

Reconstruction of the Effective Energy-deposition Vertex of Muon Showers using PMT Waveform in a Large-scale Liquid Scintillator Detector

Junwei Zhang^{1,2} · Yongpeng Zhang^{2*} · Yongbo Huang^{1†} · Jilei Xu^{2‡} · Junyou Chen^{1,2} · Yi Wang²

October 20, 2025

Abstract Cosmogenic muon-induced radioactive isotopes pose a significant background source for deep-underground low-background experiments. Although rock overburdens at underground sites substantially attenuate the cosmogenic muon flux, residual muon-induced backgrounds still require active suppression. For future multi-kiloton liquid scintillator (LS) detectors, such as the Jiangmen Underground Neutrino Observatory (JUNO), shower muons contribute to more than 88% of all muon-induced isotopes. Consequently, precise reconstruction of shower vertices is essential for implementing localized spatial vetoes. We propose a novel waveform-based method to reconstruct the shower vertex, defined as the energy-deposition centroid. By subtracting the track contributions from non-shower muons in the recorded waveforms, the isolated shower component is extracted. Subsequently, combined with a photon propagation model and an iterative optimization algorithm, the shower vertex positions are reconstructed. Simulations show that for 68% of events, the single shower vertex resolution is better than 0.16 m, 0.15 m, and 0.26 m along X, Y, and Z respectively. Furthermore, the reconstruction efficiency exceeds 96% when requiring the distance between the reconstructed and true vertices to be less than 3.0 m. This method provides a critical technical foundation for muon-induced background suppression in JUNO and other large-scale LS detectors.

1 Introduction

Cosmogenic muon-induced radioactive isotopes represent significant backgrounds for low-background experiments. Although detectors are generally shielded by being located in underground experimental facilities, where substantial rock

overburden effectively attenuates the majority of cosmogenic muons, residual muon-induced backgrounds still require additional suppression. Consequently, muon tagging techniques and corresponding veto strategies are essential. This approach is widely adopted in underground experiments (e.g., KamLAND [1], Borexino [2], Daya Bay [3], Double Chooz [4], SNO+ [5]).

The muon veto strategy typically exploits the spatio-temporal correlation between a muon track and its induced isotopes. By vetoing a volume surrounding the muon track for a specific time window, these isotopes can be effectively rejected. Consequently, the development of efficient muon tagging techniques is a prerequisite for implementing the muon veto. Muon events can be categorized according to their topology in the detector. Primary classifications include through-going muons, shower muons, and stopping muons, as illustrated in Fig. 1. Further distinctions, such as clipping muons and muon bundles, arise when considering factors like the muon's path relative to the detector and its multiplicity. Comprehensive classification schemes are detailed in [6].

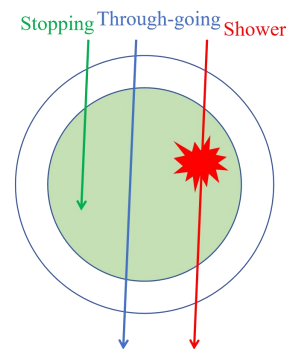


Fig. 1 Schematic of a spherical target detector and different kinds of muon events labelled with shower muon, through-going muon, and stopping muon

¹ School of Physics and Technology, Guangxi University, Nanning 530004, China · ² Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China

In prior experiments utilizing detectors with size under 10 m, the constrained size limited the muon type predominantly to through-going muons, characterized by low muon rates (typically below 1 Hz). Consequently, implementing a full-detector veto for a defined time window preserved high live time and detection efficiency. However, this strategy becomes highly inefficient for kiloton-scale LS detectors. Instead, reconstructing the muon trajectory using deposited charge and timing information, followed by vetoing a cylindrical volume centered on the reconstructed track, proved significantly more effective. At the kiloton scale, single through-going muons remain the dominant type, and their distinct linear track signatures facilitate relatively straightforward reconstruction.

The deployment of 10-kiloton class detectors, however, reveals non-negligible occurrences of muon bundles-multiple muons traversing the detector simultaneously. Furthermore, a larger fraction of incident muons induce energetic showers (hadronic or electromagnetic), classified as shower muons. These evolving characteristics create stringent demands on muon reconstruction precision and the effective rejection of muon-induced isotope backgrounds in low-background experiments. As highlighted in [7], the full-detector veto approach is inadequate under these conditions. This necessitates the development of advanced muon reconstruction and classification techniques capable of handling diverse muon event topologies, including single muons, muon bundles, and shower muons.

The Jiangmen Underground Neutrino Observatory (JUNO), a representative LS-based experiment designed to determine the neutrino mass ordering, has been completed and has begun acquiring data. JUNO will build and operate the world's largest liquid scintillator detector, with a liquid scintillator (LS) target mass of 20 kilotons. About 17600 20-inch photomultiplier tubes (PMTs) and 25600 3-inch PMTs are equipped as photosensors. Additional detector details are provided in [8, 9]. A rock overburden of approximately 650 m (1,800 m.w.e.) above the JUNO central detector (CD) reduces the muon flux by about four orders of magnitude to muon rate of approximately 0.004 counts/m²/s and an average muon energy of around 207 GeV at JUNO. However, the radioactive isotopes produced by muons remain significant contributors to backgrounds in both reactor neutrino and solar neutrino detection. Therefore, it is essential to further implement muon tagging technology and corresponding muon veto strategies to mitigate the remaining muon-induced backgrounds. For the JUNO experiment, extensive studies have been conducted on the track reconstruction of single through-going muons [10–12] and double through-going muons [6]. However, there remains a scarcity of detailed reports regarding the reconstruction of shower muons in JUNO.

The Super-K experiment conducted comprehensive analyses of shower muon physics in large water Cherenkov de-

tectors, establishing that most muon-induced isotopes originate from shower muons [13, 14]. As reported in [13], the isotope backgrounds are produced by muon-induced shower secondaries, rather than the cosmogenic muons themselves. These isotopes are predominantly distributed within several meters of the energy-deposition centroid, this indicates that reconstructing the position of the weighted center of the shower energy deposition could be more effective for background rejection compared to reconstructing the initial interaction position of the shower muon with minimal impact on detector dead time. Therefore, the "shower vertex" henceforth in this paper is defined as the weighted center of the shower energy deposition.

The Super-K experiment carried out extensive analyses [13, 14] of shower muon physics in large water Cherenkov detectors, demonstrating that the majority of muon-induced isotopes are generated by secondary particles produced in muon-induced showers, rather than by the incident cosmogenic muons themselves. These isotopes are predominantly located within several meters of the energy-deposition centroid. This spatial correlation suggests that reconstructing the weighted center of the shower energy deposition may offer a more effective strategy for background rejection, with minimal impact on detector dead time. For the sake of clarity and consistency in the following discussion, this paper henceforth defines the term "shower vertex" as the weighted center of the energy deposition of the muon shower. Based on these studies, Super-K developed advanced background rejection techniques that substantially improved physics measurement precision [15, 16]. Similarly, the KamLAND-Zen experiment implemented shower muon reconstruction in its large-scale LS detector [17]. Compared to technologies like the Liquid Argon Time Projection Chamber (LArTPC) used in the DUNE experiment, which has great tracking capability and precise measurement on energy deposits [18], LS detectors face challenges due to isotropic lighting, making it difficult to detect the profile of a shower. Consequently, reconstructing shower vertices in LS detectors is particularly challenging.

This paper presents the first comprehensive study of shower muon reconstruction in a 20-kton LS detector using the JUNO experiment as a benchmark. Our methodology provides critical technical support for muon-induced background rejection in JUNO and future large-volume, low-background experiments. The paper is structured as follows: Section 2 characterizes shower muon simulations, event topologies, and isotope yields by muon species. Section 3 details our reconstruction algorithm and analyzes PMT waveform signatures that form the foundation of our reconstruction technique. Reconstruction performance is evaluated in Section 4, with results and discussion presented in Section 5.

2 Characteristics of shower muon in LS

2.1 Simulation description

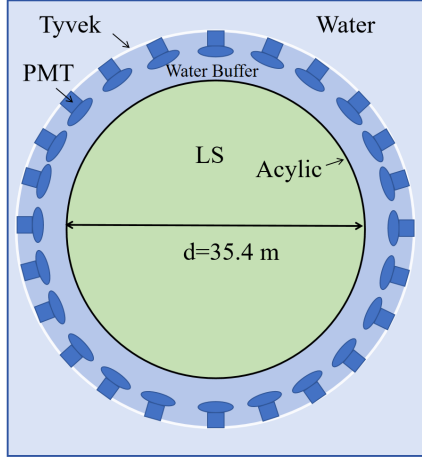


Fig. 2 A schematic view of the detector in the simulation

A simulation software framework based on Geant4 [19–22] was developed for detector simulation, using the JUNO-like detector geometry as a benchmark to investigate shower characteristics of muons in the liquid scintillator. The simulated detector layout is illustrated in Fig. 2. Muon events generated from MUSIC [23] software were imported into the simulation, then shower muons were selected for further analysis. To comprehensively record energy deposits from muons and their secondary particles, the liquid scintillator detector was segmented into voxels of size $30\text{ cm} \times 30\text{ cm} \times 30\text{ cm}$. Consequently, the shower energy, along with its deposition location and spatial distribution range, can be effectively visualized. Fig. 3 shows an example of the space extension of a shower muon going through LS. Each point is for the center position of the voxel and the colour is for the total deposited energy larger than 20 MeV in a voxel. The two-dimensional projections clearly show the lateral extension around the muon track. Along the longitudinal direction, the shower extends further forward.

Figure 4 presents the energy deposit profiles of shower muons. Figure 4(a) presents the distribution of distances (d_T) from each energy-depositing voxel to the reconstructed shower vertex. This reflects the radial spread of the shower around its energy-weighted center [24], with the inflection point near 2 m indicating the typical spatial extent of the electromagnetic and hadronic components. Figure 4(b) shows the longitudinal distribution (d_L) of energy deposition relative to the muon track direction. Values are calculated along the track, with positive (negative) (d_L) corresponding to positions ahead of (behind) the shower vertex. The asymmetric shape and inflection point illustrate the forward-peaking na-

ture of shower development due to the high-energy muon's direction. Figure 4(c) displays the transverse profile (d_T), representing the perpendicular distance from deposited energy voxels to the muon track. The observed exponential decay is consistent with expectations from particle cascade theories and provides useful insight into shower development and supports reconstruction efforts.

The inflection point near 2 m in both radial and longitudinal profiles motivates the use of a spherical volume of this radius for identifying shower development. A muon is classified as a shower muon if the total energy deposited within this sphere exceeds 1.8 GeV—a threshold selected to include the muon's minimum ionization energy (roughly 2 MeV/cm and 800 MeV over 4 m) plus an additional 1 GeV from shower secondaries. To prevent the merging of nearby showers, a minimum separation of 4 m is required between any two shower vertices. To give a clear definition, the detail explain as the following:

- **Shower vertex:** The energy-weighted centroid of deposits within a 2 m sphere.
- **Shower muon:** The muon that accumulated deposited energy larger than 1.8 GeV in a sphere with radius 2 m.
- **Shower energy:** The accumulated deposited energy subtract the muon track minimum ionization energy in a sphere with radius 2 m.

Additionally, a simplified electronics simulation was implemented to facilitate the reconstruction of shower vertices. This simulation models the PMT waveform response by superimposing single photoelectron waveform templates for each photon hit. Contributions from dark noise and pure electronic white noise were also incorporated to generate realistic overall waveform responses.

2.2 Characteristics of shower muon and isotopes yield in LS

After defining the shower muon and the shower vertex, we can analyze the characteristics of the shower muon using the information recorded in the simulation.

For shower muon events, the average energy deposition in the LS exceeds that of non-shower muons. In the spherical LS volume with 35.4 m diameter, the total deposited energy for minimally ionizing muons is approximately 7 GeV, whereas shower muons deposit $> 10\text{ GeV}$ on average. Fig. 5 presents the energy spectrum per shower within a sphere with radius 2 m. Single-shower events contain one distinct vertex along the muon trajectory through the LS, while double-shower events feature two spatially separated vertices whose energies are recorded independently. The energy distribution of individual showers in double-shower events closely matches that of single showers, indicating stochastic shower

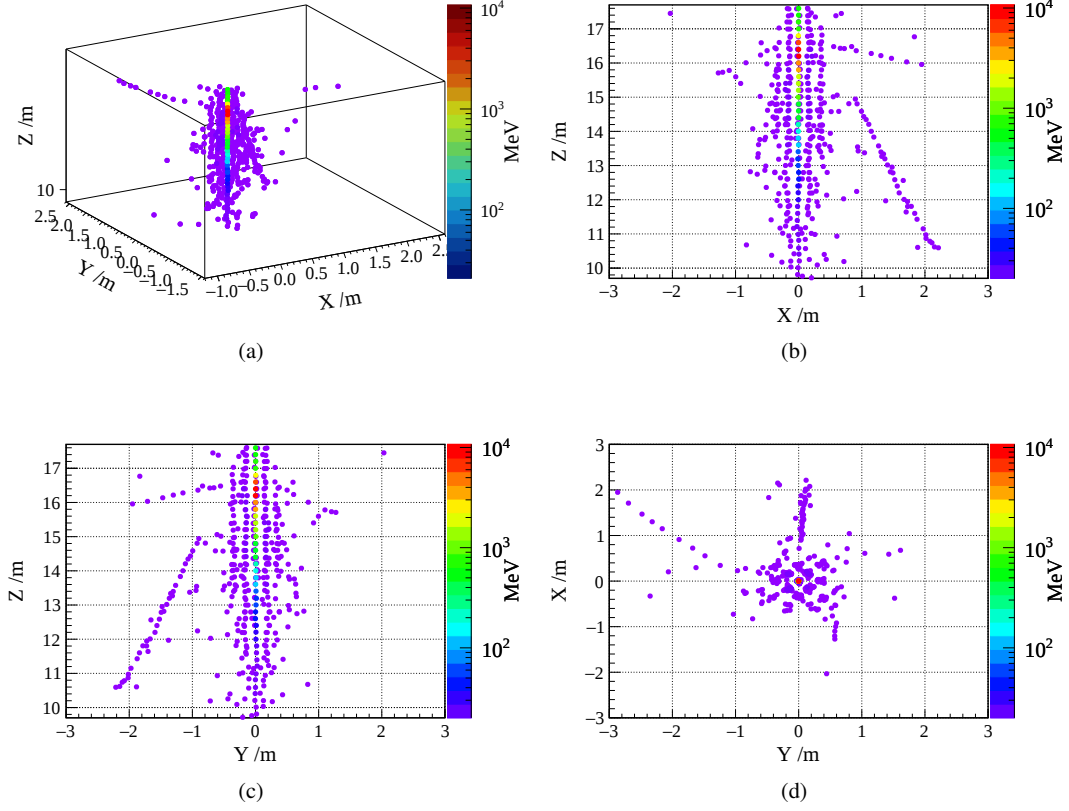


Fig. 3 An example of muon energy deposition (~ 220 GeV) extension in three dimension space. For clarity, the two dimension plots, XY, XZ, YZ also are shown

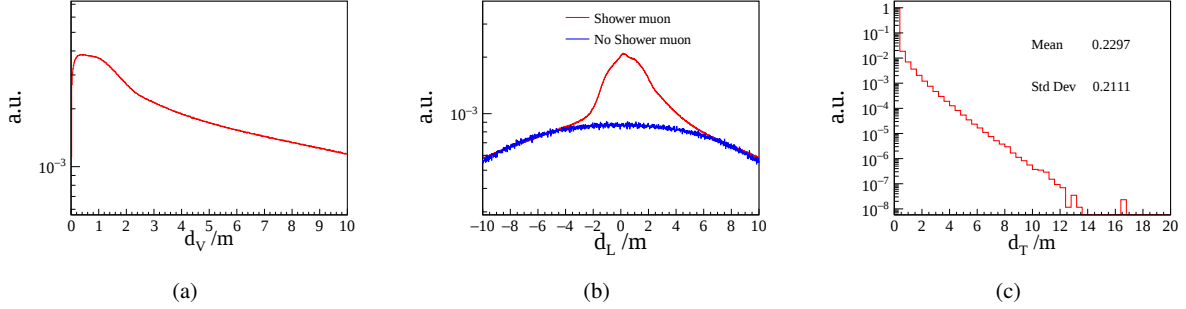


Fig. 4 The shower muon energy deposit profiles. (a) The distance (d_V) distribution of the energy deposited voxels relative to the shower vertex. (b) The longitudinal distance (d_L) distribution of the energy deposited voxels relative to the muon track. The positive (negative) values are the voxels project onto the forward (backward) place of the shower vertex in the muon track. (c) The transverse distance (d_T) distribution from energy deposited voxels to muon track

generation where double showers effectively constitute two independent single showers. This analysis focuses primarily on single-shower events, with supplementary discussion of double-shower scenarios.

Based on simulations using a LS detector with a diameter of 35.4 m, the average shower multiplicity per muon (number of muon shower) is approximately 1.6 when an energy threshold above 1.8 GeV is applied within a 2 m dis-

tance. Under these conditions, the multiplicity can reach up to 4 showers per muon in the LS. The average distance between two shower vertices is found to be about 13 m.

Table 1 shows the proportion of isotopes produced by different shower energies of muon events and by non-shower muon events. Although the shower muon events occupy the ratio of 19.8% in all muon events, they can contribute about 88.2% of the isotopes. Consequently, if the shower vertex

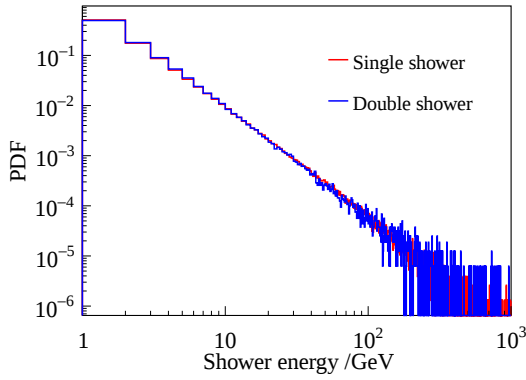


Fig. 5 The shower energy spectrum of each shower in LS

Table 1 The proportion of isotopes produced by different shower energies of single muon events and by non-shower muon events

Muon type		Muon ratio	Isotope ratio
Shower muon	All	19.8%	88.2%
	1 - 3 GeV	11.0%	12.2%
	3 - 5 GeV	4.5%	7.0%
	5 - 7 GeV	1.5%	5.1%
	7 - 9 GeV	0.8%	3.6%
	> 9 GeV	2.0%	60.3%
Through-going		76.3%	10.8%
Stop		3.9%	1.0%

can be reconstructed, then the isotope background caused by muons in neutrino analysis can be significantly reduced. As mentioned in Section 2.1, the shower vertex discussed in this paper corresponds to the weighted center of the shower energy deposition. From Fig. 6(a), it can be seen that the average distance between the shower vertex and the initial interaction position of the shower (shower start position) is about 1.9 m. Additionally, Fig. 6(b) shows the distances of muon-induced isotopes to the shower vertex and the initial shower interaction position; the proportions of isotopes within 3 m from these two positions are 84% and 60%, respectively. This also indicates that using the shower vertex as the reference point can more effectively eliminate cosmic isotopes, which is consistent with the conclusion in [13].

3 Methodology

3.1 Principle

When a muon traverses the LS (Fig. 7), scintillation photons emitted along its path propagate through the medium and subsequently impinge on the PMTs. The combined response of the PMTs and back-end electronics induces measurable waveforms. In the event of a muon-induced shower, substantial energy deposition occurs within the LS. Consequently, a significant flux of photons is generated near the shower. These photons propagate through the LS toward the

PMTs. The photon flux from the shower energy deposition introduces distinct peaks into the PMT waveforms, making the waveforms of shower events distinguishable from those of non-shower events. To investigate the waveform features attributable specifically to the shower component, the electronics simulations were performed for both showering and non-showering muons. Within these simulations, the trajectories of shower muon events and non-shower muon events were maintained near-identical by adjusting the primary muon momentum. This design ensured comparable detector responses arising solely from the track (minimum ionization) component. Non-shower muons were selected based on a calculated average dE/dx value approximating the minimum ionization energy loss rate (2 MeV/cm), derived from the total deposited energy and muon track length.

Fig. 8 shows PMT waveforms from a non-shower muon event (blue) and a shower muon event (black). As discussed in Section 2, the energy deposition of a shower muon comprises a track component and a shower component (Fig. 4(b)); both components contribute to its PMT waveform (Fig. 8). For accurate shower vertex reconstruction, waveform components directly associated with shower energy deposition must be isolated. This necessitates the prior subtraction of waveform contributions arising from the track energy deposition. By subtracting the waveform of a non-shower muon event from that of a shower muon event, interference from the track component can be effectively mitigated during shower vertex reconstruction.

Due to inherent fluctuations in processes such as energy deposition, photon propagation, and photon detection, even identical muon tracks can yield slightly different waveforms on the same PMT. To reliably identify residual waveform components generated by the shower after track subtraction, a selection threshold was implemented. Candidate peaks attributed to the shower component were required to exceed 60% of the maximum peak height observed in the subtracted waveform.

The peak information after shower and non-shower subtraction (Fig. 8) will be utilized for reconstructing the shower vertex. The number of peaks represents the number of shower vertices that can be reconstructed with the predicted peak times based on the time evolution from muon incidence to photon hitting the PMT and the observed peak times in the subtracted waveform. The χ^2 of the k^{th} peak in the subtracted waveform can be expressed as:

$$\chi^2 = \sum_i \left(\frac{T_i^{\text{pre}} - T_i^{\text{obs}}}{\sigma_i} \right)^2 \quad (1)$$

where T_i^{pre} is the predicted k^{th} peak time, T_i^{obs} is the observed k^{th} peak time and σ_i represents the uncertainty of peak time for the i^{th} PMT, which is the transit time spread (TTS) of PMT. It is assumed that the σ_i remains constant for

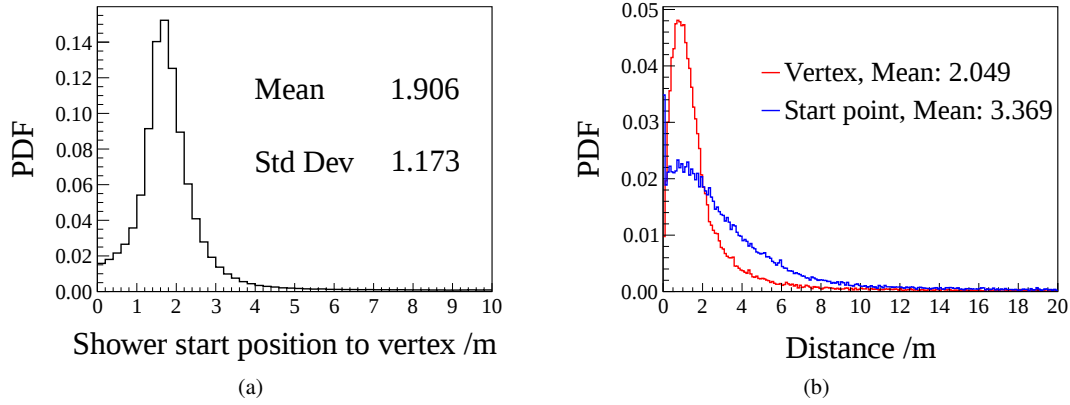


Fig. 6 (a) Distance from shower vertex to shower start position. (b) The distances of the muon-induced isotopes to the shower vertex (red line) and to the initial interaction position of the shower (blue line)

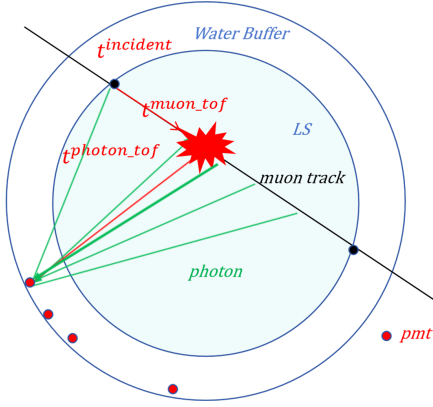


Fig. 7 Schematic diagram of a muon goes through LS, the light emission and transmits to the PMTs

a given PMT throughout the analysis. The k^{th} shower vertex parameters are reconstructed by minimizing the χ^2 function.

$$T^{\text{pre}} = t^{\text{incident}} + t^{\text{muon_tof}} + t^{\text{photon_tof}} + t^{\text{delay}} + t^{\text{offset}} \quad (2)$$

Based on the muon track and the photon propagation process shown in Fig. 7, it can be concluded that the T^{pre} is composed of muon incident time (t^{incident}), time of flight of muon ($t^{\text{muon_tof}}$), time of flight of photons ($t^{\text{photon_tof}}$), delay time caused by the PMT, cable and readout electronics (t^{delay}) and the offset of the first hit time (FHT) and peak time of the waveform (t^{offset}), as indicated in Eq. (2).

The overall reconstruction process is summarized as shown in the Fig. 9. Following electronics simulation, the PMT waveforms are obtained. The waveform of the shower muon event need to be subtracted from that of the non-shower muon event. The method for obtaining the waveform of non-shower muon events will be explained in 3.2. The TSpec-trum tool in the ROOT package [25, 26] is used to find the peaks over threshold in the subtracted waveform. In the reconstruction of a single shower vertex, PMTs with a single

peak after subtraction are selected, and the peak time is T_i^{obs} as described in Eq. 1.

From Fig. 7, it is best to have muon track information as input for calculating the $t^{\text{muon_tof}}$ and the $t^{\text{photon_tof}}$. The track of the shower muon event can be reconstructed using the fastest light method [10, 11] or machine learning method [12]. If muon track is not available, it is necessary to search throughout the entire detector volume during the minimization process of Eq.1 to reconstruct the shower vertex. The initial value of the shower vertex will be provided by the charge centroid method. During the minimization process, $t^{\text{muon_tof}}$ changes as the candidate shower vertex iterates to different positions. The propagation of photons in LS [27–36] is complicated and includes various optical processes including scintillation, Cherenkov process, absorption and re-emission, rayleigh scattering, and refraction or reflection at detector boundaries formed with acrylic and water materials. The time-of-flight may not be proportional to propagation distance. The effective refractive index n_{eff} as a function of the propagation distance which has been thoroughly studied in the literature [37]. For a certain PMT, $t^{\text{photon_tof}}$ can also be calculated based on the shower vertex position and the effective refractive index. The PMTs densely cover the entire surface of the detector, therefore, the position where the earliest photons hit the PMTs approximately corresponds to the muon incident position, and the time of this hit is also approximately the muon incident time. The FHTs of all PMTs are sorted from smallest to largest, and the average of the first five PMTs is taken as the t^{incident} . The t^{incident} calculated using the average value is closer to the true incident time, which can avoid the fluctuation when using the minimum FHT as the muon incident time. The readout electronics and signal transmission cables of the PMT can cause a time delay (t^{delay}). The t^{delay} can actually be calibrated using a laser calibration source. The t^{offset} is set as a variable to describe the difference be-

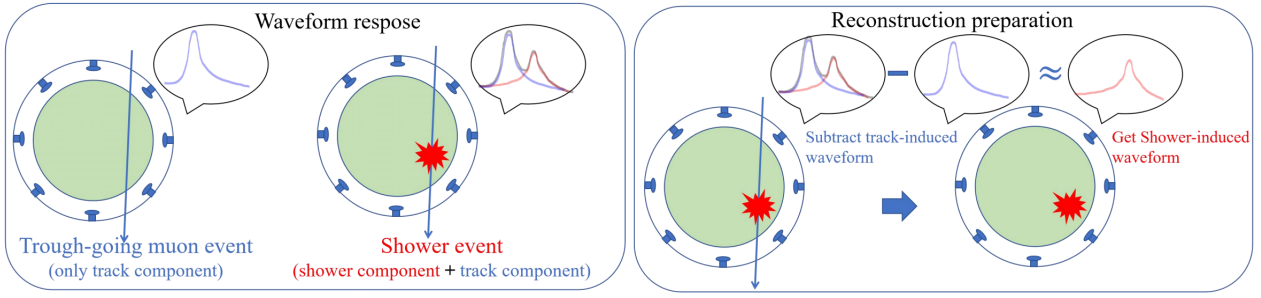


Fig. 8 PMT waveforms for a non-shower muon event (blue) and a shower muon event (black). Their trajectories were maintained near-identical by adjusting the primary muon momentum. The waveform of the shower muon event incorporates contributions from both track- and shower-related energy depositions. To enable accurate reconstruction of the shower vertex, the track-induced component is subtracted from the shower waveform by subtracting the waveform of the non-shower muon event, thereby isolating the signal attributable solely to the shower

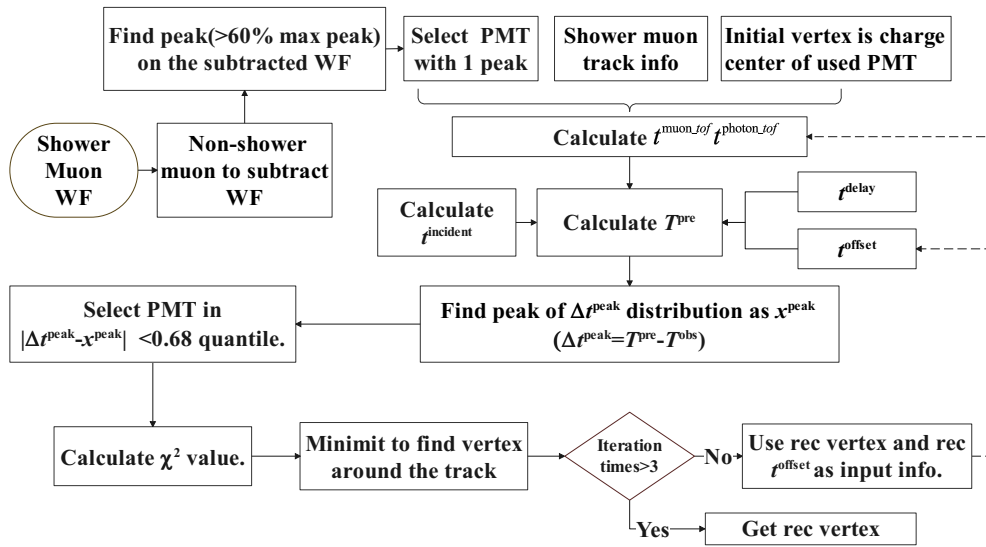


Fig. 9 Reconstruction process

tween the waveform peak time and the FHT. This initial value of t_{offset} can also be estimated from the waveform and bounded within a specified range during minimization. Based on Eq. (2), the expected time (T^{pre}) of the peak caused by the shower on the waveform of each PMT can be predicted.

The difference between T^{pre} and T^{obs} determined by peak finding in the subtracted waveform, results in a distribution of Δt^{peak} ($\Delta t^{\text{peak}} = T^{\text{pre}} - T^{\text{obs}}$), as indicated by the red curve labeled "charge center" in Fig. 10. Since the value of Δt^{peak} can assess the accuracy of the T^{pre} , it is also used to select suitable PMTs as input for shower vertex reconstruction. Through in-depth optimization studies, it was determined that PMTs within a region covering 68% of the distribution around the peak value of Δt^{peak} serve as inputs for vertex reconstruction.

Furthermore, based on the true shower vertex information and assuming the shower is a point source, the expected

distribution of true Δt^{peak} can be predicted. Comparing the expected Δt^{peak} distribution from the reconstructed vertex with the true Δt^{peak} distribution can evaluate the accuracy of the reconstructed vertex, as shown in Fig. 10. The initial Δt^{peak} distribution predicted based on the charge center differs significantly from the true Δt^{peak} distribution, indicating a large deviation of the charge center reconstructed vertex from the true shower vertex.

After calculating the Δt^{peak} distribution based on the charge center and the selected PMTs, the shower vertex can be initially reconstructed using Eq. 1. With this first reconstructed shower vertex, a new expected Δt^{peak} distribution can be calculated, represented by the blue "1st" curve in Fig. 10. As the Fig. 10 shows, the Δt^{peak} distribution calculated after the first reconstruction is closer to the true Δt^{peak} distribution, demonstrating the effectiveness and correctness of the reconstruction process.

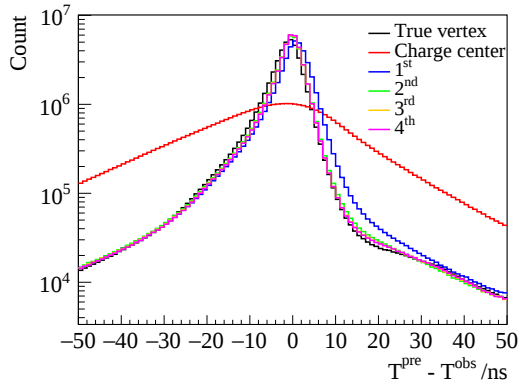


Fig. 10 $T^{\text{pre}} - T^{\text{obs}}$ distribution with different iteration times during reconstruction

Using the first reconstructed shower vertex and t^{offset} as new inputs for the reconstruction algorithm, with updated T^{pre} and the selected PMTs, a second shower vertex reconstruction based on Eq. 1 is achieved after optimization. The second Δt^{peak} distribution, compared to the first, more closely aligns with the true Δt^{peak} distribution, as seen in Fig. 10. This process is repeated iteratively. As the iteration times increase, the RMS of Δt^{peak} becomes smaller, and the mean of Δt^{peak} approaches zero. Δt^{peak} distribution tends to be similar to the distribution calculated using the true shower vertex. This indicates that the candidate shower vertex is gradually converging toward the true vertex. After four iterations, the reconstructed shower vertex's expected Δt^{peak} distribution closely matches the true Δt^{peak} distribution. Therefore, the vertex reconstructed in the fourth iteration is considered the final result of the entire reconstruction algorithm.

3.2 Method for obtaining waveform of non-shower muon

As mentioned in the previous Section 3.1, the initial momentum of shower muons in the generator information was reduced during detector simulation to obtain the waveform of non-shower muons. This adjustment suppressed shower development while aiming to preserve the track component. The dE/dx distribution can be derived from the track length and the total deposited energy. The peak position of this distribution corresponds to the well-known minimum ionizing energy of a muon, which is about 2 MeV/cm. By selecting samples within one sigma of this distribution, non-shower muon events can be obtained.

Directly identifying both shower event and non-shower muon event occurring along the exact same physical track in real experimental data is infeasible. However, several methodologies can be employed to derive the requisite waveforms of non-shower muon events from real data.

The expected waveform of the track component in a shower muon event can be derived based on physical models of

energy deposition, photon propagation process, and the response of the PMT. When a non-shower muon event passes through the LS, it deposits energy primarily through minimum ionizing energy. For simplification of calculations, the muon's track is divided into small segments of 30 cm each. The energy deposited in each segment is estimated as about 60 MeV. The midpoint of each segment is taken to be the emission point of photons produced by the energy deposition. These photons isotropic propagate through the LS and eventually hit the PMT with a certain probability. The probability of photons hitting the PMT is calculated based on the attenuation length of the LS and the spatial acceptance of the photocathode area of a given PMT. As shown in Eq. (3), where $\frac{dE}{dx}$ is the minimum ionizing energy, L_{segment} is the length of the 30 cm segments, Y is the light yield of the LS, L_{att} is the attenuation length of the LS and DE_i is the detection efficiency of the PMT for the hitting photons. The other item of Eq. (3) describes the spatial acceptance of the photocathode area of a specific PMT for receiving photons at the position of an emitted scintillation photon. In the Eq.(3), R is the radius of the PMT's photocathode surface, d is the distance between the PMT and the position of the emitted scintillation photon, and $\theta_{\text{edep}, \text{PMT}_i}$ is the angle between the line connecting a specific PMT and center of detector and the direction from the position of emitted scintillation photon to this PMT. Thus, based on Eq.(3), the number of scintillation photons produced in the j -th segment of the muon track that are converted into photoelectrons when they hit the PMTs, denoted as N_j , can be calculated.

$$N_j = Y \times \frac{dE}{dx} \times L_{\text{segment}} \times e^{-\frac{d}{L_{\text{att}}}} \times DE_i \times \frac{R_{\text{PMT}_i}^2 \times \cos \theta_{\text{edep}, \text{PMT}_i}}{4d^2} \quad (3)$$

After obtaining the number of photoelectrons (N_j) detected by the PMT, it is necessary to consider the timing information of each photon hitting the PMT and the waveform of the single photoelectron (SPE) before predicting the waveform for each PMT. The timing information of photons hitting the PMT can be predicted based on the scintillation photon emission time profile generated in the simulation and the time of flight ($t_{i,j}^{\text{tof}}$) of photons from the photon emission position to the PMT, as described in the second item of Eq. (4). The $f_{\text{scint}}(t)$ in Eq. (4) represents the scintillation photon emission time spectrum obtained through simulation [31]. Each photoelectron needs to be convolved with the transit time distribution and the waveform of the SPE, as shown in Eq. (4). TT_i is the transit time, $TT\sigma_i$ is the sigma of the transit time distribution, and W_{SPE}^i is the SPE waveform of i -th PMT. The transit time distribution and the SPE waveform of each PMT can be obtained through calibration. By combining Eq. (3) and Eq. (4), the track compo-

nent waveform (W_i) of i -th PMT in shower muon events can be predicted.

$$W_i = \sum_j N_j \times f_{\text{scint}}(t - t_{i,j}^{\text{tof}}) \otimes \text{Gaus}(TT_i, TTS_i) \otimes W_{\text{SPE}}^i \quad (4)$$

As shown by the red curve in the Fig. 11(a), this is the expected track component waveform of a PMT based on the aforementioned process. The black curve represents the corresponding waveform generated after the detector and electronic simulation. The waveform shape expected from the analytical calculation is generally consistent with the simulated waveform. However, the expected waveform from the analytical calculation may be smoother than the simulated waveform due to the no-uniform energy deposition of the muon along its track and the complexity of photon propagation in the detector.

If assuming the shower is a point source for depositing energy, the expected waveform of shower component also can be derived based on the Eq. (3) and Eq. (4). In Fig. 11(b), the red curve represents the expected waveform of shower component for a PMT caused by a shower. The waveform of shower component can be calculated using the shower vertex and the shower energy. The green curve is the waveform of shower component after subtracting the waveform of track component, which has been described in the Fig. 8. The waveform shapes obtained by both method are basically consistent. This not only demonstrates the effectiveness of the analytical calculation based method for predicting waveform but also confirms that the multi-peak structure of the waveform is indeed caused by the muon shower. Moreover, it shows that subtracting the track component waveform to eliminate the muon's track contribution to the waveform of shower muon events is a feasible approach.

Other methods for obtaining non-shower waveforms are briefly outlined as follows, but further research is needed. For example, the parameter space of reconstructed tracks, such as direction and position, can be segmented into bins aligned with the reconstruction resolution. Muons falling within the same bin are considered to share the similar track. Then, based on the dE/dx values, the non-shower muon data corresponding to those sharing the similar track with the shower muon can be selected, allowing us to obtain the corresponding track component waveforms. The second method is using machine learning techniques, such as neural networks. They can be employed to learn from a large dataset of simulated samples to generate the expected waveform for a given shower muon track.

4 Reconstruction performance

In this section, the performance of the shower vertex reconstruction algorithm is evaluated using simulated samples. As

outlined in Section 3, accurate reconstruction of the shower vertex requires the prior subtraction of the waveform contribution originating from the muon track. Several subtraction strategies can be employed. To quantify the impact of track accuracy on the performance of the proposed algorithm, we first present results obtained by subtracting the track-component waveforms using the true muon track information from simulation (Section 4.1). We then assess the reconstruction performance when the tracks of showering muon events are smeared to emulate realistic reconstruction uncertainties (Section 4.2).

4.1 Reconstruction performance of single shower

Figure 12 illustrates the discrepancies between the true and reconstructed shower vertices. For the majority of single-shower events, the reconstructed vertices cluster within a narrow region and exhibit an approximately Gaussian distribution. However, a small subset of events exhibits substantial deviations from the true vertices, indicative of reconstruction failures. The criteria for reconstruction failure and the definition of reconstruction efficiency are detailed in Section 4.3. Gaussian fits were performed near the peak positions of the three spatial component distributions (X, Y, Z). The resulting Gaussian widths (σ) are 0.09 m (X), 0.09 m (Y), and 0.18 m (Z), significantly smaller than the corresponding Root Mean Square (RMS) values of the distributions (X: 1.03 m, Y: 0.83 m, Z: 1.84 m, vertex distance: 2.15 m). This discrepancy arises primarily from the large deviations of the reconstruction failures. To more accurately characterize the reconstruction performance for the majority of well-reconstructed single-shower events (highlighted by the shaded regions in Fig. 12), the resolution is defined hereafter as the central 68% percentile of each distribution. Accordingly, the resolutions for the X, Y, and Z components of the reconstructed shower vertex position for all single-shower events are 0.16 m, 0.15 m, and 0.26 m, respectively. Similarly, as shown in Fig. 12(d) for the distribution of the distance between the reconstructed and true shower vertices, the resolution defined by the 68% percentile is 0.49 m.

Figure 13 presents the dependence of single-shower vertex reconstruction performance on shower energy, specifically examining the resolution (Fig. 13(a)) and bias (Fig. 13(b)) of the reconstructed vertex position. The spatial resolutions (defined by the 68% percentile) of the X and Y components remain below 0.2 m, with negligible biases. Moreover, the reconstruction accuracy for the X and Y components shows no significant energy dependence. In comparison, the resolution for the Z component is less precise, though it stays within 0.35 m. The bias in the reconstructed Z position stabilizes at approximately -0.2 m ($Z_{\text{true}} - Z_{\text{rec}}$) for shower energies above 2 GeV. Detailed analysis attributes the compar-

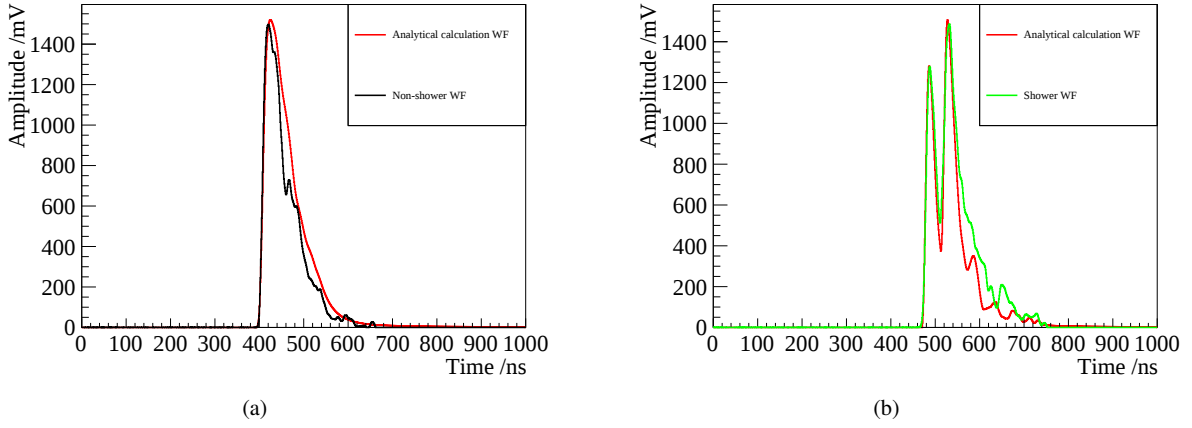


Fig. 11 (a) Waveform example for the track component of non-shower muon events. The predicted waveform from analytical calculation is similar to the waveform after electronic simulation. (b) The shower induced waveforms are generated from analytical calculation and after electronics simulation, respectively and they are consistent shape

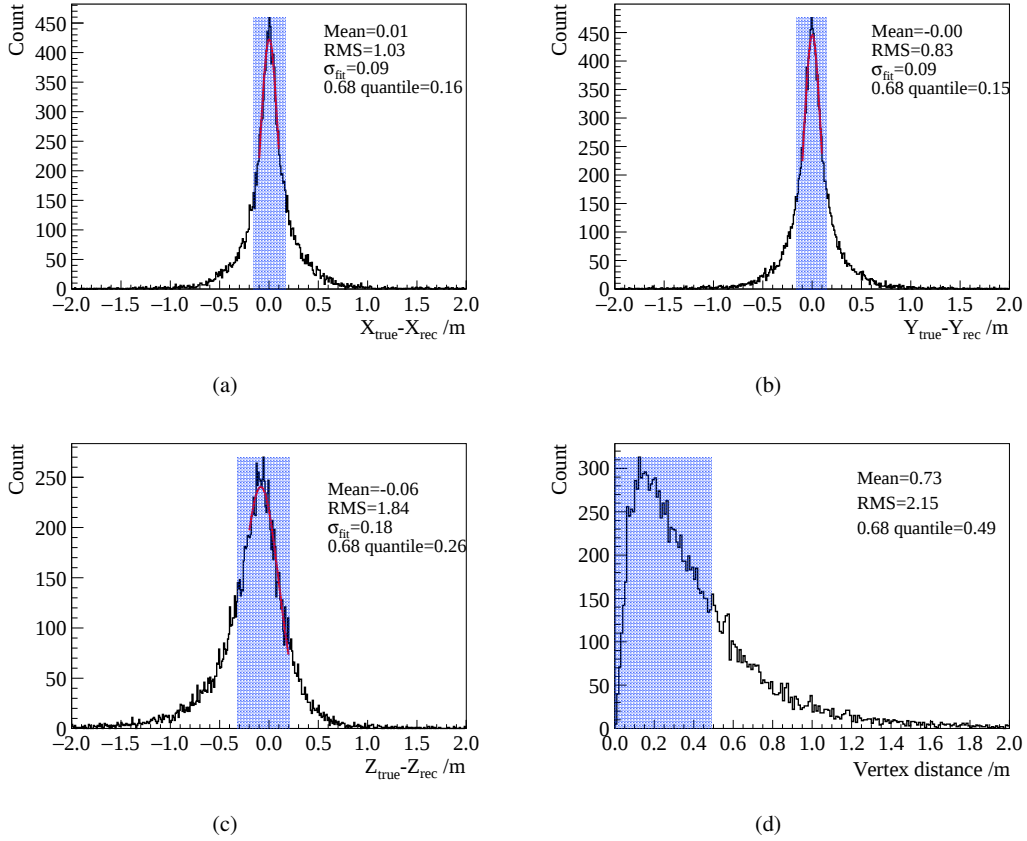
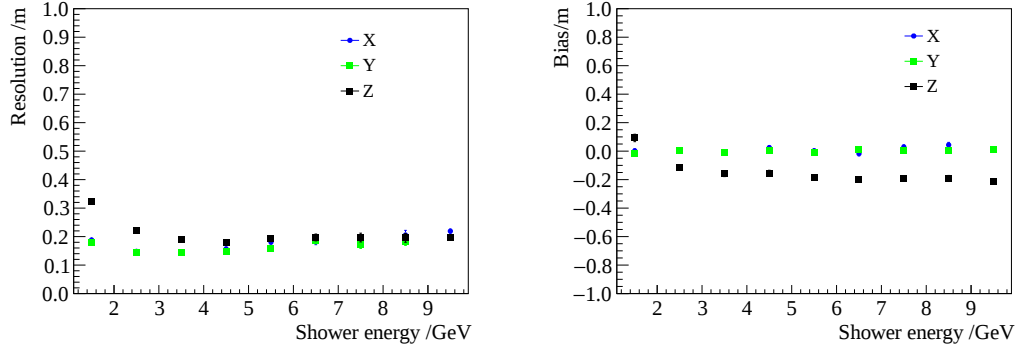


Fig. 12 The discrepancies between the true shower vertex and the reconstructed shower vertex. The red curves represent the fitting results of the local area around the peaks. The shaded regions correspond to the 68% percentile of each distribution

atively lower Z reconstruction performance primarily to the forward-peaking nature of muon energy deposition within the shower. This asymmetry in the shower spatial profile along the track direction degrades reconstruction accuracy. Additionally, the higher proportion of muons entering from

the detector top (corresponding to smaller zenith angles, θ) further contributed to this effect.

Furthermore, this effect can be understood by examining Fig. 14. The figure shows that the Z-direction resolution of the reconstructed shower vertex improves with increas-



(a) The relationship between reconstruction resolution and (b) The relationship between reconstruction bias and its shower energy

Fig. 13 The relationship between the reconstruction performance of a single shower event and its shower energy. In the figure, the shower vertex of each individual event in the simulated sample is within a range of 16 m

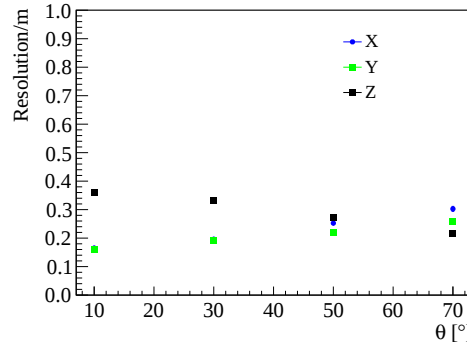
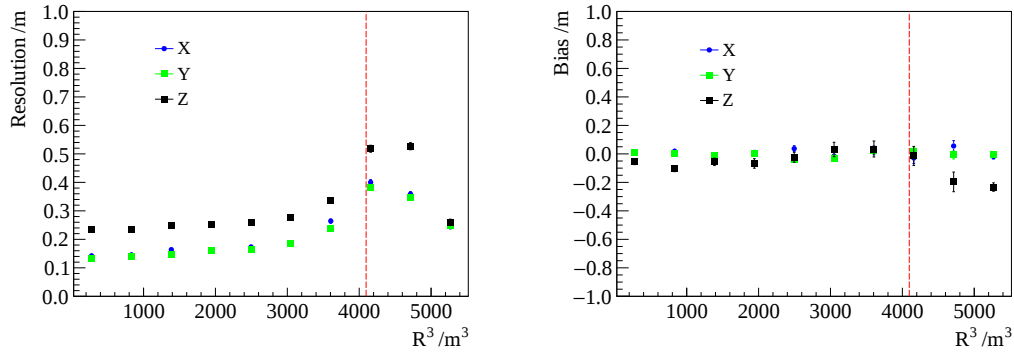


Fig. 14 The relationship between the resolution of the shower reconstruction vertex and the incident angle (θ) of the muon



(a) The relationship between reconstruction resolution and (b) The relationship between reconstruction bias and the vertex position

Fig. 15 The relationship between the reconstruction performance of single-shower vertex and its position. On each plot, red vertical dotted lines correspond to $R = 16 \text{ m}$

ing muon incident angle (θ). This improvement stems from the forward-peaking of shower energy deposition: as θ increases, the direction of peak energy deposition shifts increasingly towards the X and Y directions. Consequently, the resolutions along the X and Y axes degrade correspond-

ingly with higher θ . Notably, for $\theta > 50^\circ$, the X resolution deteriorates more rapidly than the Y resolution. Detailed investigations suggest this asymmetry is linked to the non-uniform ϕ -angle distribution of incident muons, primarily modulated by the mountain overburden. This non-uniformity

implies that at larger θ angles, a higher proportion of shower muons exhibit a more pronounced forward-peak component aligned with the X-direction. As a result, the vertex reconstruction performance (resolution) in X is slightly inferior to that in Y for $\theta > 50^\circ$.

Figure 15 illustrates the spatial dependence of single-shower vertex reconstruction performance. Reconstruction accuracy remains relatively stable for events occurring within 15 m of the detector center ($R < 15$ m; $R^3 < 3375$ m³), with Z-direction resolution maintained below ~ 0.3 m and X/Y resolutions below 0.2 m. Corresponding reconstruction biases for all spatial components remain under 0.1 m. Near the detector periphery ($R > 16$ m; $R^3 > 4096$ m³), performance degrades due to muon shower energy leakage and boundary-induced total reflection effects.

The blue dots in Fig. 16 represent the distribution of the distance between the reconstructed vertex and the true vertex in a single shower event. Figure 16(a) and Fig. 16(b) respectively show the relationship between this distance and both the shower energy and the vertex position. It can be seen that most of the distances can be controlled within 0.6 m, while they deteriorate to approximately 0.8 m only at positions near the edge of the detector ($R > 16$ m). Considering the distance relationship between the muon-induced isotopes and shower vertices (Fig. 6(b)) as described in Section 2, it can be found that the distance between the reconstructed vertex and true shower vertex is less than the distance between the muon-induced isotopes and true shower vertex. This indicates the shower vertex reconstruction algorithm developed in this paper is expected to be applicable for eliminating the muon-induced isotopes background. On the other hand, this algorithm exhibits potential for reconstructing double-shower events when selecting PMTs with two peaks to be used during the reconstruction process in the Fig. 9. The black triangles in Fig. 16 depict the reconstruction performance of the first shower vertex in a double-shower event. The overall reconstruction performance of the first shower vertex in a double shower event is around 1 m. Regarding the second shower vertex, it is not shown in the figure and its reconstruction performance is affected by interference from the first shower vertex, leading to instability. Therefore, further analysis and investigation are required in the future.

4.2 The impact of track reconstruction

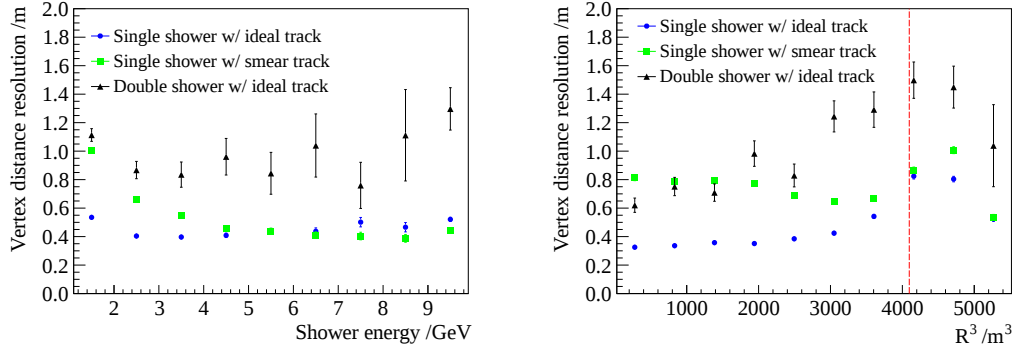
The reconstruction results in Section 4.1 assume access to the true muon track when subtracting the track-induced waveform. In real experiments, however, the reconstructed track for a shower muon event can deviate from the true trajectory. We therefore assess the impact of track reconstruction uncertainties on shower vertex reconstruction. Following previous studies [6, 10–12], we adopt a simplified model

in which the entry and exit points of the muon track in a single event are independently smeared with a Gaussian of standard deviation $\sigma = 20$ cm. The green squares in Fig. 16 show the results obtained by subtracting the waveform contributions of the smeared track, where the track-component waveforms are computed analytically using Eq. (3) and Eq. (4). The results indicate that, even with this level of track smearing, the distance between the reconstructed and true shower vertices remains within 1 m.

4.3 Reconstruction efficiency

As noted in Section 4.1, the RMS of the reconstruction residuals exceeds the Gaussian width (σ) from the fit (Fig. 12), indicating the presence of non-Gaussian tails caused by a small subset of events with large deviations from the true shower vertex. These outliers are attributable to a minor fraction of reconstruction failures. The overall vertex resolution shown in Fig. 16 remains below 1 m. To establish a clear performance benchmark, we define a successful reconstruction as one for which the distance between the reconstructed and true shower vertices is less than 3m (approximately three times the resolution); events outside this range are classified as failures. Using this criterion, Fig. 17 presents the reconstruction efficiency as a function of shower energy. The blue dots and green squares correspond to efficiencies obtained when subtracting waveform contributions computed from the true track and from a smeared track, respectively. The two sets of points largely overlap, and the efficiency approaches 100% for shower energies above 3 GeV. In particular, for the simulated sample with an initial muon energy of 100 GeV, the single-shower reconstruction efficiency reaches 96.7%. The black triangles in Fig. 17 show the efficiency for reconstructing the first vertex in double-shower events. As expected from the more challenging event topology, this efficiency is lower than for single-shower vertices. Nevertheless, within the 5–9 GeV range, the efficiency for the first vertex in double-shower events still approaches 100%.

In experimental analyses, the χ^2/NDF value serves as a helpful indicator for assessing whether the reconstruction process has been successful. Using truth information, we partitioned simulated events into two classes: those with a distance between the reconstructed and true vertices greater than 3 m (failures) and those within 3 m (successes). We find that the χ^2/NDF distribution exhibits clear separation: the majority of failures have $\chi^2/\text{NDF} > 10$, whereas only a small fraction of successful reconstructions exceed this value. This motivates adopting $\chi^2/\text{NDF} > 10$ as a quality cut to flag unreliable reconstructions in data. Future work could further improve the reconstruction performance by refining both the shower energy-deposition and optical models.



(a) The relationship between the distance from the reconstructed vertex to the true vertex at different shower energies. (b) The relationship between the distance from the reconstructed vertex to the true vertex at different positions.

Fig. 16 Reconstruction performance of the shower vertex in several scenarios. On the right plot, red vertical dotted lines correspond to $R = 16$ m

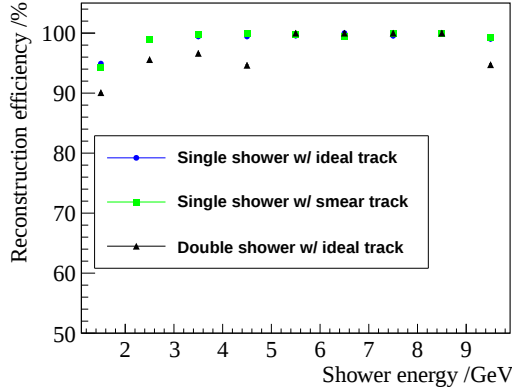


Fig. 17 Distribution of reconstruct efficiency vs shower energy

5 Discussion and conclusion

In most underground rare-event experiments, muon-induced spallation products constitute a major source of background. We have developed a method for reconstructing muon-induced shower vertices in 20 kiloton-scale LS detectors. The waveform-based algorithm first subtracts the contribution from the through-going muon track; the residual waveforms, which are primarily attributable to the shower component, are then used for vertex reconstruction. The shower vertex is defined as the energy-deposition-weighted centroid, and simulations indicate that approximately 84% of muon-induced isotopes are produced within 3 m of this point. Given that shower muons ($\sim 19.8\%$ of all muons) account for $\sim 88.2\%$ of isotope production, this method enables localized spherical vetoes that suppress the isotope background while preserving signal acceptance and maintaining high physics sensitivity.

To further improve the signal-to-background ratio for isotope identification, an optimized veto strategy has been developed. This strategy utilizes the temporal and spatial

correlation to the parent muon, including the muon track information and energy deposition profile. Since decay signals from isotopes can mimic genuine physics signals, an effective veto scheme is essential. Inspired by Ref. [38, 39], where veto time windows are varied based on the distance from the muon track, we propose applying a similar distance-dependent veto time to the reconstructed shower vertex. For the spherical veto centered on the shower vertex, increasing its radius from 3 m to 5 m is shown to enhance the fraction of vetoed isotopes from 84% to 92.5%. The optimal veto radius, however, must be determined in conjunction with the veto time window to balance background rejection against live time. Furthermore, a hybrid veto strategy can be implemented by combining a cylindrical veto volume along the muon track, a spherical veto centered on the shower vertex, and additional spherical vetoes centered on neutron capture locations. The precise veto efficiency and overall performance of this combined approach will require further optimization and study in future work.

For single-shower muon events, the algorithm attains spatial resolutions of 0.16 m (X), 0.15 m (Y), and 0.26 m (Z), with a reconstruction efficiency exceeding 96%. Performance is stable for shower energies above 2 GeV within the central detector region ($R < 16$ m), with modest degradation near the detector boundary. Importantly, when muon-track reconstruction uncertainties are included, the vertex deviation remains within 1 m. The algorithm also shows promise for resolving double-shower vertices.

The methodology developed in this work is readily adaptable to other large-scale, low-background experiments, providing a powerful tool for neutrino and dark matter searches.

Acknowledgements We thank the JUNO reconstruction and simulation working group for many helpful discussions. This work was supported by National Key R&D Program of China No. 2023YFA1606103,

National Natural Science Foundation of China No. 12494574, and the Program of Bagui Scholars Program (XF).

References

1. S. Abe, S. Enomoto, K. Furuno *et al.*, Production of Radioactive Isotopes through Cosmic Muon Spallation in KamLAND, *Phys. Rev. C* **81** (2010), 025807. <https://doi.org/10.1103/PhysRevC.81.025807>
2. G. Bellini, J. Benziger, D. Bick *et al.*, Cosmogenic Backgrounds in Borexino at 3800 m water-equivalent depth, *JCAP* **08** (2013), 049. <https://doi.org/10.1088/1475-7516/2013/08/049>
3. F.P. An, A.B. Balantekin, H.R. Band *et al.*, The muon system of the Daya Bay Reactor antineutrino experiment, *Nucl. Instrum. Meth. A* **773** (2015), 8-20. <https://doi.org/10.1016/j.nima.2014.09.070>
4. Y. Abe, J.C. dos Anjos, J.C. Barriere *et al.*, Precision Muon Reconstruction in Double Chooz, *Nucl. Instrum. Meth. A* **764** (2014), 330-339. <https://doi.org/10.1016/j.nima.2014.07.058>
5. R.W. Hackenburg, Muon reconstruction in the Daya Bay water pools, *Nucl. Instrum. Meth. A* **872** (2017), 52-63. <https://doi.org/10.1016/j.nima.2017.08.002>
6. C.F. Yang, Y.B. Huang, J.L. Xu *et al.*, Reconstruction of a muon bundle in the JUNO central detector, *Nucl. Sci. Tech.* **33** (2022) no.5, 59. <https://doi.org/10.1007/s41365-022-01049-3>
7. M. Grassi, J. Evslin, E. Ciuffoli *et al.*, Showering Cosmogenic Muons in A Large Liquid Scintillator, *JHEP* **09** (2014), 049. [https://doi.org/10.1007/JHEP09\(2014\)049](https://doi.org/10.1007/JHEP09(2014)049)
8. A. Abusleme, T. Adam, S. Ahmad *et al.*, JUNO Physics and Detector. *Prog. Part. Nucl. Phys.* **123**, 103927 (2022). <https://doi.org/10.1016/j.ppnp.2021.103927>
9. Abusleme, A., Adam, T., Ahmad, S. *et al.*, Mass testing and characterization of 20-inch PMTs for JUNO. *Eur. Phys. J. C* **82**, 1168 (2022). <https://doi.org/10.1140/epjc/s10052-022-11002-8>
10. C. Genster, M. Schever, L. Ludhova *et al.*, Muon reconstruction with a geometrical model in JUNO. *JINST* **13**, no.03, T03003 (2018). <https://doi.org/10.1088/1748-0221/13/03/T03003>
11. K. Zhang, M. He, W.D. Li *et al.*, Muon Tracking with the fastest light in the JUNO Central Detector. *Radiation Detection Technology and Methods* **2.1**(2018). <https://doi.org/10.1007/s41605-018-0040-8>
12. Y. Liu, W.D. Li, T. Lin *et al.*, Muon reconstruction with a convolutional neural network in the JUNO detector. *Radiat Detect Technol Methods* **5**, 364–372 (2021). <https://doi.org/10.1007/s41605-021-00259-4>
13. S.W. Li and J.F. Beacom, First calculation of cosmic-ray muon spallation backgrounds for MeV astrophysical neutrino signals in Super-Kamiokande, *Phys. Rev. C* **89** (2014), 045801. <https://doi.org/10.1103/PhysRevC.89.045801>
14. S.W. Li and J.F. Beacom, Spallation Backgrounds in Super-Kamiokande Are Made in Muon-Induced Showers,” *Phys. Rev. D* **91** (2015) no.10, 105005. <https://doi.org/10.1103/PhysRevD.91.105005>
15. S.W. Li and J.F. Beacom, Tagging Spallation Backgrounds with Showers in Water-Cherenkov Detectors, *Phys. Rev. D* **92** (2015) no.10, 105033. <https://doi.org/10.1103/PhysRevD.92.105033>
16. S. Locke, A. Coffani, K. Abe *et al.*, New methods and simulations for cosmogenic induced spallation removal in Super-Kamiokande-IV, *Phys. Rev. D* **110** (2024) no.3, 032003. <https://doi.org/10.1103/PhysRevD.110.032003>
17. S. Abe, S. Asami, M. Eizuka *et al.*, Measurement of cosmic-ray muon spallation products in a xenon-loaded liquid scintillator with KamLAND, *Phys. Rev. C* **107** (2023) no.5, 054612. <https://doi.org/10.1103/PhysRevC.107.054612>
18. B. Abi, R. Acciarri, Mario A. Acero *et al.* (DUNE Collaboration), Deep Underground Neutrino Experiment (DUNE), Far Detector Technical Design Report, Volume III: DUNE Far Detector Technical Coordination, *JINST* **15** (2020) no.08, T08009. <https://doi.org/10.1088/1748-0221/15/08/T08009>
19. S. Agostinelli, J. Allison, K. Amako *et al.* [GEANT4], GEANT4—a simulation toolkit. *Nucl. Instrum. Meth. A* **506**, 250-303 (2003). [https://doi.org/10.1016/S0168-9002\(03\)01368-8](https://doi.org/10.1016/S0168-9002(03)01368-8)
20. J. Allison, K. Amako, J. Apostolakis *et al.*, Geant4 developments and applications. *IEEE Trans. Nucl. Sci.* **53**, 270 (2006). <https://doi.org/10.1109/TNS.2006.869826>
21. J. Allison, J. Apostolakis, S.B. Lee *et al.*, Recent developments in Geant4. *Nucl. Instrum. Meth. A* **835**, 186-225 (2016). <https://doi.org/10.1016/j.nima.2016.06.125>
22. Lin, T., Hu, Y., Yu, M. *et al.*, Simulation software of the JUNO experiment. *Eur. Phys. J. C* **83**, 382 (2023). <https://doi.org/10.1140/epjc/s10052-023-11514-x>
23. P. Antonioli, C. Ghatti, E.V. Korolkova, V.A. Kudryavtsev, G. Sartorelli, A three-dimensional code for muon propagation through the rock: MUSIC. *Astropart. Phys.* **7**, no.4, 357-368 (1997). [https://doi.org/10.1016/S0927-6505\(97\)00035-2](https://doi.org/10.1016/S0927-6505(97)00035-2)
24. S.Y. Zhang, Y.B. Huang, M. He *et al.*, Sub-GeV events energy reconstruction with 3-inch PMTs in JUNO, *Nucl. Sci. Tech.* **36** (2025) no.5, 84. <https://doi.org/10.1007/s41365-025-01678-4>
25. R. Brun and F. Rademakers, ROOT: An object oriented data analysis framework. *Nucl. Instrum. Meth. A* **389**, 81-86 (1997). [https://doi.org/10.1016/S0168-9002\(97\)00048-X](https://doi.org/10.1016/S0168-9002(97)00048-X)
26. CERN ROOT. An open-source data analysis framework, <https://root.cern.ch/root/html526/TSpectrum2.html>;

- 2022 [accessed 15 March 2022].
27. X. Zhou, Q. Liu, M. Wurm *et al.*, Rayleigh scattering of linear alkylbenzene in large liquid scintillator detectors. *Rev. Sci. Instrum.* **86**, no.7, 073310 (2015). <https://doi.org/10.1063/1.4927458>
 28. L. Gao, B.X. Yu, Y.Y. Ding *et al.*, Attenuation length measurements of a liquid scintillator with LabVIEW and reliability evaluation of the device. *Chin. Phys. C* **37**, 076001 (2013). <https://doi.org/10.1088/1674-1137/37/7/076001>
 29. R. Zhang, D.W. Cao, C.W. Loh *et al.*, Using monochromatic light to measure attenuation length of liquid scintillator solvent LAB. *Nucl. Sci. Tech.* **30**, no.2, 30 (2019). <https://doi.org/10.1007/s41365-019-0542-1>
 30. M. Wurm, F.von Feilitzsch, M. Goeger-Neff *et al.*, Optical Scattering Lengths in Large Liquid-Scintillator Neutrino Detectors. *Rev. Sci. Instrum.* **81**, 053301 (2010). <https://doi.org/10.1063/1.3397322>
 31. Y. Zhang, Z.Y. Yu, X.Y. Li *et al.*, A complete optical model for liquid-scintillator detectors. *Nucl. Instrum. Meth. A* **967**, 163860 (2020). <https://doi.org/10.1016/j.nima.2020.163860>
 32. X.F. Ding, L.J. Wen, X. Zhou *et al.*, Measurement of the fluorescence quantum yield of bis-MSB. *Chin. Phys. C* **39**, no.12, 126001 (2015). <https://doi.org/10.1088/1674-1137/39/12/126001>
 33. C. Buck, B. Gramlich and S. Wagner, Light propagation and fluorescence quantum yields in liquid scintillators. *JINST* **10**, no.09, P09007 (2015). <https://doi.org/10.1088/1748-0221/10/09/P09007>
 34. H.M. O’Keeffe, E. O’Sullivan and M.C. Chen, Scintillation decay time and pulse shape discrimination in oxygenated and deoxygenated solutions of linear alkylbenzene for the SNO+ experiment. *Nucl. Instrum. Meth. A* **640**, 119-122 (2011). <https://doi.org/10.1016/j.nima.2011.03.027>
 35. H.S. Zhang, M. Beretta, S. Cialdi *et al.*, Refractive index in the JUNO liquid scintillator. *Nucl. Instrum. Methods Phys. Res. A* **1068**, 169730 (2024). <https://doi.org/10.1016/j.nima.2024.169730>
 36. M. Beretta, F. Houria, F. Ferraro *et al.*, Fluorescence emission of the JUNO liquid scintillator. *J. Instrum.* **20**, no.05, P05009 (2025). <https://doi.org/10.1088/1748-0221/20/05/P05009>
 37. G.H. Huang, W. Jiang, L.J. Wen *et al.*, Data-driven simultaneous vertex and energy reconstruction for large liquid scintillator detectors. *Nucl. Sci. Tech.* **34**, no.6, 83 (2023). <https://doi.org/10.1007/s41365-023-01240-0>
 38. A. Abusleme, T. Adam, S. Ahmad *et al.*, Feasibility and physics potential of detecting 8B solar neutrinos at JUNO. *Chinese Phys. C* **45** 023004 (2021). <https://doi.org/10.1088/1674-1137/abd92a>
 39. Abusleme, A., Adam, T. *et al.*, JUNO sensitivity to invisible decay modes of neutrons. *Eur. Phys. J. C* **85**, 5 (2025). <https://doi.org/10.1140/epjc/s10052-024-13638-0>