint on the dynamics of generalized K-essence.

From the parameter inference perspective, all scenarios remain consistent with *Planck* 2018, DESI DR1 BAO, and BBN data. They systematically predict slightly lower S_8 and higher H_0 relative to Λ CDM, leading to modest shifts toward the region favored by SH0ES and weak lensing surveys. The kinetic and exponential cases reduce the Hubble tension to the $\sim 3\sigma$ level, while the quadratic potential remains redundant with the kinetic limit. In addition, all models prefer values of the cold dark matter density ω_{dm} below the Λ CDM baseline, compensating for the scalar's relativistic contribution at early times and preserving consistency with CMB acoustic peaks and BAO distances.

Although current data do not provide a statistical preference for K-essence over Λ CDM, the coherent parameter shifts and theoretical robustness of the framework highlight its potential as a unifying description of dark matter and dark energy. Recent work has suggested that extensions involving direct couplings to matter may further alleviate the H_0 and S_8 tensions by modifying the effective gravitational coupling at different epochs [52]. Future observations with next-generation galaxy surveys such as DESI DR2 [47], Euclid [45], and the Vera C. Rubin Observatory [46], together with upcoming CMB experiments such as CMB-S4 [48] and the Simons Observatory [49], will be decisive to test these models beyond the linear regime and to search for distinctive signatures in gravitational lensing, clustering, and non-linear structure formation.

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